Wiki Authoring and Semantics of Mathematical Document Structure

The ITP 2011 Workshop on Mathematical Wikis (MathWikis-2011)

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Overview

- Introduction of Matherial
  - Wiki engine of Matherial
  - Authoring Documents on Matherial
  - Publishing Documents on the Web
- Describing structure of a Document
  - Documents consisting of several Resources
  - Revisiting OAI-ORE to describe an Aggregation
  - RDF schema

Document Authoring with *Matherial*

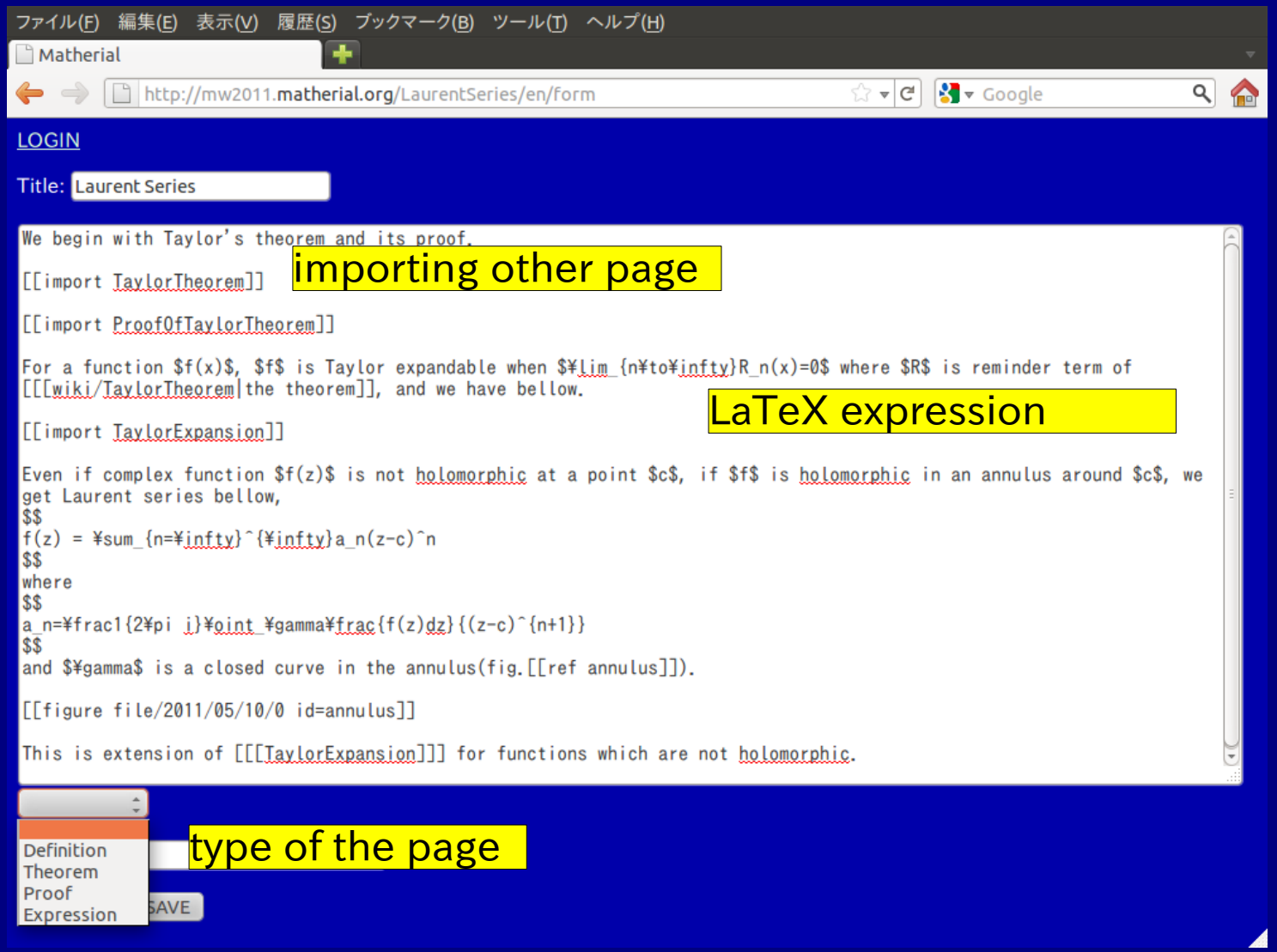
Matherial

is a Contents Management System for Mathematics

Features of Matherial

- Authoring a Document
  - on Wiki Engine
  - which aggregates several Documents
- File Storage
- Blog
- Discussion Forum

Wiki engine of Matherial



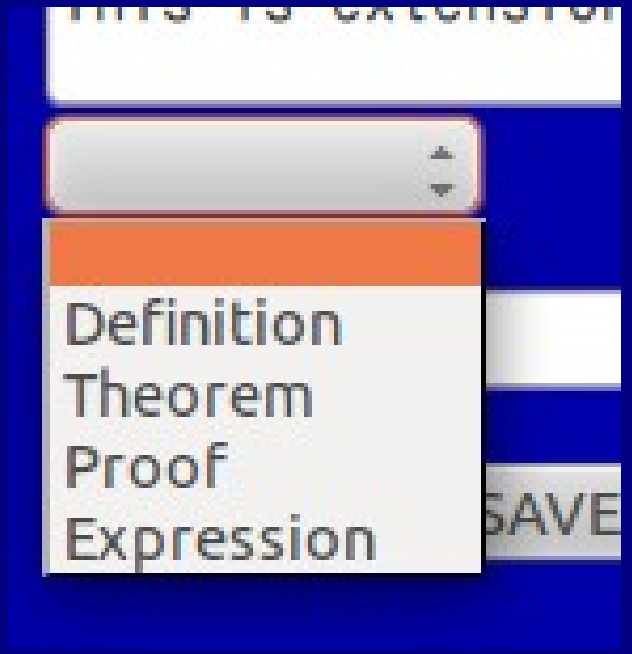
Wiki engine of Matherial

- Mathematical Expressions
  - Input
    - LaTeX notation
  - Output
    - “Presentation” MathML

```
e when $¥lim_{n¥to¥infty}R_n(x)=0$ where $R$ is
ve bellow.
```

Wiki engine of Matherial

- Type of Page
  - General Page
  - Mathematical
    - Definition
    - Theorem
    - Proof
    - Expression



Wiki engine of Matherial

- *Import*ing (or transcluding)
  - Mathematical Resources
    - Sharing Def., Th., Pr. and Expression
  - Image files
    - Embedding with their description

```
[[import TaylorTheorem]]

[[import ProofOfTaylorTheorem]]
```

```
[[figure file/2011/05/10/0 id=annulus]]
```

Wiki engine of Matherial

ファイル(F) 編集(E) 表示(V) 履歴(S) ブックマーク(B) ツール(T) ヘルプ(H)

Matherial

http://mw2011.matherial.org/LaurentSeries

Google

LOGIN

EDIT DISCUSSION NEW PAGE XML(NLM-DTD) SRC

Laurent Series

We begin with Taylor's theorem and its proof.

Taylor Theorem

Let f be a function which is defined on the interval (a, b) and suppose the nth derivative $f^{(n)}$ exists on (a, b). Then for all x and x_0 in (a, b),

$$R_n(x) = \frac{f^{(n)}(y)}{n!}(x - x_0)^n$$

with y strictly between x and x_0 (y depends on the choice of x). $R_n(x)$ is the nth remainder of the Taylor series for $f(x)$.

(Original text is http://planetmath.org/encyclopedia/TaylorsTheorem.html, retrieved at 2011.05.08)

Proof of Taylor Theorem

Let $f(x)$, $a < x < b$ be a real-valued, n-times differentiable function, and let $a < x_0 < b$ be a fixed base-point. We will show that for all $x \neq x_0$ in the domain of the function, there exists a ξ, strictly between x_0 and x such that

$$f(x) = \sum_{k=0}^{n-1} f^{(k)}(x_0)\frac{(x - x_0)^k}{k!} + f^{(n)}(\xi)\frac{(x - x_0)^n}{n!}.$$

Fix $x \neq x_0$ and let R be the remainder defined by

$$f(x) = \sum_{k=0}^{n-1} f^{(k)}(x_0)\frac{(x - x_0)^k}{k!} + R\frac{(x - x_0)^n}{n!}.$$

Next, define

For a function $f(x)$, f is Taylor expandable when $\lim_{n\to\infty} R_n(x) = 0$ where R is reminder term of the theorem, and we have bellow.

Taylor Expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x - a)^n$$

Even if complex function $f(z)$ is not holomorphic at a point c, if f is holomorphic in an annulus around c, we get Laurent series bellow,

$$f(z) = \sum_{n=\infty}^{\infty} a_n(z - c)^n$$

where

$$a_n = \frac{1}{2\pi i}\oint_\gamma \frac{f(z)dz}{(z - c)^{n+1}}$$

and γ is a closed curve in the annulus(fig.annulus).

Annulus for Laurent Series

Annulus for Laurent Series is shown.

This is extension of TaylorExpansion for functions which are not holomorphic.

