Formal Reasoning 2015
Test 3: Languages and automata
(21/10/15)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. Give a regular expression that defines the language: (20 points)

$$L_1 := \{w \in \{a,b\}^* \mid w \text{ contains an even number of } a\text{'s}\}$$

2. Give a right linear grammar G_2 that defines the language: (15 points)

$$L_2 := \{w \in \{a,b\}^* \mid w \text{ contains both an even number of } a\text{'s and an even number of } b\text{'s}\}$$

3. Give a finite automaton M_3 that matches the context-free grammar G_3: (20 points)

$$\begin{aligned} S &\to AB \\ A &\to aA \mid \lambda \\ B &\to bB \mid \lambda \end{aligned}$$

   In particular $L(M_3) = \mathcal{L}(G_3)$ must hold.

4. (a) Let $|w|_a$ be the number of occurrences of the symbol a in word w, so for example $|abccb|_b = 2$, $|S|_S = 1$ and $|S|_a = 0$. Somebody claims that: (10 points)

$$P(w) := \ w \text{ contains } aa \ \text{ and/or } \ 2|w|_S + 2|w|_A + |w|_a \leq 2$$

   is an invariant for the context-free grammar G_4:

$$\begin{aligned} S &\to BAB \\ A &\to aaA \mid \lambda \mid aBa \\ B &\to bB \mid \lambda \mid bBaaA \end{aligned}$$

   Is this person right? Explain your answer.

   (b) Somebody else claims that: (10 points)

$$\mathcal{L}(G_4) = \{w \in \{a,b\}^* \mid w \text{ contains an even number of } a\text{'s}\}$$

   Is this person right? If not, provide a word which is contained in exactly one of these two languages. Explain your answer. (Hint: have a look at $P(w)$ in Exercise 4a.)

5. (a) We define: (10 points)

$$L_5 := \{w \in \{a,b\}^* \mid w \text{ starts with an } a\}$$

   Explain why:

$$L_5^* = L_5 \cup \{\lambda\}$$

   (b) Give two languages L and L' over alphabet $\Sigma = \{a,b\}$ such that: (5 points)

$$L \cap L' = \emptyset \quad L^* \neq \Sigma^* \quad L'^* \neq \Sigma^* \quad L \cup L' \neq \Sigma^* \quad L^* \cup L'^* = \Sigma^*$$

   Explain your answer. (If you can't manage to comply to all these requirements, try to comply to as many as possible.)