Wiki Authoring and Semantics of Mathematical Document Structure

The ITP 2011 Workshop on Mathematical Wikis (MathWikis-2011)

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August 27th, 2011

Overview

- Introduction of Matherial
 - Wiki engine of Matherial
 - Authoring Documents on Matherial
 - Publishing Documents on the Web
- Describing structure of a Document
 - Documents consisting of several Resources
 - Revisiting OAI-ORE to describe an Aggregation
 - RDF schema

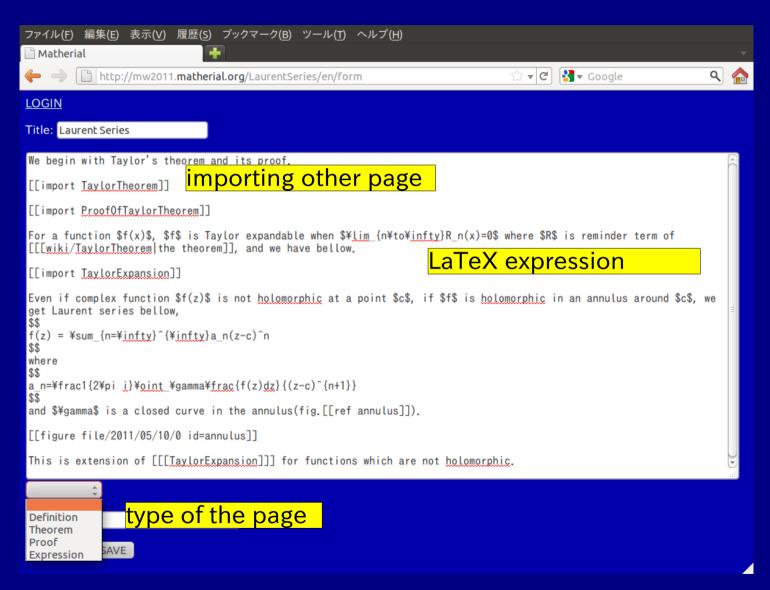
Document Authoring with *Matherial*

Matherial

is a Contents Management System for Mathematics

Features of Matherial

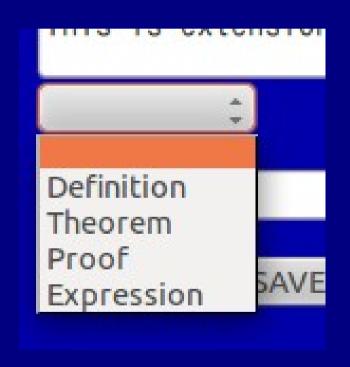
- Authoring a Document
 - on Wiki Engine
 - which aggregates several Documents
- File Storage
- Blog
- Discussion Forum



- Mathematical Expressions
 - Input
 - LaTeX notation
 - Output
 - "Presentation" MathML

```
e when $\frac{1}{\lim_{n\to\frac{1}{\infty}}R_n(x)=0\times where $R\times is noted to be the content of the con
```

- Type of Page
 - General Page
 - Mathematical
 - -Definition
 - -Theorem
 - -Proof
 - -Expression



- Importing (or transcluding)
 - Mathematical Resources
 - Sharing Def., Th., Pr. and Expression
 - Image files
 - Embedding with their description

```
[[import TaylorTheorem]]
[[import ProofOfTaylorTheorem]]
```

```
[[figure file/2011/05/10/0 id=annulus]]
```



Laurent Series

We begin with Taylor's theorem and its proof.

Taylor Theorem

Let f be a function which is defined on the interval (a, b) and suppose the nth derivative $f^{(n)}$ exists on (a, b). Then for all x and x_0 in (a, b),

$$R_n(x) = \frac{f^{(n)}(y)}{n!} (x - x_0)^n$$

with y strictly between x and x_0 (y depends on the choice of x). $R_n(x)$ is the nth remainder of the Taylor series for f(x).

(Original text is http://planetmath.org/encyclopedia/TaylorsTheorem.html, retrieved at 2011.05.08)

Proof of Taylor Theorem

Let f(x), a < x < b be a real-valued, n-times differentiable function, and let $a < x_0 < b$ be a fixed base-point. We will show that for all $x \ne x_0$ in the domain of the function, there exists a ξ , strictly between x_0 and x such that

$$f(x) = \sum_{k=0}^{n-1} f^{(k)}(x_0) \frac{(x-x_0)^k}{k!} + f^{(n)}(\xi) \frac{(x-x_0)^n}{n!}.$$

Fix $x \neq x_0$ and let R be the remainder defined by

$$f(x) = \sum_{k=0}^{n-1} f^{(k)}(x_0) \frac{(x-x_0)^k}{k!} + R \frac{(x-x_0)^n}{n!}.$$

Next, define

For a function f(x), f is Taylor expandable when $\lim_{n\to\infty} R_n(x) = 0$ where R is reminder term of the theorem, and we have bellow.