Two types of Documents

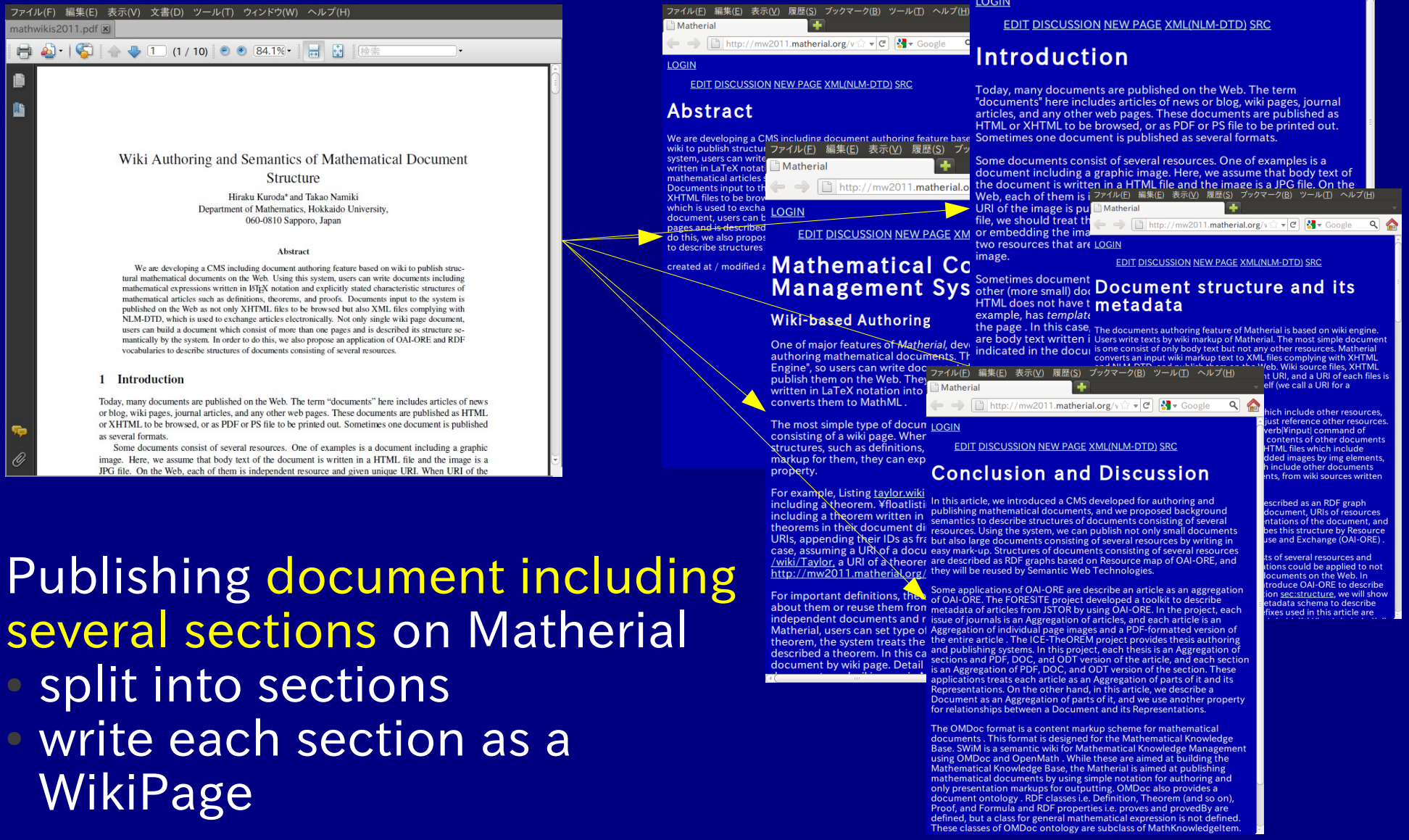
- Matherial covers two types of Documents
  - A Wiki page
  - An Aggregation of Wiki pages

A Wiki page as a Document

- has Body text
  - Wiki notation → HTML and NLM-DTD
- import other resources
  - other page
  - image

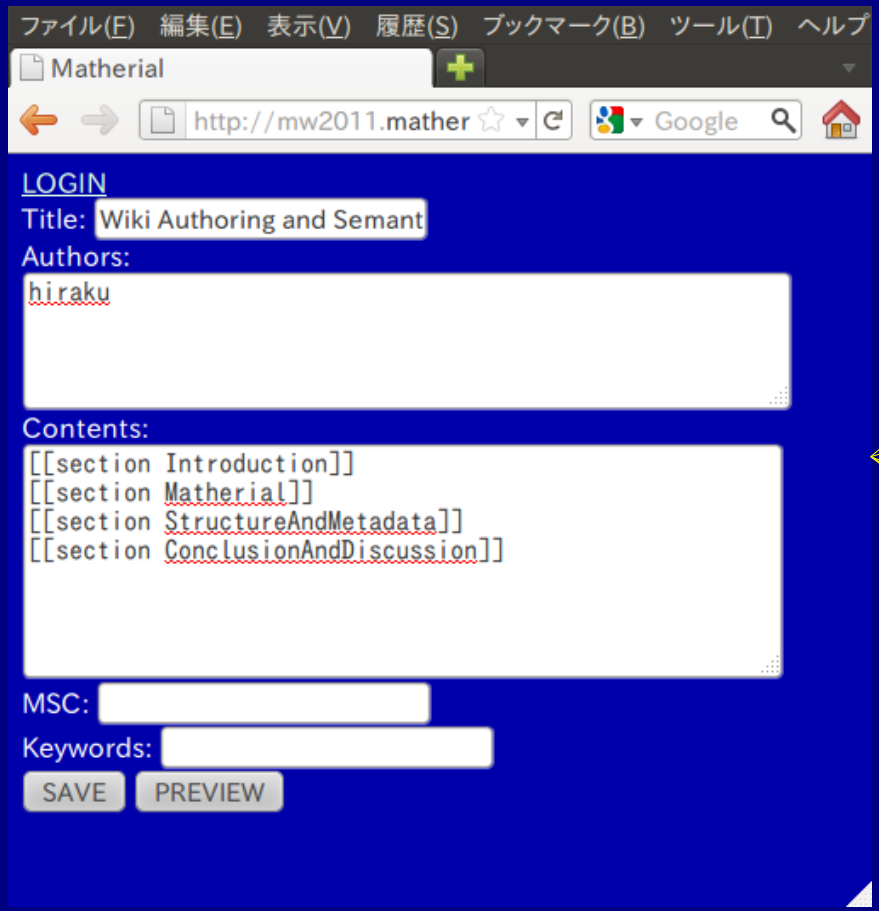
→ parts of documents

An Aggregation of Wiki pages



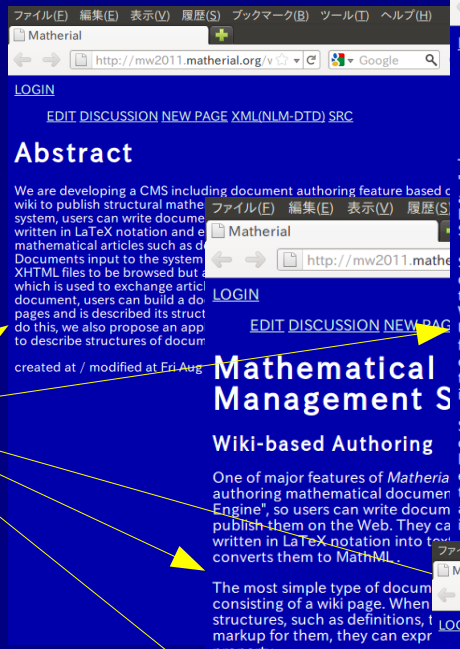
- Publishing document including several sections on Matherial
  - split into sections
  - write each section as a WikiPage

