Formal Reasoning 2018
Test Blocks 1, 2 and 3: Additional Test
(09/01/19)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question the maximum score is indicated. Good luck!

1. Does the following hold? (20 points)

$$(\exists x \in D\,(P(x) \to Q(x))) \vDash ((\exists x \in D\,P(x)) \to (\exists x \in D\,Q(x)))$$

   Explain your answer.

2. (a) We define: (20 points)

$$L_{a,b} := \{uavbw \mid u, v, w \in \{a, b\}^* \text{ and } |u| + |w| = |v|\}$$

   For example $ababaabb \in L_{a,b}$, with $u = ab$, $v = baa$ and $w = b$. Note that all words in $L_{a,b}$ have even length.

   Show that the language $L_{a,b}$ is context-free.

   (b) We define: (20 points)

$$L_2 := \{uu \mid u \in \{a, b\}^*\}$$

   For example $abbabb \in L_2$, with $u = abb$. Note that all words in L_2 have even length.

   Show that the language $\overline{L_2}$ is context-free.

3. Given a graph $G_1 = \langle V_1, E_1\rangle$ and a graph $G_2 = \langle V_2, E_2\rangle$, we say that G_1 is a *subgraph* of G_2 if and only if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$. Count the number of subgraphs of the complete graph on five points K_5. (30 points)

   If you give an expression for this number in terms of binomial coefficients and arithmetic operations, but do not have time to compute this to an explicit number, you can still get partial points for this exercise.