Taylor Expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

Even if complex function f(z) is not holomorphic at a point c, if f is holomorphic in an annulus around c, we get Laurent series bellow,

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-c)^n$$

where

$$a_n = \frac{1}{2 \pi i} \oint \int_{\gamma} \frac{f(z) dz}{(z-c)^{n+1}}$$

and γ is a closed curve in the annulus(fig.annulus).



Annulus for Laurent Series

Annulus for Laurent Series is shown.

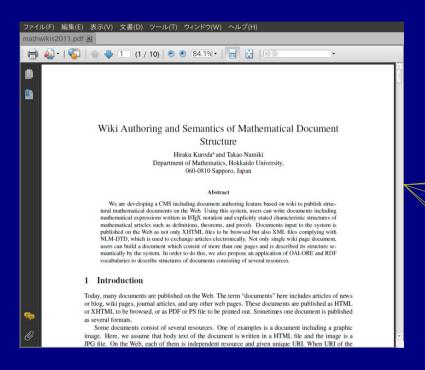
This is extension of <u>TaylorExpansion</u> for functions which are not holomorphic.

Two types of Documents

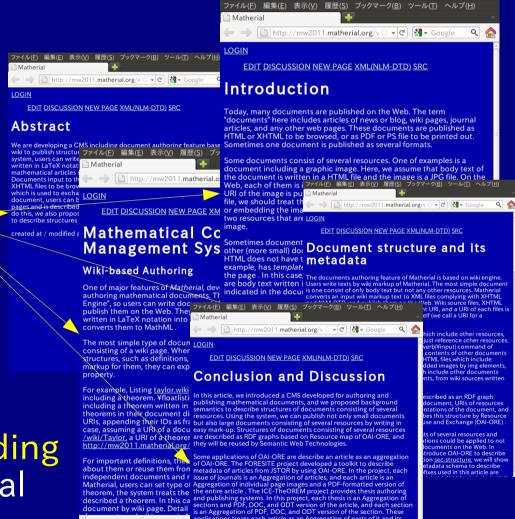
- Matherial covers two types of Documents
 - A Wiki page
 - An Aggregation of Wiki pages

A Wiki page as a Document

- has Body text
 - Wiki notation → HTML and NLM-DTD
- import other resources
 - other page
 - image
 - → parts of documents



- Publishing document including several sections on Matherial
 - split into sections
- write each section as a WikiPage



phications treats each article as an Aggregation of parts of it and its presentations. On the other hand, in this article, we describe a ocument as an Aggregation of parts of it, and we use another property r relationships between a Document and its Representations.

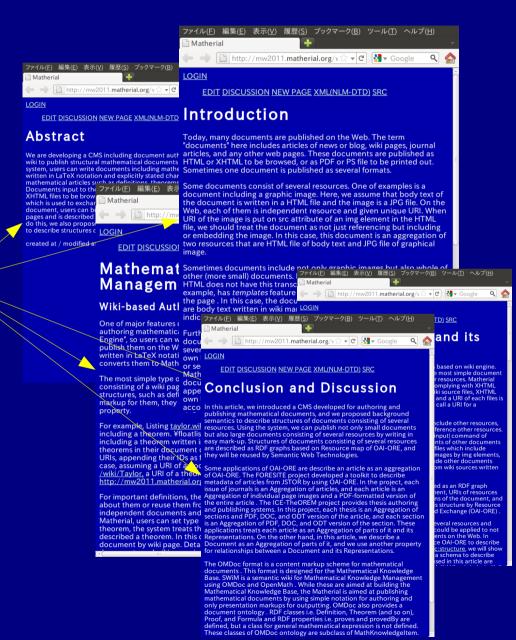
The OMDoc format is a content markup scheme for mathematical documents. This format is designed for the Mathematical Knowledge Base. SWiM is a semantic wiki for Mathematical Knowledge Managem

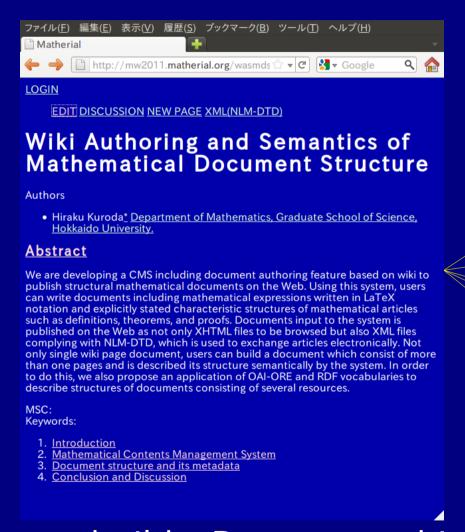
Base. SWIM is a semantic wint for Mathematical Knowledge Manageme using OMDoc and OpenMath. While these are aimed at building the Mathematical Knowledge Base, the Matherial is aimed at publishing mathematical documents by using simple notation for authoring and only presentation markups for outputting. OMDoc also provides a document ontology. RDF classes ie. Definition, Theorem (and so on), Proof, and Formula and RDF properties i.e. proves and provedBy are defined, but a class for nearest mathematical expression is not defined.