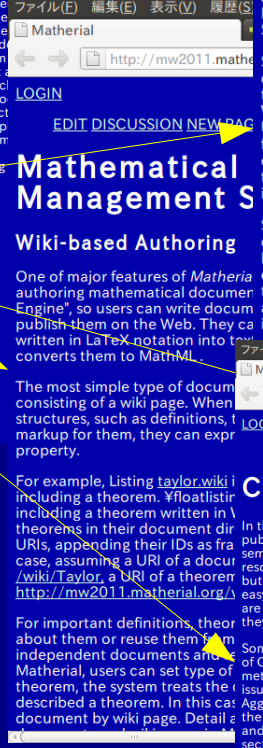
An Aggregation of Wiki pages

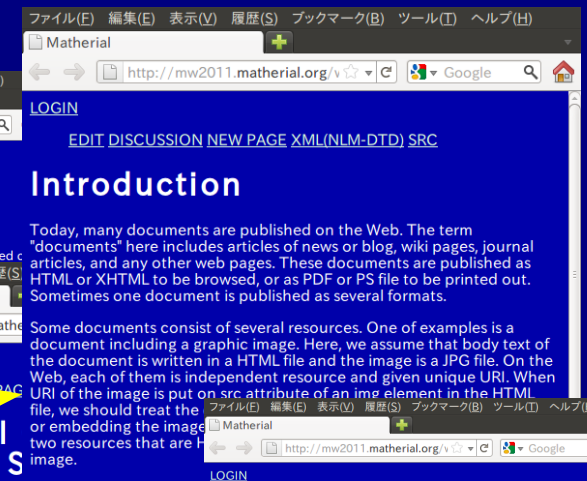


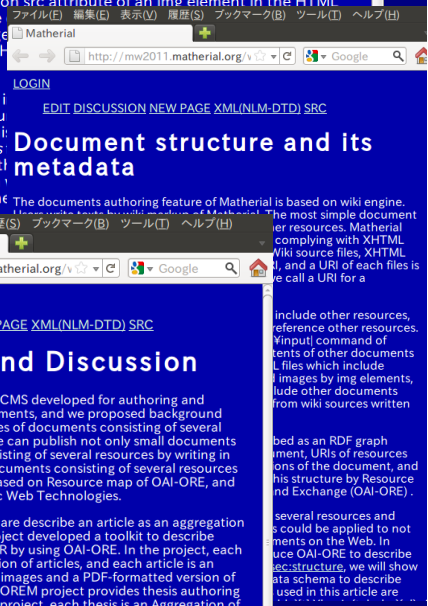
Enumerating Wiki pages to aggregate

An Aggregation of Wiki pages









.. to build a Document which is an aggregation of Wiki Pages

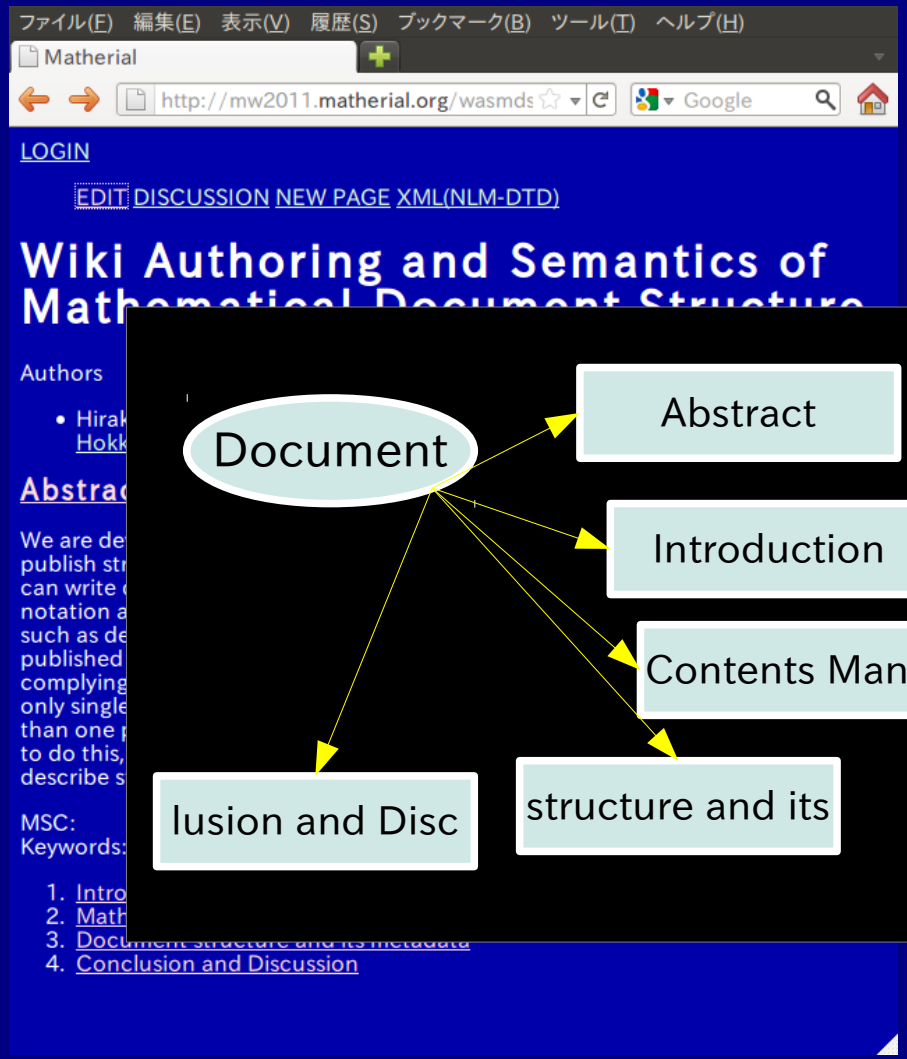
Authoring Mathematical Document

User can author full-text mathematical document using by easy Wiki notaion with structure of Mathematical Documents

Representations of a document in Matherial

- XHTML with MathML, RDFa for
  - browsed by human
  - describing metadata by RDFa
    - Structure between Resources
    - General Information
- NLM-DTD
  - using Archiving and Interchange Tag Set
  - XML will be transcluded from other systems

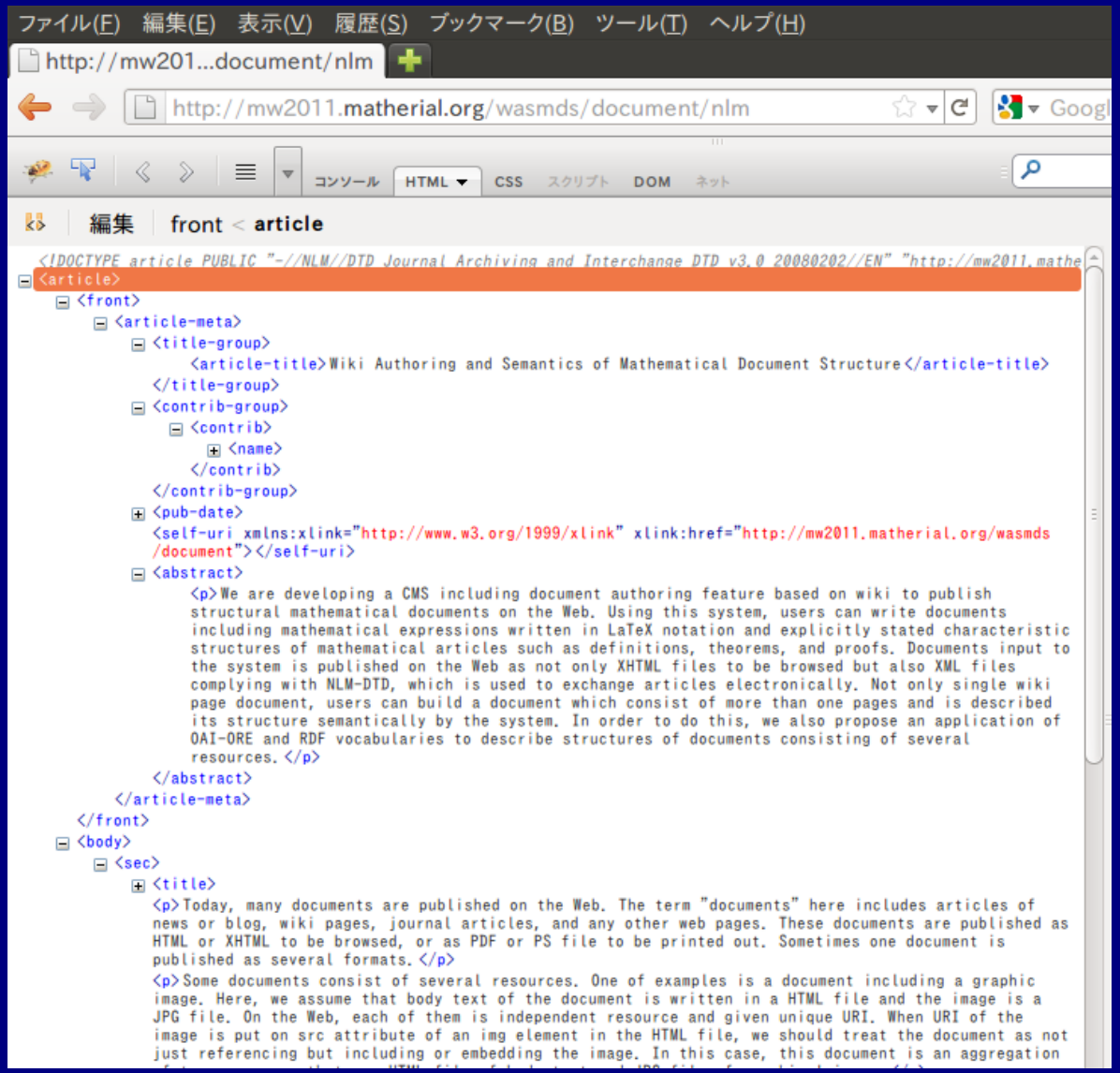
Representations of a document in Matherial



