

**Formal Reasoning 2015**  
**Test 5: Modal Logic**  
(16/12/15)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. The test is *closed book*. Good luck!

1. (a) Explain the difference in the meaning of the two sentences below using formulas of modal logic. (10 points)
- *I know that it rains.*
  - *I know whether it rains.*

You have to choose yourself which version of modal logic fits these sentences best and which ‘dictionary’ you use.

- (b) What is the name of the modal logic you used in item 1a? (5 points)
- (c) What is in this logic the interpretation of  $\Box f$  and  $\Diamond f$ ? (10 points)
2. (a) Draw the parse tree for this formula in modal logic: (10 points)

$$\Diamond a \wedge \neg \Box a \rightarrow a \wedge \neg a$$

- (b) Write the formula of question 2a according to the official grammar in the course notes. (10 points)
- (c) Give a Kripke-model  $\mathcal{M}_2$  such that the formula of question 2a is true, i.e. a Kripke model in which we have: (10 points)

$$\mathcal{M}_2 \models \Diamond a \wedge \neg \Box a \rightarrow a \wedge \neg a$$

Use the graphical representation with circles and arrows to show you model. And explain your answer.

- (d) Write model  $\mathcal{M}_2$  of question 2c as triple  $\langle W, R, V \rangle$ . (10 points)
- (e) Show that: (10 points)

$$\not\models \Diamond a \wedge \neg \Box a \rightarrow a \wedge \neg a$$

3. Give an LTL-formula that describes the situation in which the following two properties hold: (15 points)
- On each moment either  $a$  or  $b$  is true, but not both.
  - For both  $a$  and  $b$  it holds that they are at most three times in a row true. After that, the other one should become true again.

Explain your answer. (If you can’t find a single formula that describes both properties at once, try to formalize at least one of the properties.)