Formal Reasoning 2018
Test Block 1: Propositional and Predicate Logic
(24/09/18)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. $$\neg a \wedge b \vee a \rightarrow b \leftrightarrow a$$

   (a) Write this formula according to the official grammar from the course notes. (10 points)

   (b) Give the full truth table of this formula. (10 points)

2. *I only am happy if I am well rested, because else I am tired.*

   Formalize this English sentence as a formula of propositional logic using the following dictionary as well as possible: (20 points)

H	I am happy
W	I am well rested
T	I am tired

3. The following statement holds:

   $$\text{If } \vDash f \vee g \text{ and } f \vDash h \text{ and } g \vDash h, \text{ then } \vDash h.$$

   Explain what this statement *says* in terms of truth tables or models. Note that you do not have to *show* that this statement holds. (10 points)

4. *The prime numbers are the numbers greater than one that are not a product of two numbers greater than one.*

   Formalize this English sentence as a formula of predicate logic using the following dictionary as well as possible: (20 points)

N	the domain of numbers
u	the number one
$Pr(x)$	x is a prime number
$Lt(x, y)$	$x < y$
$M(x, y, z)$	$x \times y = z$

5. $$\forall x \in D\, P(x) \vDash \exists x \in D\, P(x)$$

   Does this statement hold? Explain your answer. (10 points)

6. Give an interpretation I_6 in the model $M_6 = (\mathbb{N}, +, 1, =, \leq)$ under which the following formula is true: (10 points)

   $$\forall x \in D\, \exists y, z \in D\, [R(x, y) \wedge R(y, z) \wedge \neg R(x, z)]$$

   Note that you do not need to explain your answer.