XHTML+MathML+RDFa

RDF-Graph is embedded by using RDFa

Representations of a document in Matherial



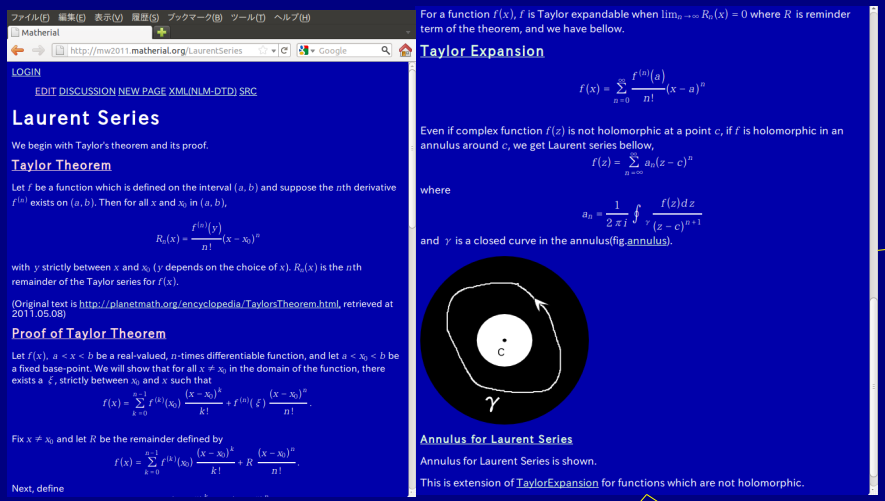
- NLM-DTD
  - Journal Archiving and Interchange Tag Suite
  - by National Library for Medicine
  - 3 Tag Set
    - Archiving and Interchange
    - Jounal Publishing
    - Article Authoring
- Matherial outputs Archiving and Interchange
  - including all text of the document
  - transcluded from other systems

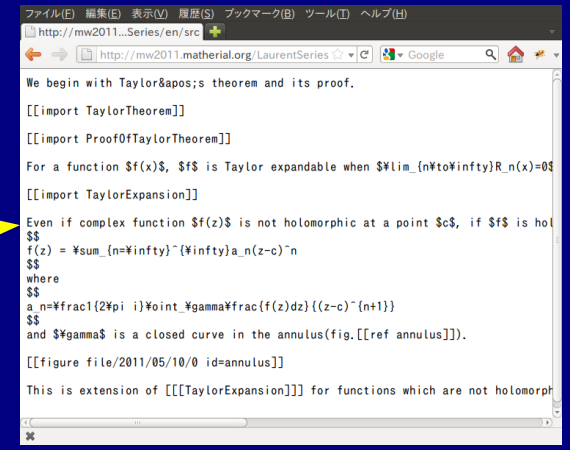
Describing Structure of a Document

Two types of Document

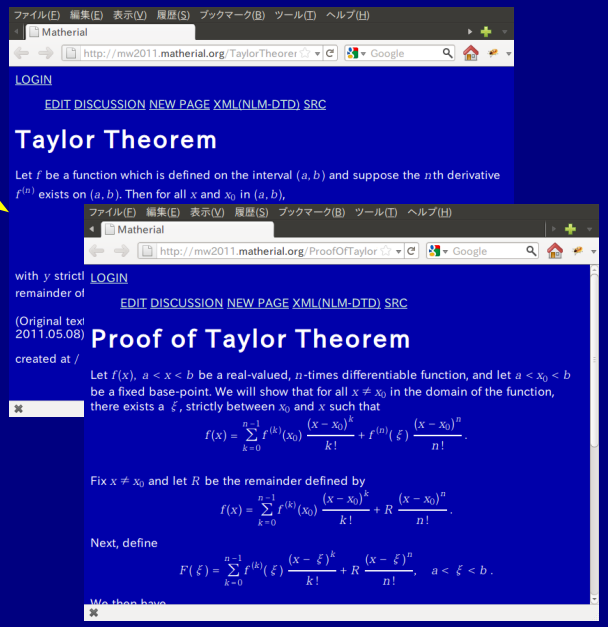
- A Wiki (or Web) page type of Document
- Aggregation of pages

Wiki Page as Aggregation





source text



imported pages

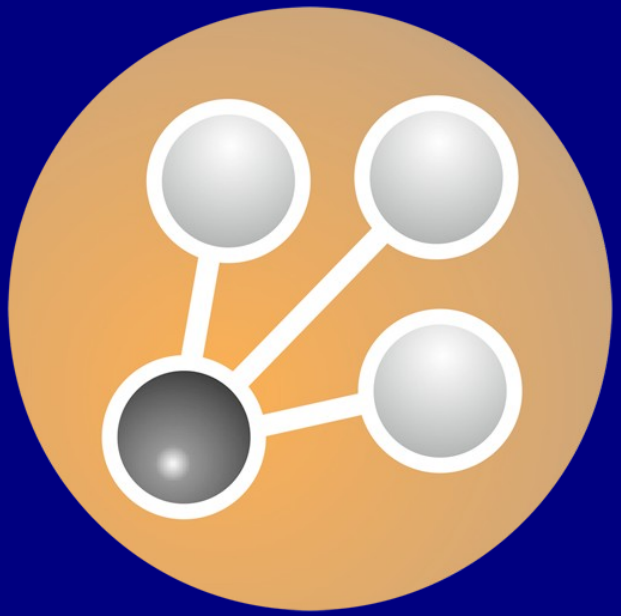
image files

Open Archives Initiative Object Reuse and Exchange

or

OAI-ORE

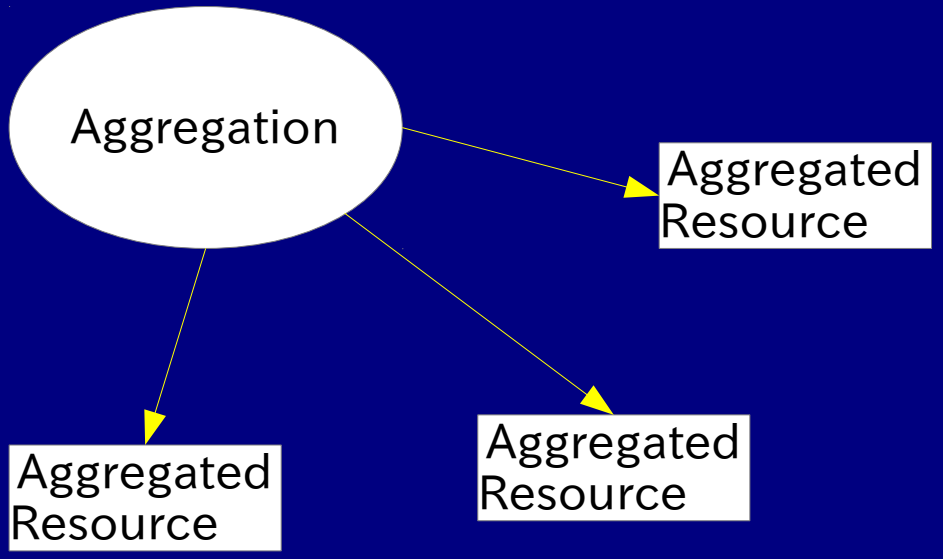
(http://www.openarchives.org/ore/)



(http://www.openarchives.org/ore/logos/logos)

Aggregation of OAI-ORE

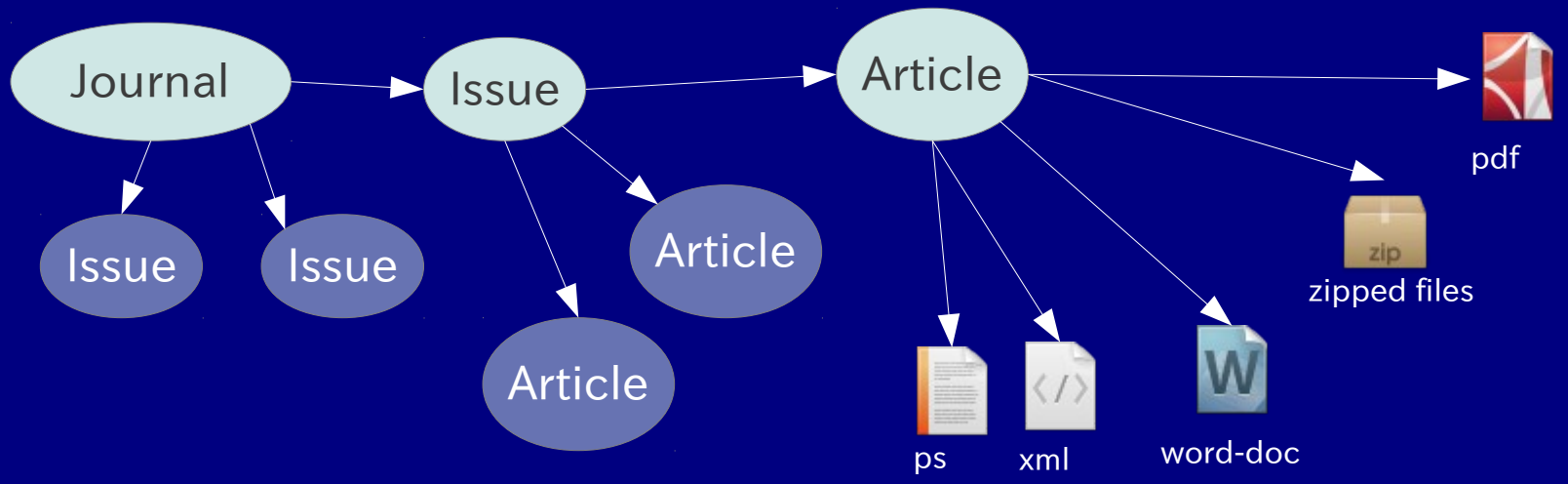
ore:aggregates —Rdfs:subPropertyOf→ dcterms:hasPart



- OAI-ORE provides a mechanism for describing an Aggregation as RDF graph.

