Introduction to Matlab

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Contents

Introduction **1**
Preliminaries . . . 3

1 Getting started with MATLAB **4**
1.1 Input via the command-line . . . 4
1.2 `help`-facilities . . . 5
1.3 Interrupting a command or program . . . 6
1.4 Path . . . 6
1.5 Workspace issues . . . 6
1.6 Saving and loading data . . . 7

2 Basic syntax and variables **7**
2.1 MATLAB as a calculator . . . 7
2.2 An introduction to floating-point numbers . . . 8
2.3 Assignments and variables . . . 9

3 Mathematics with vectors and matrices **10**
3.1 Vectors . . . 10
3.1.1 Colon notation and extracting parts of a vector . . . 11
3.1.2 Column vectors and transposing . . . 12
3.1.3 Product, divisions and powers of vectors . . . 13
3.2 Matrices . . . 16
3.2.1 Special matrices . . . 16
3.2.2 Building matrices and extracting parts of matrices . . . 17
3.2.3 Operations on matrices . . . 20

4 Visualization **22**
4.1 Simple plots . . . 22
4.2 Several functions in one figure . . . 24
4.3 Other 2D plotting features - *optional* . . . 24
4.4 Printing . . . 25
4.5 3D line plots . . . 26
4.6 Plotting surfaces . . . 26
4.7 Animations - *optional* . . . 27

5 Control flow **28**
5.1 Logical and relational operators . . . 28
5.1.1 The command `find` . . . 30
5.2 Conditional code execution . . . 31
5.3 Loops . . . 33
5.4 Evaluation of logical and relational expressions in the control flow structures . . . 35

6 Numerical analysis **36**
6.1 Curve fitting . . . 36
6.2 Interpolation . . . 37
6.3 Evaluation of a function . . . 37
6.3.1 Inline functions . . . 38
6.4 Integration and differentiation . . . 39
6.5 Numerical computations and the control flow structures . . . 40

7 Script and function m-files **40**
7.1 Script m-files . . . 40
7.2 Function m-file . . . 41
7.2.1 Special function variables . . . 43
7.2.2 Local and global variables . . . 44
7.2.3 Indirect function evaluation - *optional* . . . 45
7.3 Scripts vs. functions . . . 46

8 Text **47**

8.1 Character strings . . . 47
8.2 Text input and output . . . 49

9 Cell arrays and structures - *optional* **52**
9.1 Cell arrays . . . 52
9.2 Structures . . . 53

10 Optimizing the performance of MATLAB **code** **54**
10.1 Vectorization - speed-up of computations . . . 55
10.2 Array preallocation . . . 56
10.3 MATLAB'S tricks and tips . . . 56

11 File input/output operations **59**
11.1 Text files . . . 60
11.2 Binary files - *optional* . . . 61

12 Writing and debugging MATLAB **programs** **62**
12.1 Structural programming . . . 62
12.2 Debugging . . . 64
12.3 Recommended programming style . . . 65

Introduction

During this course you will learn how to use MATLAB, to design, and to perform mathematical computations. You will also get acquainted with basic programming. If you learn to use this program well, you will find it very useful in future, since many technical or mathematical problems can be solved using MATLAB.

This text includes all material (with some additional information) that you need to know, however, many things are treated briefly. Therefore, "The Student Edition of Matlab" user's guide should be used as a complementary book during the course. A sign of a book accompanied by page numbers, placed in the left margin, indicates where you can find more information. Read the suggested pages to better understand the concepts discussed. Where no reference to the book appears, it means that only this text explains some ideas, since they might be more about general programming than about MATLAB. Exercises are marked with , also in the left margin. The end of an exercise is marked with ■.

The text of this course includes some optional parts for volunteers or for more advanced students. Such a part is enclosed between two horizontal lines, with the word 'optional' on the first line. There is also an 'intermezzo' part, which is not essential, but provides some important information. If you already know how to program in a language, like Pascal or C, then you may find it easier or more interesting to do the optional parts as well. However, if you do not have any idea about programming, please feel free to skip those fragments. It is important that you spend enough time to learn the MATLAB basics.

Please test after each section whether you have sufficient understanding of the issues discussed. Use the lists provided below.

Sections 1-2.

You should be able to:

- recognize built-in variables;
- define variables and perform computations using them;
- perform basic mathematical operations;
- know how to suppress display with `;` (semicolon);
- use the `format` command to adjust the Command window;
- add and remove variables from the workspace; check which variables are currently present in the workspace;
- use on-line `help` to get more information on a command and know how to use the `lookfor` command;
- use the `load` and `save` commands to read/save data to a file;
- access files at different directories (manipulate `path`-changing commands);

Section 3.