defined, but a class for general mathematical expression is not defined. These classes of OMDoc ontology are subclass of MathKnowledgeItem

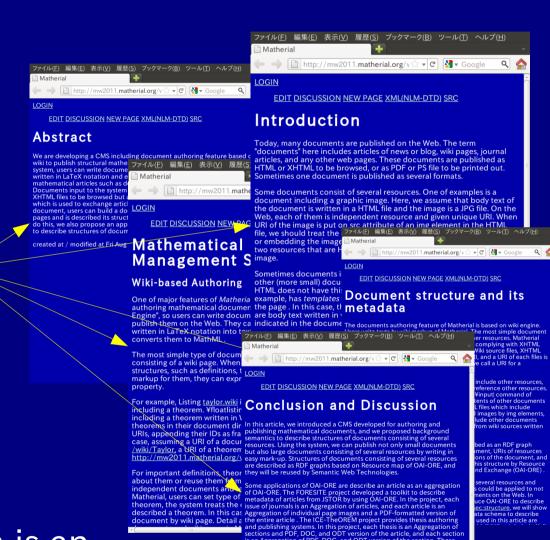


Enumerating Wiki pages to aggregate





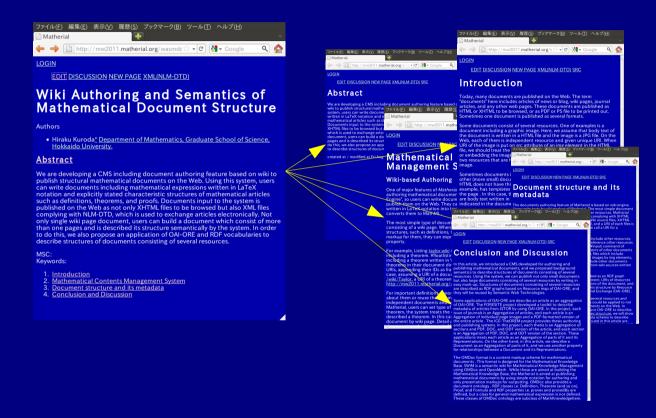
.. to build a Document which is an aggregation of Wiki Pages



documents. This format is designed for the Mathematical Knowledge Bases. SWIM is a semantic will for Mathematical Knowledge Manageme using DMDoc and OpenMath. While these are aimed at building the Mathematical Knowledge Base, the Matherial is aimed at publishing mathematical documents by using simple notation for authoring and only presentation markups for outputting. OMDoc also provides a document ontology. RDF classes i.e. Definition, Theorem (and so on), Proof, and Formula and RDF properties i.e. proves and provedBy are defined, but a class for general mathematical expression is not defined.

These classes of OMDoc ontology are subclass of MathKr

Authoring Mathematical Document

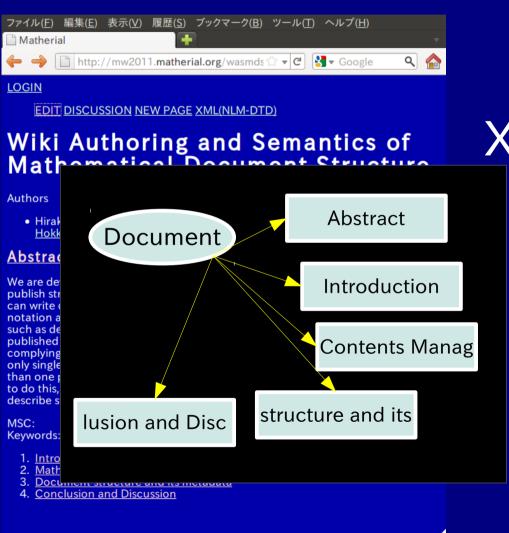


User can author full-text mathematical document using by easy Wiki notaion with structure of Mathematical Documents

Representations of a document in Matherial

- XHTML with MathML, RDFa for
 - browsed by human
 - describing metadata by RDFa
 - Structure between Resources
 - General Information
- NLM-DTD
 - using Archiving and Interchange Tag
 Set
 - XML will be transcluded from other systems