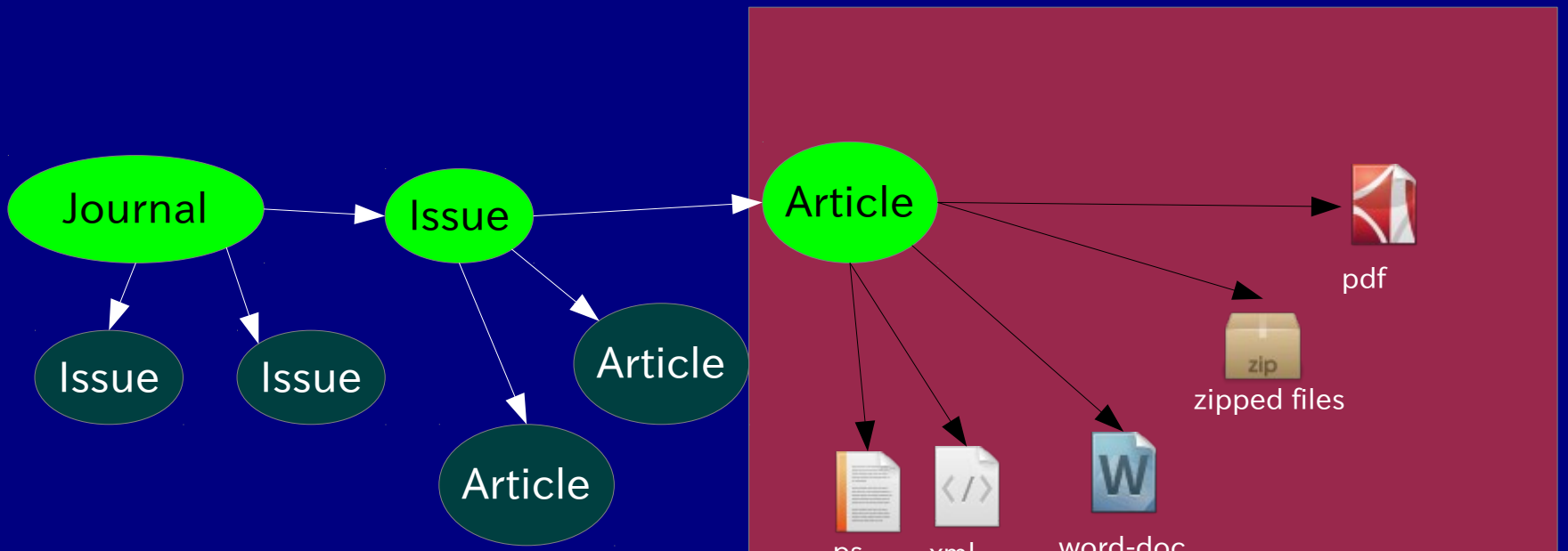
Structure of Journal, Issue, Article and Representations

- A journal article is a major example of OAI-ORE aggregation.



- We think that a journal article is also an example of Document.

A Question



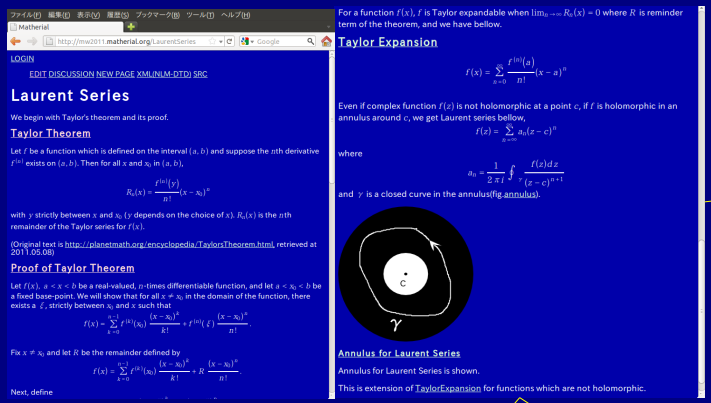
Is it appropriate to describe an article as an aggregation of its representations?
(a PDF as a part of document?)

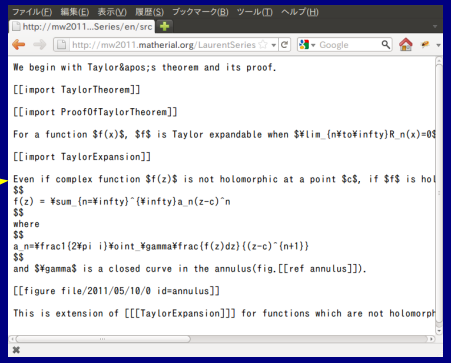
Document as an Aggregation

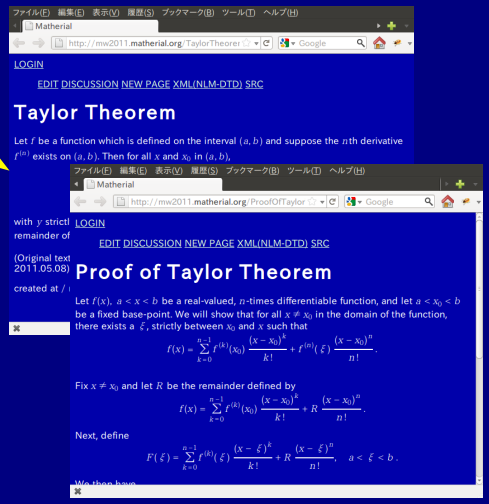
An article (or A Document) should be described as an Aggregation of constituent resources.

- Parts, Chapters, Sections, …
- Embedded images
- Content (body text) of article

Constituents of a Wiki Page







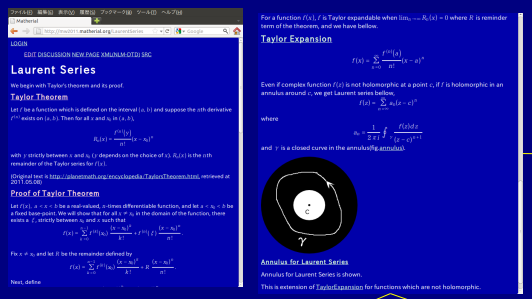
This Wiki Page is an Aggregation of wiki-source and imported Resources

Main Content of a Document

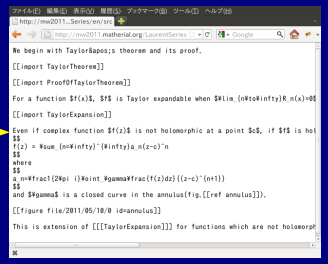
- The Wiki source file is a special member of the Document. This includes
  - Main content of Document
    - body text
  - Indication of embedding Resources
    - [[import TaylorTheorem]]
    - [[figure file/2011/5/10/0 id=annul]]

Main Content of a Document

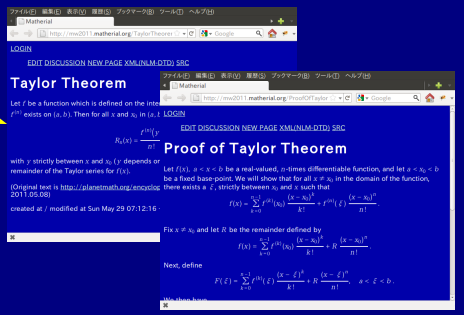
mt:hasContent —rdf:subPropertyOf→ ore:aggregates



mt:hasContent



ore:aggregates



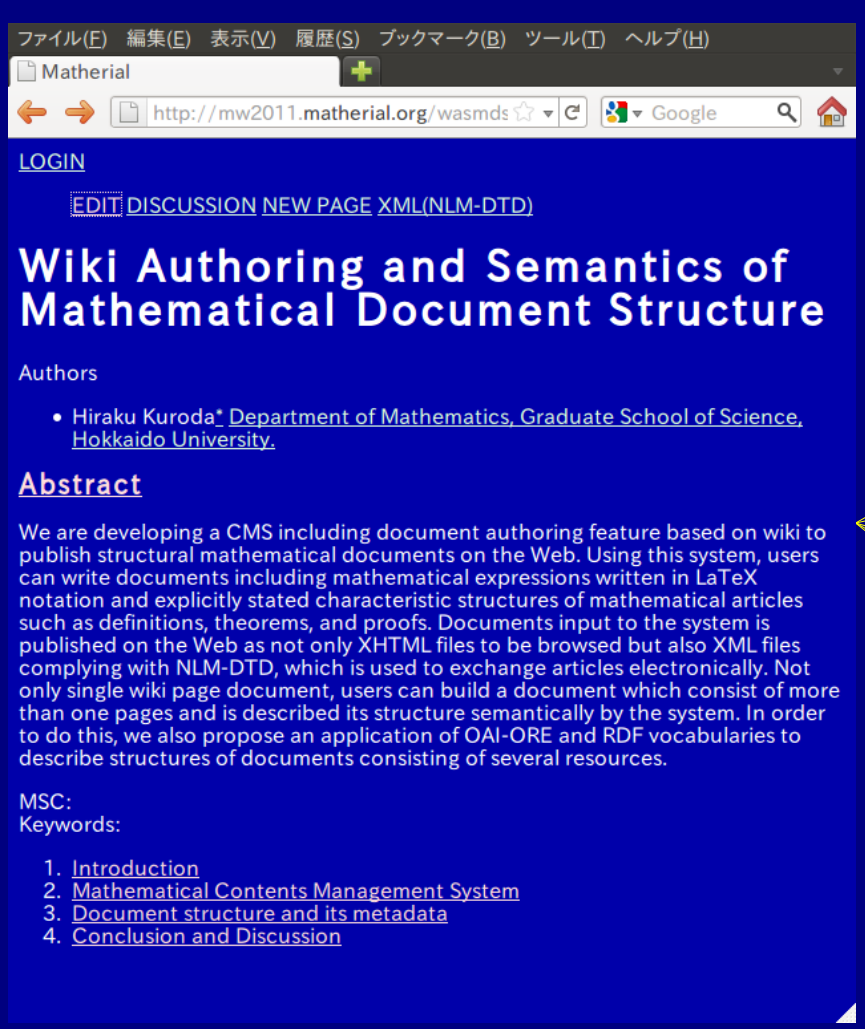
mt (http://www.matherial.org/terms/) is for our new terms of RDF.

