Formal Reasoning 2017
Test Blocks 1, 2 and 3: Additional Test
(10/01/18)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question the maximum score is indicated. Good luck!

1. We have the following dictionary to talk about a Kripke model $\mathcal{M}$:

domains:	W	the worlds of $\mathcal{M}$
	A	the atomic propositions
constants:	x_0	the world x_0
	a	the atomic proposition a
predicates:	$R(x, y)$	y is a successor of x
	$V(x, y)$	the atomic proposition y is true in world x

   Translate to formulas of predicate logic using this dictionary:

   (a) $x_0 \Vdash \Box a \to \Diamond a$ (15 points)

   (b) $\mathcal{M}$ is serial (15 points)

2. Let a graph $G = \langle V, E \rangle$ and a node $v_0 \in V$ be given. (20 points)

   From this we recursively define sets $V_n \subseteq V$ for all $n \geq 0$ by:

$$V_0 = \{v_0\}$$
$$V_{n+1} = V_n \cup \{v \mid v \text{ is a neighbor of a node in } V_n\} \qquad \text{for all } n \geq 0$$

   Prove with induction that for all $n > 0$, if there is a path of length n from v_0 to a node v, then $v \in V_n$.

3. Prove that (20 points)

$$\left\{ {n \atop n-2} \right\} = \binom{n}{3} + \frac{1}{2}\binom{n}{2}\binom{n-2}{2}$$

   for $n \geq 4$ using a combinatorial argument.

4. Show that if $w \in L(M)$, where M is a deterministic finite automaton with n states and $|w| \geq n$, then one can write w as the concatenation (20 points)

$$w = uvu'$$

   where $v \neq \lambda$ and such that $uv^k u' \in L(M)$ for all $k \geq 0$.

   Hint: Consider the states that M subsequently goes through when processing w.