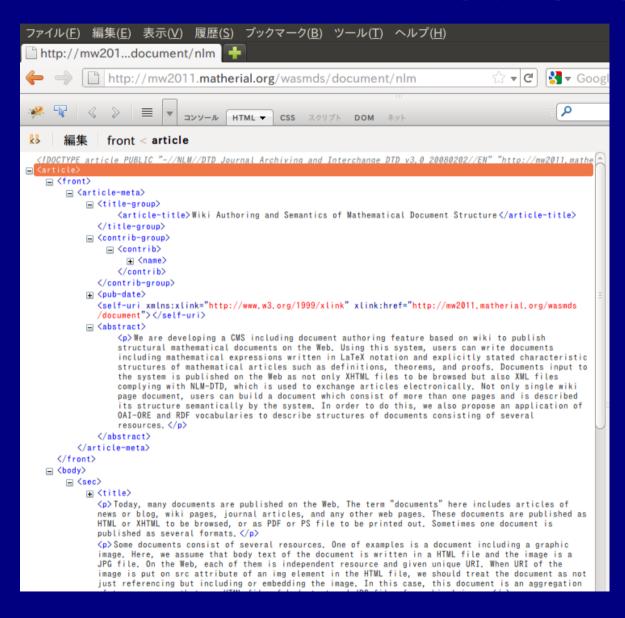
Representations of a document in Matherial



XHTML+MathML+RDFa

RDF-Graph is embedded by using RDFa

Representations of a document in Matherial



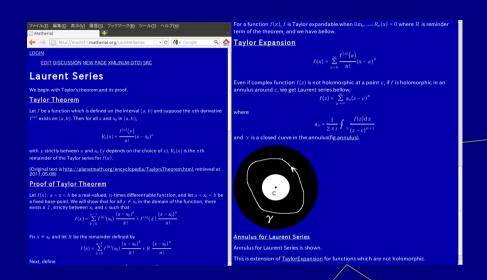
- NLM-DTD
 - Journal Archiving and Interchange Tag Suite
 - by National Library for Medicine
 - 3 Tag Set
 - Archiving and Interchange
 - Jounal Publishing
 - Article Authoring
- Matherial outputs Archiving and Interchange
 - including all text of the document
 - transcluded from other systems

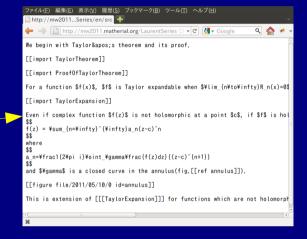
Describing Structure of a Document

Two types of Document

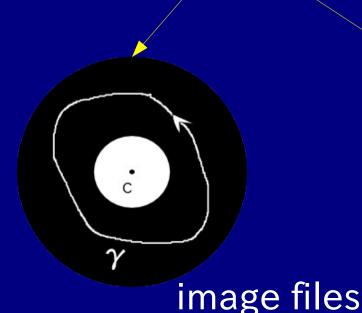
- A Wiki (or Web) page type of Document
- Aggregation of pages

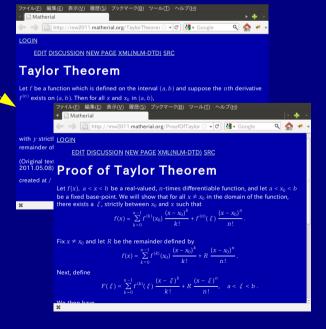
Wiki Page as Aggregation





source text



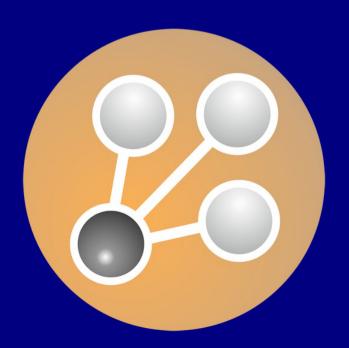


imported pages

Open Archives Initiative Object Reuse and Exchange

or

OAI-ORE (http://www.openarchives.org/ore/)



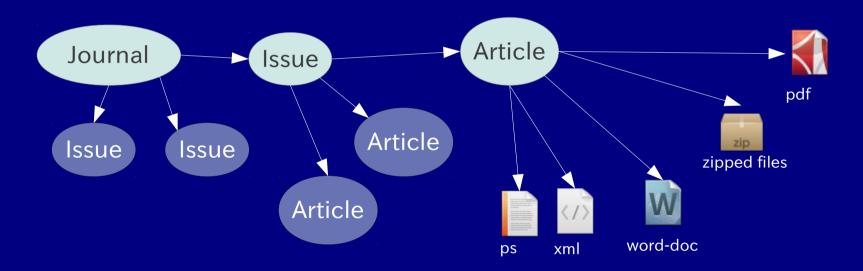
Aggregation of OAI-ORE

Aggregated Resource Rdfs:subPropertyOf Aggregated Resource Aggregated Resource

•OAI-ORE provides a mechanism for describing an Aggregation as RDF graph.