You should be able to:

- create vectors and matrices with direct assignment (using `[ ]`);
- use `linspace` to create vectors;
- create random vectors and matrices;
- create matrices via the commands: `eye` `ones`, `zeros` and `diag`;
- build a larger matrix from smaller ones;
- use colon notation to create vectors and extract ranges of elements from vectors and matrices;
- extract elements from vectors and matrices with subscript notation, e.g. `x(5), A(i,j)`;
- apply transpose operators to vectors and matrices;
- perform legal addition, subtraction, and multiplication operations on vectors and matrices;
- understand the use of dot operators, like `.*`, `./`,... and know why they are different from the regular `*`, `/`,... operators;
- delete elements from vectors and matrices;
- compute inner products and the Euclidean length of vectors;
- create and manipulate complex vectors and matrices.

Section 4.

You should be able to:

- use the `plot` command to make simple plots;
- know how to use `hold on/off`

- plot several functions in one figure either in one graphical window or by creating a few smaller ones (the use of `subplot`);
- add a title, grid and a legend, describe the axes, change the range of axes;
- use logarithmic axes;
- make simple 3D line plots;
- plot surfaces, contours, change colors;
- send figures to the printer or print them to a file;
- **optional:** make some fancy plots;
- **optional:** create MATLAB animations.

Section 5.

You should be able to:

- use relational operators: `<, <=, >, >=, ==, ~=` and logical operators: `&, |` and `~`;
- understand the logical addressing;
- fully understand how to use the command `find`, both on vectors and matrices;
- use `if...end`, `if...elseif...end` and `if...elseif...else...end` and `switch` constructs;
- use `for`-loops and `while`-loops and know the difference between them;
- understand how logical expressions are evaluated in the control flow structures.

Section 6.

You should be able to:

- create and manipulate MATLAB polynomials;
- fit a polynomial to data;
- interpolate the data;
- evaluate a function;
- create inline functions;
- integrate and differentiate a function;
- **optional:** understand how to make approximations of Taylor expansions with the given precision.

Section 7.

You should be able to:

- edit and run an m-file (both functions and scripts);
- identify the differences between scripts and functions;
- understand the concept of local and global variables;
- create a function with one or more input arguments and one or more output arguments;
- use comment statements to document scripts and functions;
- **optional:** use the `feval` command - know how to pass a function name as an input argument to another function.

Sections 8-9.

You should be able to:

- create and manipulate string variables, e.g. compare two strings, concatenate them, find a substring in a string, convert a number/string into a string/number etc;
- use freely and with understanding the text input/output commands: `input`, `disp` and `fprintf`;
- **optional:** operate on cell arrays and structures;

Section 10.

You should be able to:

- preallocate memory for vectors or matrices and know why and when this is beneficial;
- replace basic loops with vectorized operations;
- use colon notation to perform vectorized operations;
- understand the two ways of addressing matrix elements using a vector as an index: traditional and logical indexing;
- use array indexing instead of loops to select elements from a matrix;

- use logical indexing and logical functions instead of loops to select elements from matrices;
- understand MATLAB'S tricks.

Section 11.

You should be able to:

- perform low level input and output with `fopen`, `fscanf` and `fclose`;
- understand how to operate on text files (input/output operations);
- get more understanding on the use of `fprintf` while writing to a file;
- **optional:** understand how to operate on binary files (input/output operations);

Section 12.

You should be able to:

- know and understand the importance of structural programming and debugging;
- know how to debug your program;
- have an idea how to write programs using the recommended programming style.

Preliminaries

Below you find a few basic definitions on computers and programming. Please get acquainted with them since they introduce key concepts needed in the coming sections:

- A *bit* (short for *bi*nary digi*t*) is the smallest unit of information on a computer. A single bit can hold only one of two values: 0 or 1. More meaningful information is obtained by combining consecutive bits into larger units, such as byte.
- A *byte* - a unit of 8 bits, being capable of holding a single character. Large amounts of memory are indicated in terms of kilobytes (1024 bytes), megabytes (1024 kilobytes), and gigabytes (1024 megabytes).
- *Binary system* - a number system that has two unique digits: 0 and 1. Computers are based on such a system, because of its electrical nature (charged versus uncharged). Each digit position represents a different power of 2. The powers of 2 increase while moving from the right most to the left most position, starting from $2^0 = 1$. Here is an example of a binary number and its representation in the decimal system:
$$10110 = 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 1 * 2^1 + 0 * 2^0 = 16 + 0 + 4 + 2 + 0 = 24$$
- *Data* is information represented with symbols, e.g. numbers, words, signals or images.
- A *command* is a instruction to do a specific task.
- An *algorithm* is a sequence of instructions for the solution of a specific task in a finite number of steps.
- A *program* is the implementation of an algorithm suitable for execution by a computer.
- A *variable* is a container that can hold a value. For example, in the expression: `x+y`, `x` and `y` are variables. They can represent numeric values, like `25.5`, characters, like `'c'` or character strings, like `'Matlab'`. Variables make programs more flexible. When a program is executed, the variables are then **replaced** with real data. That is why the same program can process different sets of data.

