

Formal Reasoning 2015
Exam
(27/01/16)

Before you read on, write your name, student number and study on the answer sheet! This exam consists of fifteen exercises (three exercises for each block of the course notes) and each of these exercises is worth six points. The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. Consider the following formula in the propositional logic:

$$\neg a \vee b \leftrightarrow a \rightarrow b$$

- (a) Write this formula according to the official grammar in the course notes. (3 points)
- (b) Give the truth table of this formula. (3 points)
2. Consider the following fragment of the song *Nederwriet* by the Dutch band *Doe Maar*:¹ (6 points)

Because the leaves, they give you a headache. [...]
So these leaves, you throw them away, away, away!
And you only smoke the covers of the seeds
and... or... and/or the flowers.

Give a formula in the propositional logic that resembles the meaning of these lyrics as well as possible. Use the dictionary:

SL you smoke leaves
SC you smoke covers of seeds
SF you smoke flowers
H you get a headache

3. Consider the following statement:

$$f \equiv a \rightarrow b$$

- (a) What is the meaning of this statement? (3 points)
- (b) Give a formula f in the propositional logic that complies to this statement, but which does *not* contain the connective ' \rightarrow '. Explain your answer. (3 points)

¹This text is part of this exam because of the language construction 'and/or' that is being used and not because of the other lyrics of the song. The lecturers of this course strongly reject any kind of drug use.

4. Give a formalization of the fragment of the lyrics by *Doe Maar* from exercise 2 in the predicate logic, and use the dictionary: (6 points)

B	domain of the members of the band <i>Doe Maar</i>
P	domain of the parts of a cannabis plant
j	you = the person about whom this song is
$L(x)$	x is a leave
$C(x)$	x is a cover of a seed
$F(x)$	x is a flower
$S(x, y)$	x smokes y
$H(x)$	x gets a headache

(For example, the formula $F(j)$ has the meaning ‘you are a flower’.)

5. Give a formula in the predicate logic that expresses that the band *Doe Maar* has more than two members. Use the dictionary from the previous exercise. (6 points)
6. Give an interpretation I_6 in a model M_6 for which the following formula holds: (6 points)

$$\forall x \in D \exists y \in D (R(x, y) \wedge \forall y' \in D (R(x, y') \rightarrow y' = y))$$

Explain your answer.

7. Give a language L_7 with alphabet $\{a, b\}$ for which $L_7^* \neq L_7$ but $L_7 L_7 = L_7$. (6 points)
Explain your answer.
8. Give a finite automaton that recognizes the language (6 points)

$$L_8 := \mathcal{L}((a \cup b)^* a a (a \cup b)^*)$$

9. Consider the following context-free grammar G_9 :

$$S \rightarrow Sa \mid \lambda$$

- (a) Is this grammar right-linear? Explain your answer. (2 points)
- (b) Is the language $\mathcal{L}(G_9)$ regular? Explain your answer. (2 points)
- (c) Somebody claims that (2 points)

$$P(w) := w \text{ does not contain } b$$

is an invariant of G_9 that can be used to prove that

$$abba \notin \mathcal{L}(G_9)$$

Is this a correct claim? Explain your answer.

10. (a) Give a connected planar bipartite graph G_{10} with eight vertices, such that each vertex has degree three. Explain your answer. (Make sure that you do not forget any properties!) (2 points)
- (b) Give a *not* connected planar graph G'_{10} with eight vertices, where every vertex has degree three. Explain your answer. (Make sure that you do not forget any properties!) (2 points)
- (c) Indicate for each graph whether it has a Hamilton cycle or not. (2 points)

11. Consider the following recursive definition of the sequence of numbers a_n :

$$a_0 = 3$$

$$a_{n+1} = 3a_n - 3 \quad \text{voor } n \geq 0$$

- (a) Compute a_5 using this recursive definition. (3 points)
- (b) Prove that (3 points)

$$a_n = \frac{3}{2}(3^n + 1)$$

for all $n \geq 0$.

12. (a) Indicate in Pascal's triangle where you can find the binomial coefficients $\binom{6}{k}$. (3 points)
- (b) Compute (3 points)

$$\binom{6}{0} + \binom{6}{1}3 + \binom{6}{2}3^2 + \binom{6}{3}3^3 + \binom{6}{4}3^4 + \binom{6}{5}3^5 + \binom{6}{6}3^6$$

13. Give a formula in the epistemic logic that formalizes the following sentence as well as possible: (6 points)

Either I know that Klaas is the mole, or I know that Klaas is not the mole, but I don't know which one of these two it is.

Use as dictionary:

K Klaas is the mole

14. Give a Kripke model \mathcal{M}_{14} in which the formula of the modal logic (6 points)

$$\Box(a \vee b) \rightarrow \Box a \vee \Box b$$

is not true. Explain your answer.

15. Give an LTL formula that describes the situation in which always if a is true and b is one moment later true, c will be true another moment later. (6 points)