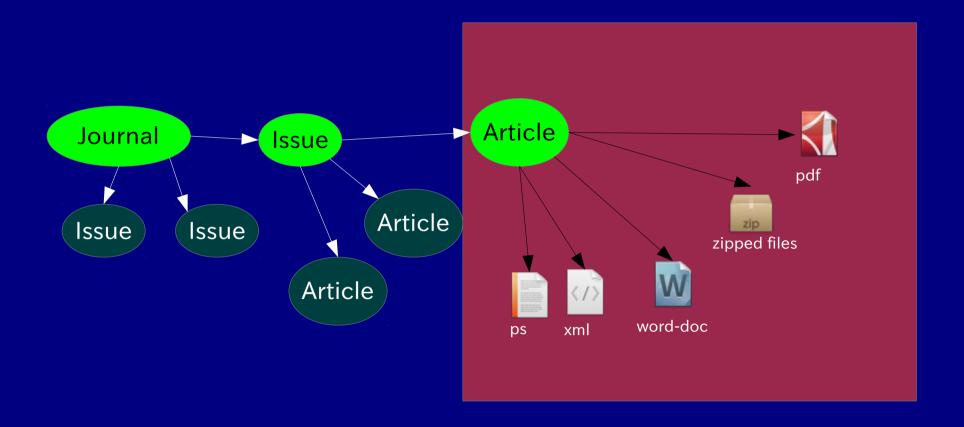
Structure of Journal, Issue, Article and Representations

 A journal article is a major example of OAI-ORE aggregation.



We think that a journal article is also an example of Document.

A Question



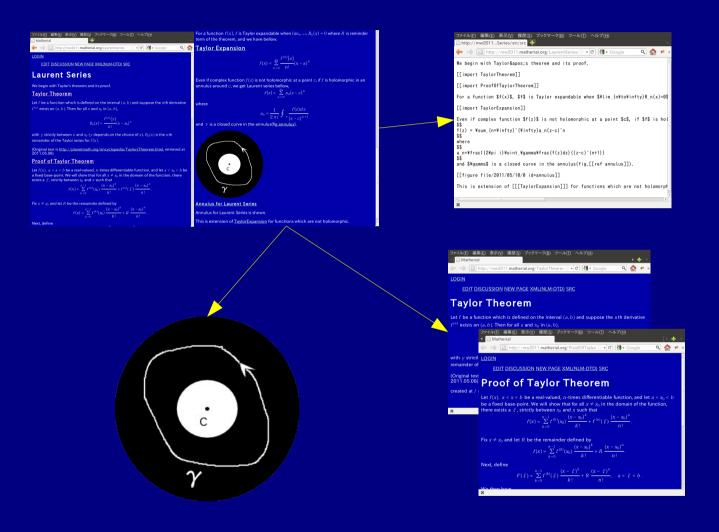
Is it appropriate to describe an article as an aggregation of its representations? (a PDF as a part of document?)

Document as an Aggregation

An article (or A Document) should be described as an Aggregation of constituent resources.

- Parts, Chapters, Sections, …
- Embedded images
- Content (body text) of article

Constituents of a Wiki Page

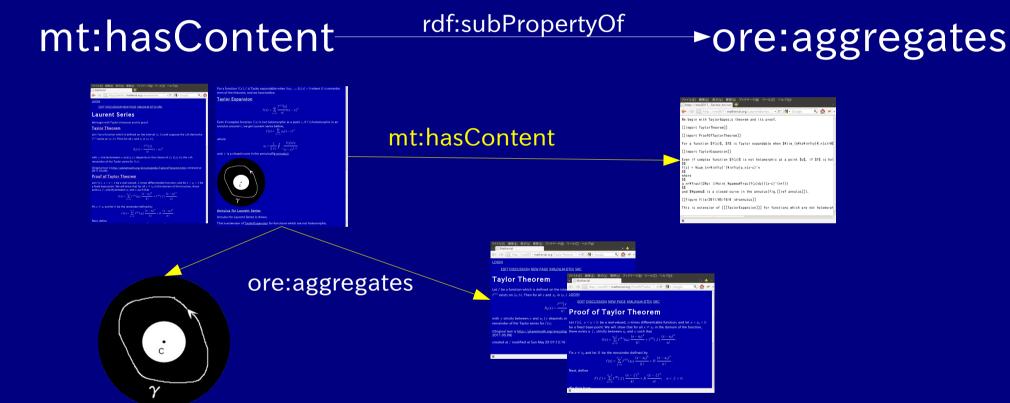


This Wiki Page is an Aggregation of wiki-source and imported Resources

Main Content of a Document

- The Wiki source file is a special member of the Document. This includes
 - Main content of Document
 - body text
 - Indication of embedding Resources
 - [[import TaylorTheorem]]
 - [[figure file/2011/5/10/0 id=annul]]

Main Content of a Document

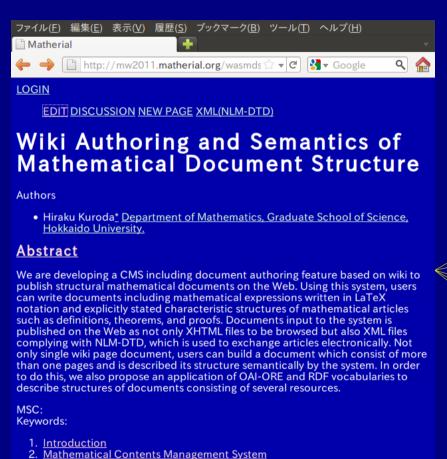


mt (http://www.matherial.org/terms/) is for our new terms of RDF.

