Formal Reasoning 2014
Test Block 3: Languages & Automata
(22/10/14)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. Give a regular expression for the language (15 points)

$$L_1 := \{w \in \{a,b\}^* \mid w \text{ contains } ab \text{ and } w \text{ contains } ba\}$$

2. We define the language $L_2 := \mathcal{L}((aa \cup ab)^*)$.

   (a) Give a state transition diagram for a (deterministic) finite automaton that accepts the language L_2. (15 points)

   (b) Write the same automaton also as a quintuple $\langle \Sigma, Q, q_0, F, \delta \rangle$. (5 points)

3. Give a context-free grammar for the language (15 points)

$$L_3 := \{uvcv^R u^R \mid u \in \{a,c\}^*, v \in \{b,c\}^*\}$$

(Note that for instance $accbbcbbcca \in L_3$, where $u = ac$ and $v = cbb$.)

4. Let G_4 be this context-free grammar:

$$S \to aB \mid aaa \mid bS$$
$$B \to abS \mid bB \mid \lambda$$

   (a) Is G_4 right-linear? Explain your answer. (10 points)

   (b) Somebody claims that

$$P(w) := w \text{ does not contain } aaaa$$

   is an invariant that shows that $aaaa \notin \mathcal{L}(G_4)$. Is this claim correct? Explain your answer. (Note: if it is not a proper invariant, you don't have to provide a proper one.) (10 points)

5. Let L be a regular language.

   (a) Does it hold that if $L \subseteq L'$, then L' also has to be a regular language? Explain your answer. (5 points)

   (b) Does it hold that if $L' \subseteq L$, then L' also has to be a regular language? Explain your answer. (5 points)

6. Does the equality below holds for any language L? (10 points)

$$L^* = \{\lambda\} \cup LL^*$$

Explain your answer.