Problem [B2 from IMO 1972]

f and g are real-valued functions defined on the real line. For all x and y,

$$f(x+y) + f(x-y) = 2f(x)g(y).$$

f is not identically zero and $|f(x)| \leq 1$ for all x. Prove that $|g(x)| \leq 1$ for all x.