Wiki page as Aggregation

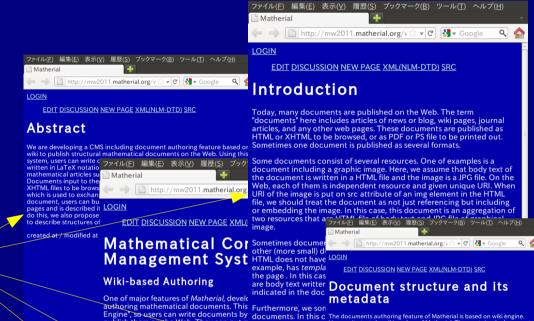
- A Document which has its own content is an Aggregation of
 - Document Content
 - Wiki-source, HTML, XML, Plain text, ···
 - Embedded (Transcluded)
 Resources
 - Image file, other Document and any objects

about an Aggregation of Documents



Document structure and its metadata

4. Conclusion and Discussion



Engine , Sousers can write documents of documents. In this coupling publish them on the Web. They can ファイル(E) 編集(E) 表示(Y) 履歴(S) ブックマーク(B) ツール(II) ヘルブ(H) written in LaTeX notation into texts converts them to MathML. The most simple type of document

consisting of a wiki page. When use LOGIN structures, such as definitions, the markup for them, they can express

EDIT DISCUSSION NEW PAGE XML(NLM-DTD) SRC For example, Listing taylor, wiki is a . Conclusion and Discussion

including a theorem. Finoaustingtic including a theorem written in Wiki lin this article, we introduced a CMS developed for authoring and theorems in their document direct I URIs, appending their IDs as fragm emantics to describe structures of documents consisting of several resources. Using the system, we can publish not only small documents with IT and the proposed background by the company of the control case, assuming a ON of a document but also targe documents consisting of several resources /wiki/Taylor, a URI of a theorem its easy mark-up. Structures of documents consisting of several resources http://mw2011.matherial.org/wiki, are described as RDF graphs based on Resource map of OAI-ORE, and they will be reused by Semantic Web Technologies.

d as an RDF graph

The will be reused by Semantic web Technologies.

Some applications of OAI-ORE are describe an article as an aggregation of OAI-ORE, the FORESITE project developed a toolkit to describe independent documents and referr metadata of articles from JSTOR by using OAI-ORE. In the project, each theorem, the system treats the doc Aggregation of individual page images and a PDF-formatted version of described a theorem. In this case, I, the entire article. The ICE-TheOREM project provides thesis authoring document by wiki page. Detail about the project provides the sistence of the article, and each section is an Aggregation of PDF, DOC, and ODT version of the article, and each section is an Aggregation of parts of it and its Representations. On the other hand, in this article, we describe a Document as an Aggregation of parts of it, and we use another property

The OMDoc format is a content markup scheme for mathematical documents. This format is designed for the Mathematical Knowledge Base. SWIM is a semantic wilk for Mathematical Knowledge Managemei using OMDoc and OpenMath. While these are aimed at building the Mathematical Knowledge Base, the Matherial is aimed at publishing mathematical documents by using simple notation for authoring and only presentation markups for outputting. OMDoc also provides a document ontology. RDF classes i.e. Definition, Theorem (and so on), Proof, and Formula and RDF properties i.e. proves and provedBy are defined, but a class for general mathematical expression is not defined these classes of OMDoc notations are such class of Math Knowledgeltem. These classes of OMDoc ontology are subclass of MathKno

This Document is an Aggregation of

- http://.../wasmds/Abstract
- http://.../wasmds/ConclusionAndDiscussion
- http://.../wasmds/Introduction
- http://.../wasmds/Matherial
- http://.../wasmds/StructureAndMetadata

(This list is sorted into alphabetical order)

What order do we read them?

Order of Members

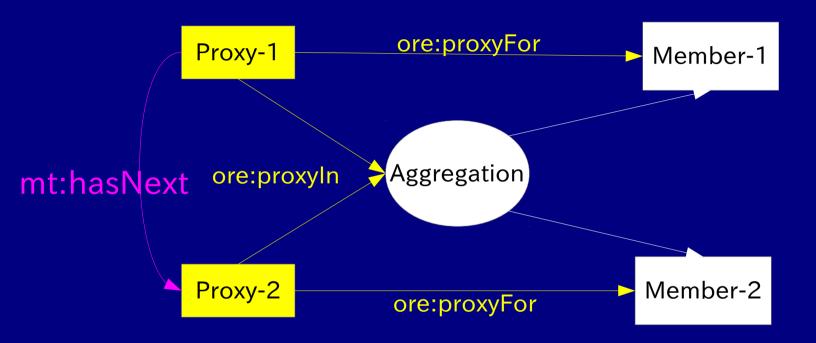
- We define a new term to describe order of members
- Order relationships should not be described between members directly.

