# Formal Reasoning 2015
# Uitwerkingen Test 1: Propositional logic
**(09/09/15)**

In the first three exercises we use the following interpretation of the atomic propositions:

| | |
|---|---|
| $R$ | it rains |
| $S$ | the sun shines |
| $RB$ | there is a rainbow |

1. Give two propositions that respectively resemble the meaning of the following two sentences:

   (a) *There is a rainbow, because the sun shines and it rains.*

   $$RB \wedge S \wedge R$$

   (b) *There is only a rainbow if the sun shines and it rains.*

   $$RB \to S \wedge R$$

   If you think that for these sentences multiple interpretations are 'defendable', then please provide those different options in your answer, and indicate which one of these options is your preferred solution. (10 + 10 points)

2. Give a correct English sentence that resembles the meaning of the following proposition as well as possible:

   (15 points)

   $$(\neg R \to R) \to R$$

   *If it follows from the fact that it doesn't rain that it rains, then it rains.*

3. Is the proposition from exercise 2 logically true? Explain your answer.
   (10 points)

   Yes, it is logically true. Let's have a look at the corresponding truth table:

| $R$ | $\neg R$ | $\neg R \to R$ | $(\neg R \to R) \to R$ |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |

   The column for this proposition has only ones in it, so this proposition is logically true.

4. Give the full truth table of the proposition: (15 points)

$$(a \to b) \leftrightarrow (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

To prevent the table from being to wide, we use some abbreviations in its header row:

$$\begin{array}{rcc} f_1 & := & (\neg a \wedge \neg b) \\ f_2 & := & (\neg a \wedge b) \\ f_3 & := & (a \wedge b) \end{array}$$

Now this is the truth table:

| $a$ | $b$ | $a \to b$ | $\neg a$ | $\neg b$ | $f_1$ | $f_2$ | $f_3$ | $f_2 \vee f_3$ | $f_1 \vee f_2 \vee f_3$ | $(a \to b) \leftrightarrow f_1 \vee f_2 \vee f_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

5. Write the proposition (10 points)

$$(\neg(a \wedge (a) \wedge a))$$

according to the official grammar in the course notes.

$$\neg(a \wedge (a \wedge a))$$

6. Give a model $v$ in which the following proposition is true: (10 points)

$$(a \wedge (b \to c)) \leftrightarrow \neg a$$

The unique model in which this proposition is true is:

$$\begin{array}{c} v(a) = 1 \\ v(b) = 1 \\ v(c) = 0 \end{array}$$

This model corresponds to the only row in the truth table

| $a$ | $b$ | $c$ | $b \to c$ | $a \wedge (b \to c)$ | $\neg a$ | $(a \wedge (b \to c)) \leftrightarrow \neg a$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 |

that has a 1 in the final column.

7. Let $f$ and $g$ be arbitrary propositions. If it holds that $\not\models f$, or $\models g$, or both, does it imply that $\models (\neg f \vee g)$ always holds? Explain your answer.

(10 points)

No, this is not true. Take for instance as a counter example $f = a$ and $g = b$. Now if we look at the corresponding truth table

| $a$ | $b$ | $\neg a$ | $\neg a \vee b$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 |

we see that $\not\models f$ holds, because $a$ does not have only ones in its column. Automatically it follows that '$\not\models f$, or $\models g$, or both' also holds. However, $\models \neg f \vee g$ does not hold, because we see that the column for $\neg a \vee b$ does not exclusively contain ones. Hence this implication is not true.