  Every variable has a name (called the *variable name*) and a *data type*. A variable's data type indicates the sort of value that the variable represents (see below).
- A *constant* is a value that never changes. That makes it the opposite of a variable. It can be a numeric value, a character or a string.
- A *data type* is a classification of a particular type of information. The most basic data types are:
  - *integer*: a whole number; a number that has no fractional part, e.g. 3.
  - *floating-point*: a number with a decimal point, e.g. 3.5 or 1.2e-16 (this stands for $1.2 * 10^{-16}$).
  - *character*: readable text character, e.g. 'p'.
- A *bug* is an error in a program, causing the program to stop running, not to run at all or to provide wrong results. Some bugs can be very subtle and hard to find. The process of finding and removing bugs is called *debugging*.
- A *file* is a collection of data or information that has a name, stored in a computer. There are many different types of files: data files, program files, text files etc.
- An *ASCII file* is a standardized, readable and editable plain text file.
- A *binary file* is a file stored in a format, which is computer-readable but not human-readable. Most numeric data and all executable programs are stored in binary files. MATLAB binary files are those with the extension `'*.mat'`.

1 Getting started with MATLAB

MATLAB is a tool for mathematical (technical) calculations. First, it can be used as a scientific calculator. Next, it allows you to plot or visualize data in many different ways, perform matrix algebra, work with polynomials or integrate functions. Like in a programmable calculator, you can create, execute and save a sequence of commands in order to make your computational process automatic. It can be used to store or retrieve data. In the end, MATLAB can also be treated as a user-friendly programming language, which gives the possibility to handle mathematical calculations in an easy way. In summary, as a computing/programming environment, MATLAB is especially designed to work with data sets as a whole such as vectors, matrices and images. Therefore, PRTOOLS, a toolbox for Pattern Recognition purposes, and DIPLIB, a toolbox for Image Processing, have been developed under MATLAB.

Under Windows, you can start MATLAB by double clicking on the MATLAB icon that should be on the desktop of your computer; on unix systems, type `matlab` at the command line. Running MATLAB creates one or more windows on your screen. The most important is the *Command Window*, which is the place you interact with MATLAB, i.e. it is used to enter commands and display text results. The string `>>` is the MATLAB prompt (or `EDU>>` for the Student Edition). When the Command Window is active, a cursor appears after the prompt, indicating that MATLAB is waiting for your command. MATLAB responds by printing text in the Command Window or by creating a *Figure Window* for graphics. To exit MATLAB use the command `exit` or `quit`.

1.1 Input via the command-line

2-10 MATLAB is an interactive system; commands followed by `Enter` are executed immediately. The results are, if desired, displayed on screen. However, execution of a command will be possible if the command is typed according to the rules. Table 1 shows a list of commands used to solve indicated mathematical equations (a, b, x and y are numbers). Below you find basic information to help you starting with MATLAB :

- Commands in MATLAB are executed by pressing `Enter` or `Return`. The output will be displayed on screen immediately. Try the following:

```
>> 3 + 7.5
>> 18/4
>> 3 *  7
```

  Note that spaces are **not** important in MATLAB.

- The result of the last performed computation is ascribed to the variable `ans`, which is an example of a MATLAB built-in variable. It can be used in the next command. For instance:

```
>> 14/4
ans =
     3.5000
>> ans^(-6)
ans =
   5.4399e-04
```

  `5.4399e-04` is a computer notation of $5.4399 * 10^{-4}$ (see **Preliminaries**). Note that `ans` is always overwritten by the last command.

- You can also define your own variables. Look how the information is stored in the variables `a` and `b`:

```
>> a = 14/4
a =
     3.5000
>> b = a^(-6)
b =
   5.4399e-04
```

  Read **Preliminaries** to better understand the concept of variables. You will learn more on MATLAB variables in section **2.3**.