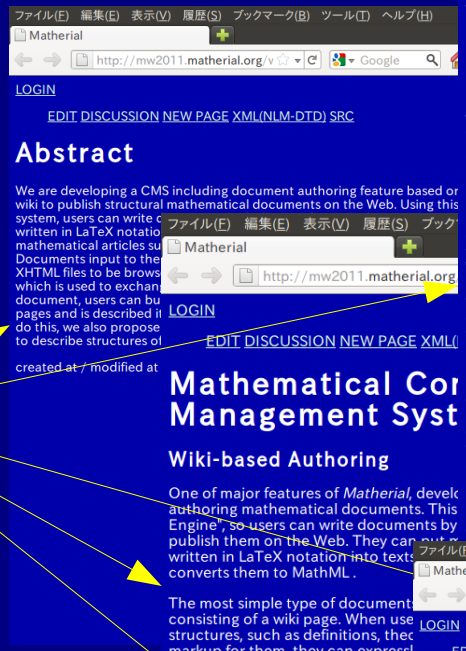
Wiki page as Aggregation

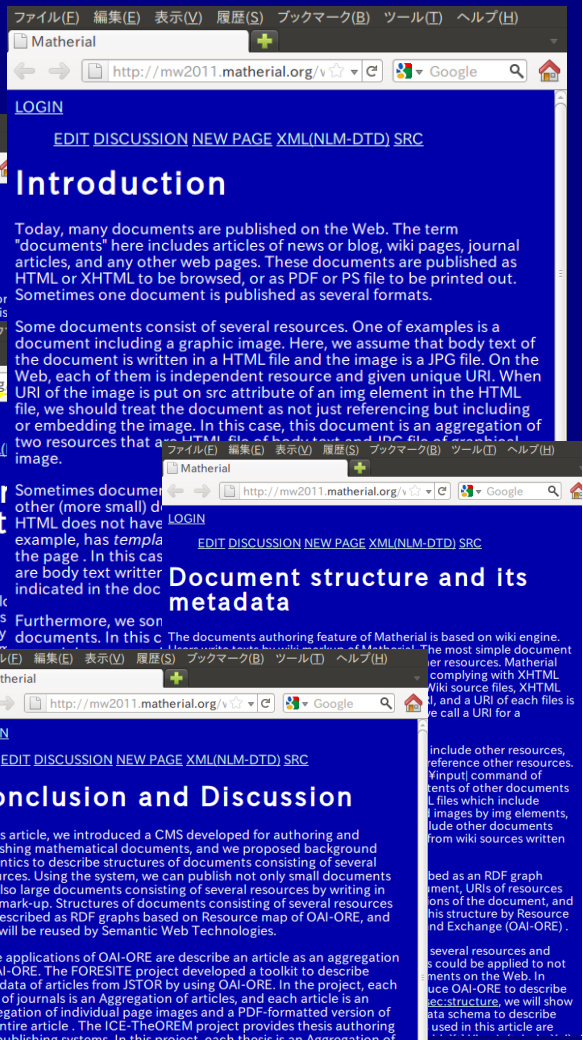
- A Document which has its own content is an Aggregation of
  - Document Content
    - Wiki-source, HTML, XML, Plain text, …
  - Embedded (Transcluded) Resources
    - Image file, other Document and any objects

about an Aggregation of Documents

An Aggregation of Wiki pages







An Aggregation of Wiki pages

This Document is an Aggregation of

- http://.../wasmds/Abstract
- http://.../wasmds/ConclusionAndDiscussion
- http://.../wasmds/Introduction
- http://.../wasmds/Matherial
- http://.../wasmds/StructureAndMetadata

(This list is sorted into alphabetical order)

What order do we read them?

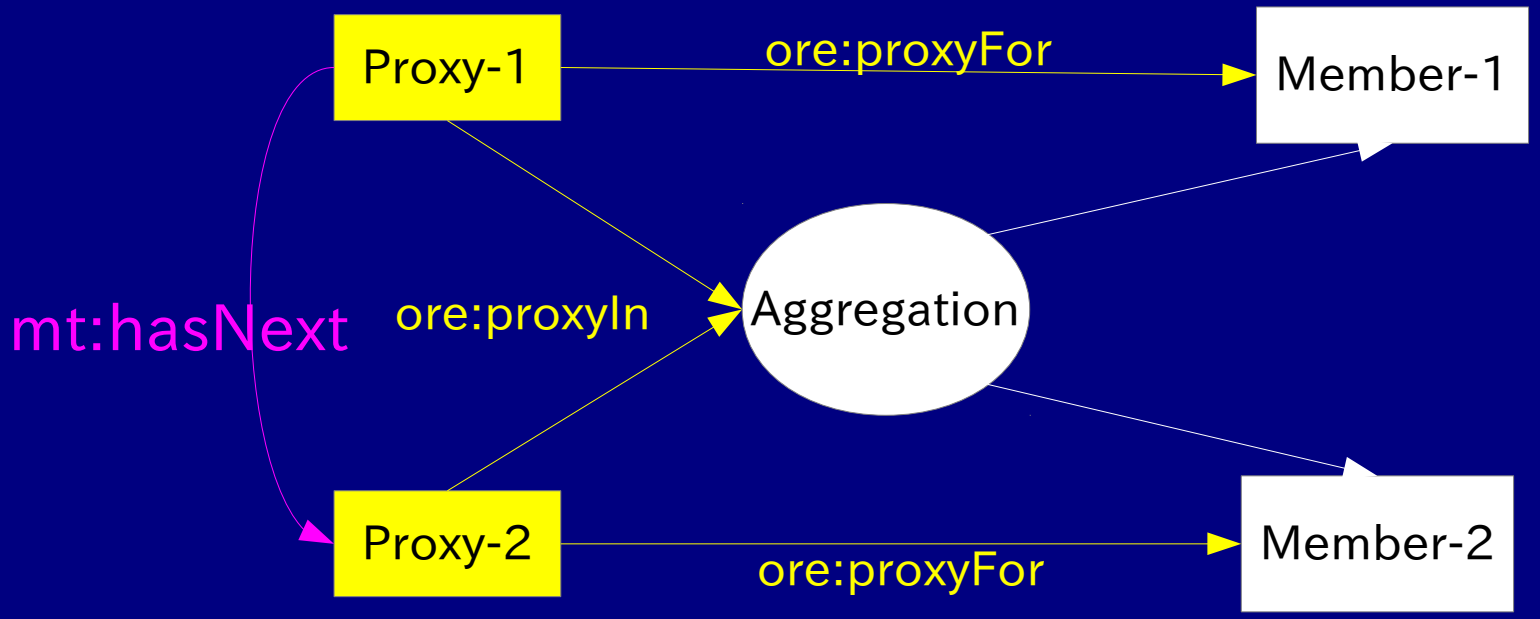
Order of Members

- We define a new term to describe order of members
- Order relationships should not be described between members directly.

Order of Members

We should not directly describe a property between members when the property is valid only in an Aggregation.

Proxy of ORE



Proxy of ORE is provided to describe a property which is valid only in an aggregation.

“Member-1 hasNext Member-2 in the Aggregation.”