Order of Members



We should not directly describe a property between members when the property is valid only in an Aggregation.

Proxy of ORE



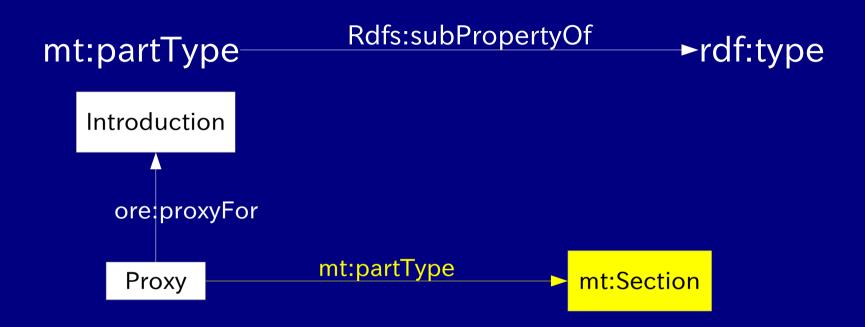
Proxy of ORE is provided to describe a property which is valid only in an aggregation.

"Member-1 hasNext Member-2 in the Aggregation."

about *hasNext*

- hasNext is used in documents of OAI-ORE.
 - x:hasNext
 - That is only for example and not defined formally.
- We need to describe order of members of Document which doesn't have its own content.
 - Therefore we define mt:hasNext.

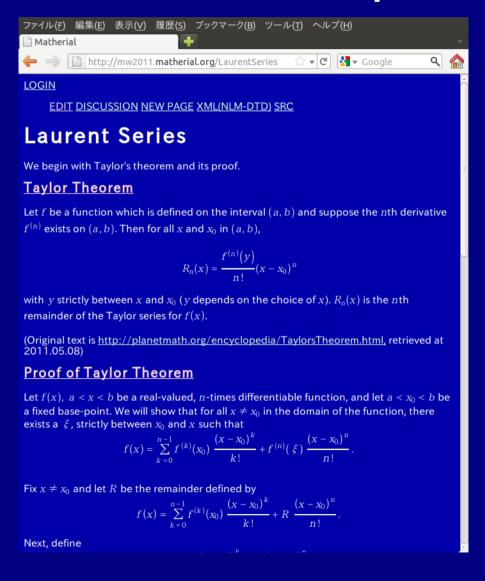
Type of a Member



- The term mt:partType is a property to describe a type or a role of members in an Aggregation
- mt:part takes sub classes of mt:PartType for object of a triple.
 - mt:Part, mt:Chapter and so on

about A Document and its Representatios

A Document and its Representations



This Document has three Representation.

Wiki-SRC

- Document Content (Member of Aggregation)
- and the simplest Representation of this Document

XHTML NLM-DTD

Aggregation of Wiki Page

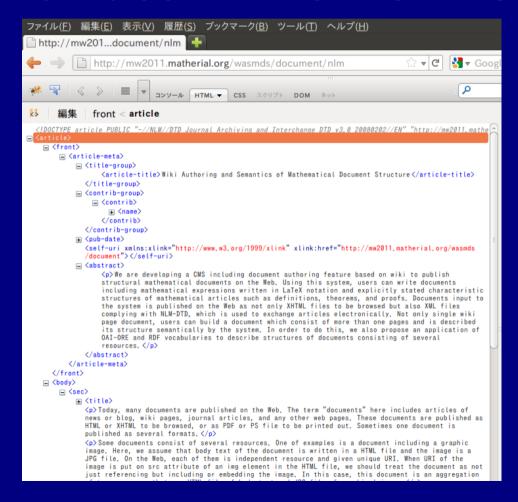


- This Document does not have its own Content
- Each member
 Document has their own Document
 Content.

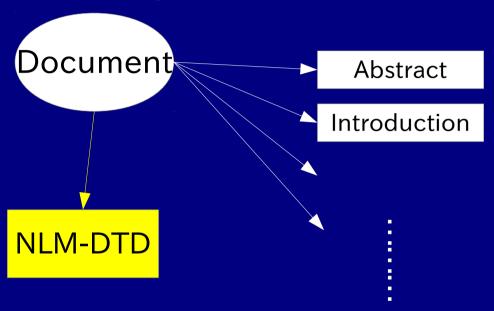
(This page is one of Representations, but not content of the Document)

Aggregation of Wiki Page

has NLM-DTD version of Representation, which includes ALL text of the Document.



