Formal Reasoning 2018
Test Block 2: Languages & Automata
(07/11/18)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score ten points. Good luck!

1. Let be given a language L_1 such that $L_1 = L_1^R$. Does it hold for each $w \in L_1$ that $w = w^R$?

   If so, explain why. If not, give a counterexample.

2. Give a regular expression r_2 such that

   $$\mathcal{L}(r_2) = \{w \in \{a, b\}^* \mid w \text{ contains } aa\}$$

3. (a) Give a deterministic finite automaton M_3 such that

   $$L(M_3) = \{w \in \{a, b\}^* \mid w \text{ contains } aa\}$$

   (b) Give a deterministic finite automaton M_3' such that

   $$L(M_3') = \{w \in \{a, b\}^* \mid w \text{ does not contain } aa\}$$

   (c) Give a non-deterministic finite automaton M_3'' with at most two states such that

   $$L(M_3'') = \{w \in \{a, b\}^* \mid w \text{ does not contain } aa\}$$

4. Give a context free grammar G_4 such that

   $$\mathcal{L}(G_4) = \{w \in \{a, b\}^* \mid w \text{ does not contain } aa\}$$

5. The grammar G_5 is defined as:

   $$G_5 = \langle \{a, b\}, \{S, B\}, \{S \to B, S \to aB, B \to bS, B \to \lambda\} \rangle$$

   (a) Give a production that shows that

   $$b \in \mathcal{L}(G_5)$$

   (b) Someone claims that the following property is an invariant for this grammar:

   $$P_5(w) := w \text{ does not contain } aS \text{ or } aa$$

   Explain why this claim is not correct.

6. Does there exist a non-deterministic finite automaton

   $$M_6 = \langle \Sigma, Q, q_0, F, \delta \rangle$$

   for which $F = Q$ and $Q \neq \emptyset$, but for which it does not hold that $L(M_6) = \Sigma^*$?

   If so, give an example. If not, explain why.