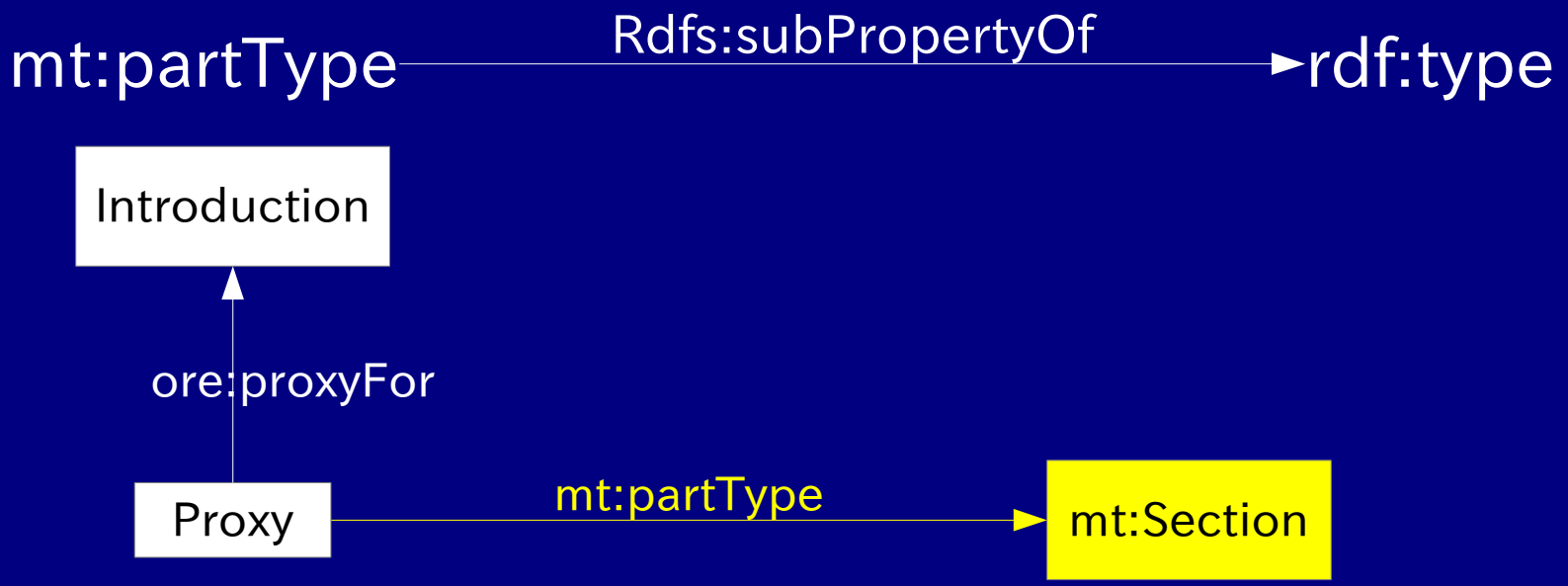
about *hasNext*

- *hasNext* is used in documents of OAI-ORE.
  - x:hasNext
  - That is only for example and not defined formally.

- We need to describe order of members of Document which doesn't have its own content.

  Therefore we define mt:hasNext.

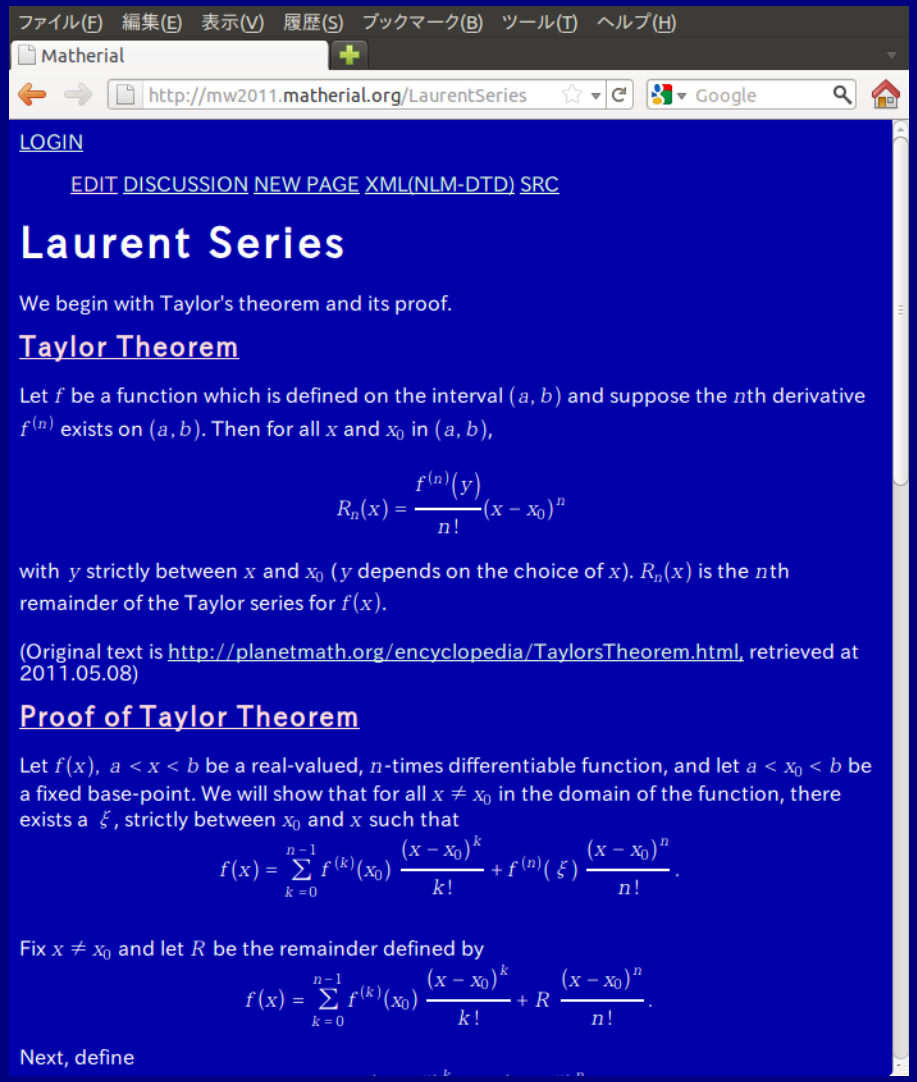
Type of a Member



- The term mt:partType is a property to describe a type or a role of members in an Aggregation
- mt:part takes sub classes of mt:PartType for object of a triple.
    - mt:Part, mt:Chapter and so on

about A Document and its Representatios

A Document and its Representations



This Document has three Representation.

Wiki-SRC

- Document Content (Member of Aggregation)
- and the simplest Representation of this Document

XHTML

NLM-DTD

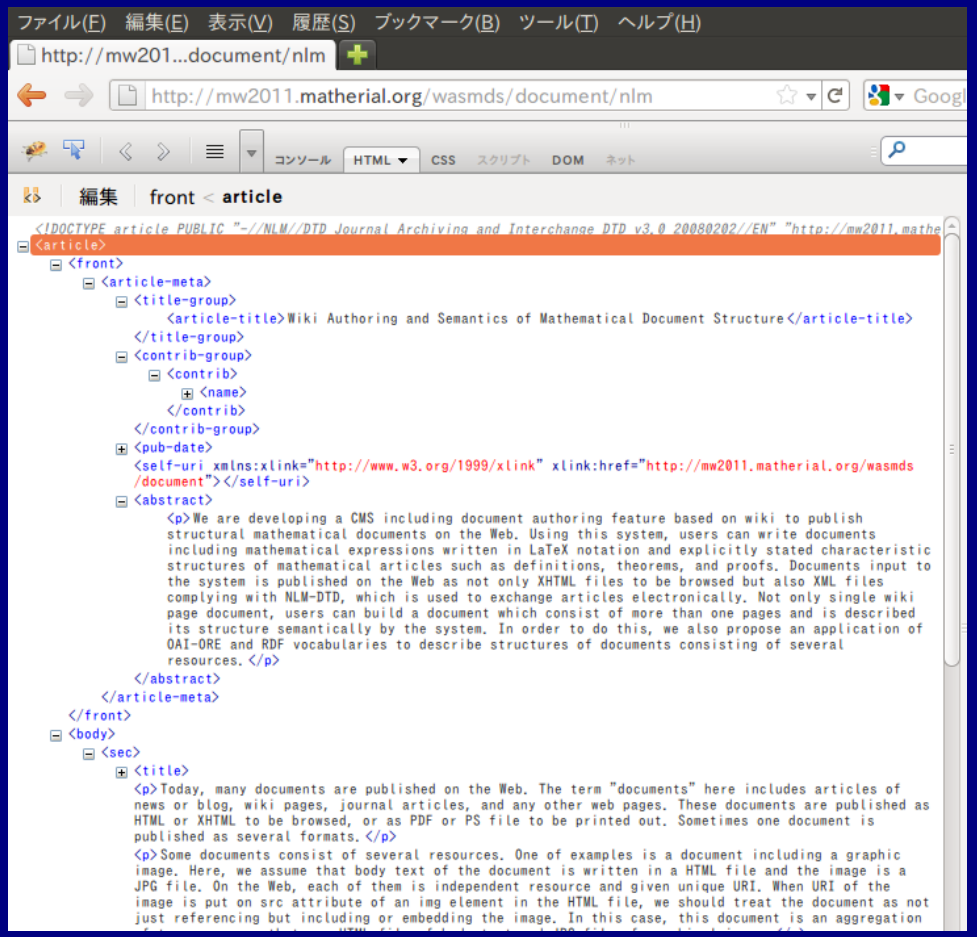
Aggregation of Wiki Page

- This Document does not have its own Content
- Each member Document has their own Document Content.

