

Formal Reasoning 2014
Test Block 2: Predicate Logic
(30/09/14)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

The first three exercises use the following ‘dictionary’.

| | |
|-----------|-----------------------|
| P | the domain of persons |
| $W(x)$ | x is a woman |
| $H(x)$ | x is happy |
| $L(x, y)$ | x loves y |

1. Give a formula of predicate logic that approximates the meaning of the following sentence as well as possible:

- (a) *There exists at least one happy person.* (10 points)
- (b) *There exists exactly one happy person.* (10 points)
- (c) *There exist exactly two happy persons.* (10 points)

2. Give a formula of predicate logic that approximates the meaning of the following Dutch sentence as well as possible: (15 points)

A man is happy if there is a woman who loves him whom he also loves.

3. Consider the following formula of predicate logic:

$$\forall x \in P [\neg \exists y \in P L(y, x) \wedge W(x) \rightarrow \neg H(x)]$$

- (a) Write this formula according to the official grammar from the course notes. (10 points)
 - (b) Give an English sentence that corresponds to this formula as well as possible. (15 points)
4. What does $f \models g$ means in predicate logic? Give the definition in terms of interpretations within models. (10 points)
5. Give an interpretation I_5 within a model M_5 such that the following formula is not true.

$$\left(\forall x \in D (P(x) \vee Q(x)) \right) \rightarrow \left((\forall x \in D P(x)) \vee (\forall x \in D Q(x)) \right)$$

Explain your answer. (10 points)