A Document and its Representations

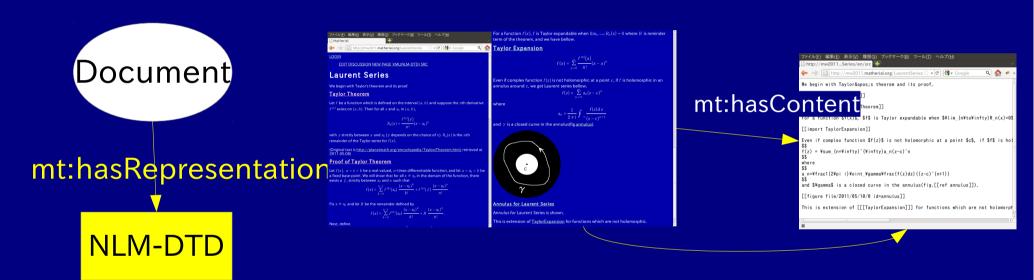


The document is an aggregation of 5 sections, not aggregation of Representation(s).

We need different way to describe between Document and Representations.

Representations of a Document

mt:hasRepresentation Rdfs:subPropertyOf dcterms:relation

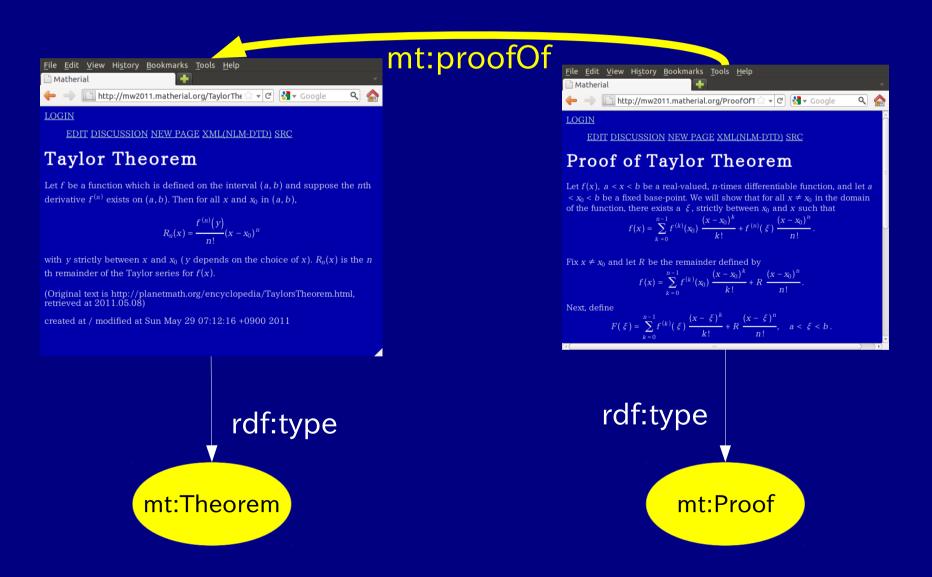


mt:hasRepresentation

- The term mt:hasRepresentation is a property to describe relationships between a Document and its Representations.
- A Document-Content member of the Document can be a Representation of the Document.

about Document Types for Mathematics

Document Types for Mathematics



Document Types for Mathematics

- Mathematical Resource Type mt:MathematicalObject
 - mt:Expression, mt:Definition, mt:Theorem and mt:Proof
- A Relationship between a theorem and its proof

mt:proofOf Rdfs:subPropertyOf

dcterms:relation

Overlapping with OMDoc Ontology

- OMDoc Ontology provides similar vocabulary
 - RDF classes i.e. Definition, Theorem, Proof and so on.
 - Each class is sub class of *MathKnowledgeItem*
 - Any mathematical knowledge item that can be expressed in OMDoc

(http://omdoc.org/ontology)

 If a document is a MathKnowledgeItem, the document may be expected to be represented in OMDoc.

Overlapping with OMDoc Ontology

- Matherial focuses to publish Documents using by presentation markups and not support OMDoc.
 - Mathematical Documents on Matherial is not represented in OMDoc
- This is because we define similar terms in our namespace.

- Matherial, CMS for mathematical documents
 - Authoring two types of document that consist of several Resources.
 - A Wiki Page type of Document
 - An Aggregation of Wiki Pages.
 - Mathematical expressions in LaTeX notation
 - Type of a Mathematical document
 - Publishing full-text documents into several Representations with their metadata.
 - XHTML + MathML and RDFa
 - NLM-DTD

- Semantics of Mathematical Document Structure
 - We defined new terms to describe Structure of Documents.
 - mt:hasContent to describe that an aggregating document has its own content resource.
 - mt:hasNext and mt:partType to describe properties of member-document of the aggregating document.
 - mt:hasRepresentation to describe relationships between the Document and its Representations

- Semantics of Mathematical Document Structure
 - We defined terms
 - mt:Defintion, mt:Theorem, mt:Proof, mt:proofOf and so on to describe type and relationship of mathematical documents which are represented in presentation markup, but not content markup.