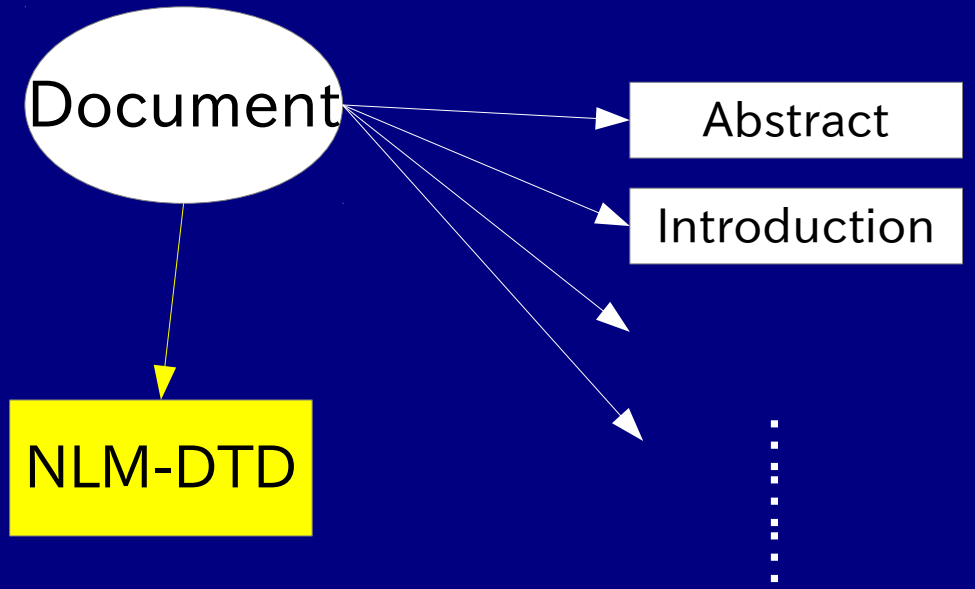
(This page is one of Representations, but not content of the Document)

Aggregation of Wiki Page

has NLM-DTD version of Representation, which includes ALL text of the Document.



A Document and its Representations

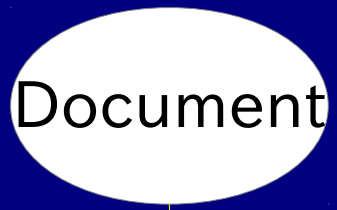


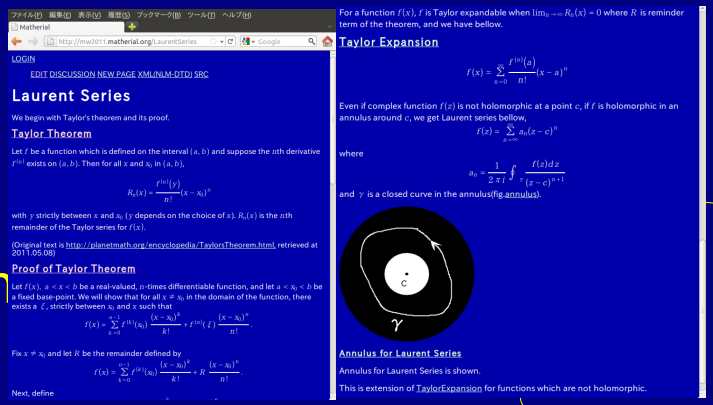
The document is an aggregation of 5 sections, not aggregation of Representation(s).

We need different way to describe between Document and Representations.

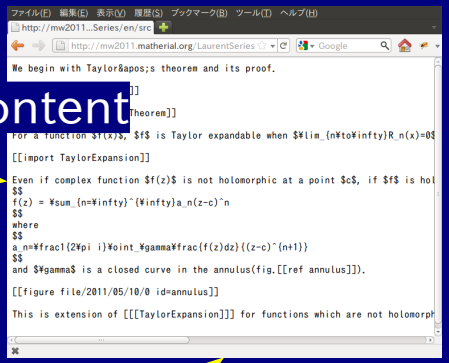
Representations of a Document

mt:hasRepresentation —Rdfs:subPropertyOf→ dcterms:relation





mt:hasContent

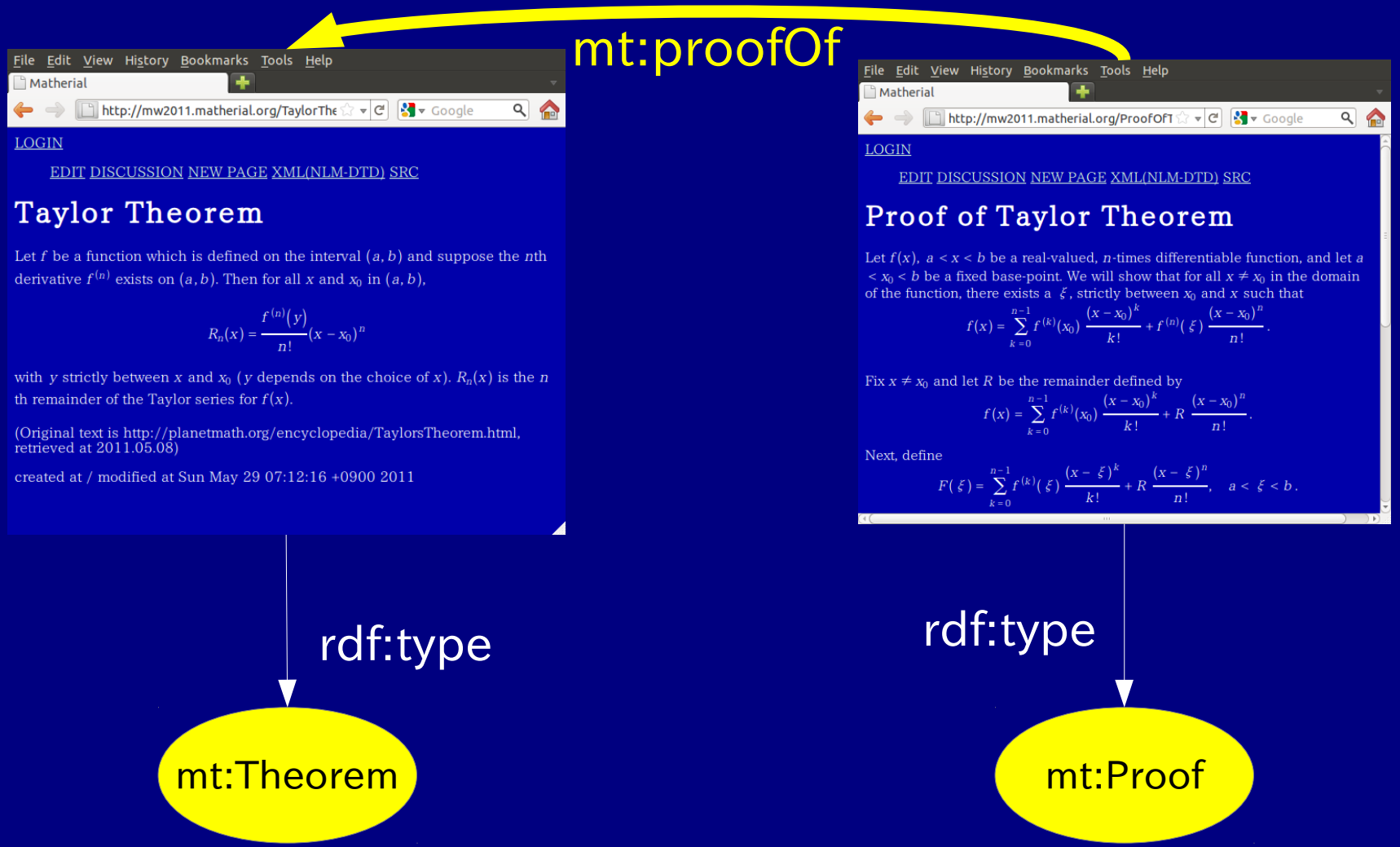


mt:hasRepresentation

- The term mt:hasRepresentation is a property to describe relationships between a Document and its Representations.
- A Document-Content member of the Document can be a Representation of the Document.

about Document Types for Mathematics

Document Types for Mathematics



Document Types for Mathematics

- Mathematical Resource Type mt:MathematicalObject
    - mt:Expression, mt:Definition, mt:Theorem and mt:Proof
- A Relationship between a theorem and its proof

Overlapping with OMDoc Ontology

- OMDoc Ontology provides similar vocabulary
  - RDF classes i.e. Definition, Theorem, Proof and so on.
  - Each class is sub class of *MathKnowledgeItem*
  - *Any mathematical knowledge item that can be expressed in OMDoc*

    (http://omdoc.org/ontology)
  - If a document is a MathKnowledgeItem, the document may be expected to be represented in OMDoc.

Overlapping with OMDoc Ontology

- Matherial focuses to publish Documents using by presentation markups and not support OMDoc.
    - Mathematical Documents on Matherial is not represented in OMDoc
- This is because we define similar terms in our namespace.

Conclusion

Conclusion

- Matherial, CMS for mathematical documents
  - Authoring two types of document that consist of several Resources.
    - A Wiki Page type of Document
    - An Aggregation of Wiki Pages.
    - Mathematical expressions in LaTeX notation
    - Type of a Mathematical document
  - Publishing full-text documents into several Representations with their metadata.
    - XHTML + MathML and RDFa
    - NLM-DTD

Conclusion

- Semantics of Mathematical Document Structure
  - We defined new terms to describe Structure of Documents.
    - mt:hasContent to describe that an aggregating document has its own content resource.
    - mt:hasNext and mt:partType to describe properties of member-document of the aggregating document.
    - mt:hasRepresentation to describe relationships between the Document and its Representations

Conclusion

- Semantics of Mathematical Document Structure
  - We defined terms
    - mt:Defintion, mt:Theorem, mt:Proof, mt:proofOf and so on to describe type and relationship of mathematical documents which are represented in presentation markup, but not content markup.