Formal Reasoning 2017
Test Block 3: Discrete Mathematics and Modal Logic
(20/12/17)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score ten points. Good luck!

1. (a) Give two non-isomorphic graphs G_1 and G_2 that have six vertices such that each vertex has degree three.
   (b) Give the chromatic number of your graphs G_1 and G_2. Explain your answer.
   (c) Explain whether your graph G_1 has an Euler path and whether your graph G_2 has a Hamilton path. In both cases, if such a path exists, give it explicitly and if such a path doesn't exist, explain why it doesn't exist.

2. We define the so-called *subfactorial* of n, denoted as $!n$, by this recursive definition:

$$\begin{aligned} !0 &= 1 \\ !(n+1) &= (n+1)\cdot !n + (-1)^{n+1} \qquad \text{for } n \geq 0 \end{aligned}$$

   (a) Compute $!2$ and explain how you did this.
   (b) Prove by induction that $!n$ is even whenever n is odd and that $!n$ is odd whenever n is even, for all natural numbers n.

3. In how many ways can we distribute five distinguishable objects into three non-empty indistinguishable groups? Write your answer in terms of Stirling numbers of the second kind and give a sufficiently large part of the triangle for these Stirling numbers to make sure that all numbers you use are visible and marked.

4. (a) Using the dictionary

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   give an English rendering of the formula $S \to \neg\Box\neg S$ according to doxastic logic.
   (b) A formula f is called *true in the logic D* if f is true in all serial Kripke models. The notation for this is $\vDash_D f$. Show that $\vDash_D S \to \neg\Box\neg S$ does not hold.
   (c) What is the counterpart of the formula $S \to \neg\Box\neg S$ in LTL, if you may only use the operators $\mathcal{F}$, $\mathcal{X}$, $\mathcal{U}$, $\neg$, $\wedge$, $\vee$, $\to$ and $\leftrightarrow$, and the new formula must have the exact same meaning as the original one? *Note:* You don't have to use all these operators, but you are not allowed to use operators that aren't listed.