

Formal Reasoning 2015
Test 2: Predicate logic
(23/09/15)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

In the first two exercises we use the following interpretation:

M	the domain of men
W	the domain of women
k	Koos
j	Joris
$T(x, y)$	x is taller than y

1. Give for each of these sentences a formula of the predicate logic with equality:

- (a) *Koos and Joris are equally tall.* (10 points)
- (b) *Men are not women.* (10 points)
- (c) *There is exactly one man who is taller than Koos.* (10 points)
- (d) *The tallest man is taller than the tallest woman.* (10 points)

2. Consider the following formula of the predicate logic with equality:

$$\forall x, y \in W [x \neq y \rightarrow T(x, y) \vee T(y, x)]$$

- (a) Write this formula according to the official grammar in the course notes. (10 points)
 - (b) Give an English sentence that resembles the meaning if this formula as well as possible. (15 points)
3. Within the propositional logic the concepts $\models f$, $f \models g$ and $f \equiv g$ are defined. In the course notes only the first two are defined within the predicate logic. Give a definition $f \equiv g$ within the predicate logic that is in line with the other two already known definitions. (15 points)

4. Give an interpretation I_4 in a model M_4 making the following formula true:

$$(\forall x \in D ((\exists y \in D \neg R(x, y)) \wedge ((\exists y \in D R(x, y)) \rightarrow (\forall y \in D R(x, y))))$$

Explain your answer. (10 points)