Formal Reasoning 2018
Solutions Test Block 2: Languages & Automata
(07/11/18)

1. Let be given a language L_1 such that $L_1 = L_1^R$. Does it hold for each $w \in L_1$ that $w = w^R$?

   If so, explain why. If not, give a counterexample.

   No, it doesn't hold. Let $L_1 = \{ab, ba\}$. Then $L_1^R = \{ba, ab\}$. So clearly $L_1 = L_1^R$. However, take $w = ab \in L_1$ then $w^R = ba$, hence clearly $w \neq w^R$.

2. Give a regular expression r_2 such that

$$\mathcal{L}(r_2) = \{w \in \{a, b\}^* \mid w \text{ contains } aa\}$$

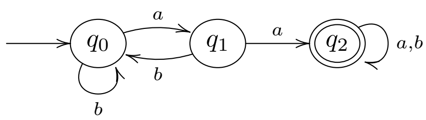
   Take for instance

$$r_2 = (a \cup b)^* aa(a \cup b)^*$$

3. (a) Give a deterministic finite automaton M_3 such that

$$L(M_3) = \{w \in \{a, b\}^* \mid w \text{ contains } aa\}$$

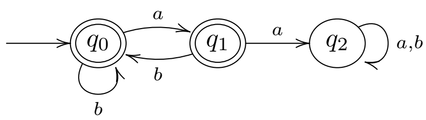
   Let M_3 be:



   (b) Give a deterministic finite automaton M_3' such that

$$L(M_3') = \{w \in \{a, b\}^* \mid w \text{ does not contain } aa\}$$

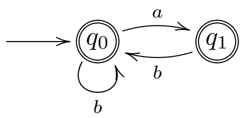
   Let M_3' be M_3 where the final and non-final nodes are swapped. All words that were accepted by M_3 are by definition not accepted in M_3' and vice versa.



   (c) Give a non-deterministic finite automaton M_3'' with at most two states such that

$$L(M_3'') = \{w \in \{a, b\}^* \mid w \text{ does not contain } aa\}$$

   Let M_3'' be M_3' where we remove the sink and all transactions going to it. Words that would have gone to the sink, now end in a *deadlock* situation, which also means that they are not accepted.



4. Give a context free grammar G_4 such that

$$\mathcal{L}(G_4) = \{w \in \{a,b\}^* \mid w \text{ does not contain } aa\}$$

We can do this by transforming the automaton from question 3b to a context free grammar. This leads to:

$$\begin{array}{lcl} S & \rightarrow & aA \mid bS \mid \lambda \\ A & \rightarrow & aB \mid bS \mid \lambda \\ B & \rightarrow & aB \mid bB \end{array}$$

However, since B represents a sink, we can omit this part of the grammar without changing the generated language and we get:

$$\begin{array}{lcl} S & \rightarrow & aA \mid bS \mid \lambda \\ A & \rightarrow & bS \mid \lambda \end{array}$$

And by substituting the possibilities for non-terminal A we get:

$$S \rightarrow a \mid abS \mid bS \mid \lambda$$

Note that grammar G_5 also works!

Another nice solution that does not originate from the finite automaton is:

$$S \rightarrow SbS \mid a \mid \lambda$$

5. The grammar G_5 is defined as:

$$G_5 = \langle \{a,b\}, \{S,B\}, \{S \rightarrow B, S \rightarrow aB, B \rightarrow bS, B \rightarrow \lambda\} \rangle$$

   (a) Give a production that shows that

   $$b \in \mathcal{L}(G_5)$$

   $$S \rightarrow B \rightarrow bS \rightarrow bB \rightarrow b$$

   (b) Someone claims that the following property is an invariant for this grammar:

   $$P_5(w) := w \text{ does not contain } aS \text{ or } aa$$

   Explain why this claim is not correct.

   Let $v = aBa$. Then $P_5(v)$ holds because aBa does not contain aS and it does not contain aa. However, because of the rule $B \rightarrow \lambda$, we have that $v \rightarrow v'$ where $v' = aa$ and $P_5(v')$ clearly doesn't hold, since it contains aa.

6. Does there exist a non-deterministic finite automaton

$$M_6 = \langle \Sigma, Q, q_0, F, \delta \rangle$$

for which $F = Q$ and $Q \neq \emptyset$, but for which it does not hold that $L(M_6) = \Sigma^*$? If so, give an example. If not, explain why.

Yes, there exists such a non-deterministic finite automaton. Namely, the answer to question 3c.