- When the command is followed by a semicolon '`;`', the output is suppressed. Check the difference between the following expressions:

```
>> 3 + 7.5
>> 3 + 7.5;
```

- It is possible to execute more than one command at the same time; the separate commands should then be divided by commas (to display the output) or by semicolons (to suppress the output display), e.g.:

```
>> sin(pi/4), cos(pi); sin(0)
ans =
      0.7071
ans =
      0
```

  Note that the value of `cos(pi)` is not printed.
- By default, MATLAB displays only 5 digits. The command `format long` increases this number to 15, `format short` reduces it to 5 again. For instance:

```
>> 312/56
ans =
    5.5714
>> format long
>> 312/56
ans =
   5.57142857142857
```

- The output may contain some empty lines; this can be suppressed by the command `format compact`. In contrast, the command `format loose` will insert extra empty lines.

256
- To enter a statement that is too long to be typed in one line, use three periods '`...`' followed by `Enter` or `Return`. For instance:

```
>> sin(1) + sin(2) - sin(3) + sin(4) - sin(5) + sin(6) - ...
   sin(8) + sin(9) - sin(10) + sin(11) - sin(12)
ans =
    1.0357
```

- MATLAB is case sensitive, for example, a is written as `a` in MATLAB; `A` will result then in an error.
- All text after a percent sign `%` until the end of a line is treated as a comment. Enter e.g. the following:

```
>> sin(3.14159)            % this is an approximation of sin(pi)
```

  You will notice that some examples in this text are followed by comments. They are meant for you and you should skip them while typing those examples.
- Previous commands can be fetched back with the $\boxed{\uparrow}$-key. The command can also be changed, the $\boxed{\leftarrow}$ and $\boxed{\rightarrow}$-keys may be used to move around in a line and edit it. In case of a long line, `Ctrl-a` and `Ctrl-e` might be useful; they allow to move the cursor at the beginning or the end of the line, respectively.
- To recall the most recent command starting from e.g. `c`, type `c` at the prompt followed by the $\boxed{\uparrow}$-key. Similarly, `cos` followed by the $\boxed{\uparrow}$-key will find the last command starting from `cos`.

Since MATLAB executes the command immediately, it might be useful to have an idea of the expected outcome. You might be surprised how long it takes to print out a 1000×1000 matrix!

1.2 `help`-facilities

MATLAB provides assistance through extensive online help. The `help` command is the simplest way to get
349-359 help. It displays the list of all possible topics. To get a more general introduction to `help`, try:

```
>> help help
```

If you already know the topic or command, you can ask for a more specified help. For instance:

```
>> help ops
```

gives information on the operators and special characters in MATLAB. The topic you want help on must be exact and spelled correctly. The `lookfor` command is more useful if you do not know the exact name of the command or topic. For example:

Mathematical notation	MATLAB command
$a+b$	`a + b`
$a-b$	`a - b`
ab	`a * b`
$\frac{a}{b}$	`a / b` or `b \ a`
x^b	`x^b`
$\sqrt{x}$	`sqrt(x)` or `x^0.5`
$\|x\|$	`abs(x)`
π	`pi`
$4 \cdot 10^3$	`4e3` or `4*10^3`
i	`i` or `j`
$3-4i$	`3-4*i` or `3-4*j`
$e,\ e^x$	`exp(1)`, `exp(x)`
$\ln x$, $\log x$	`log(x)`, `log10(x)`
$\sin x$, $\arctan x$, ...	`sin(x)`, `atan(x)`,...

Table 1: Translation of mathematical notation to MATLAB commands.

```
>> lookfor inverse
```

displays a list of commands, with a short description, for which the word `inverse` is included in its help-text. You can also use an incomplete name, e.g. `lookfor inv`. Besides the `help` and `lookfor` commands, there is also a separate mouse driven help. The `helpwin` command opens a new window on screen which can be
353-354 browsed in an interactive way.

Exercise 1.

- Is the inverse cosine function, known as cos^{-1} or *arccos*, one of the MATLAB's elementary functions?
- Does MATLAB have a mathematical function to calculate the greatest common divisor?
- Look for information on logarithms.

Use `help` or `lookfor` to find out. ■

1.3 Interrupting a command or program

Sometimes you might spot an error in your command or program. Due to this error it can happen that the command or program does not stop. Pressing `Ctrl-C` (or `Ctrl-Break` on PC) forces MATLAB to stop the process. Sometimes, however, you may need to press a few times. After this the MATLAB prompt (`>>`) re-appears. This may take a while, though.

1.4 Path

In MATLAB, commands or programs are contained in m-files, which are just plain text files and have an
37-39 extension `'.m'`. The m-file must be located in one of the directories which MATLAB automatically searches. The list of these directories can be listed by the command `path`. One of the directories that is always taken into account is the *current working directory*, which can be identified by the command `pwd`. Use `path`, `addpath` and `rmpath` functions to modify the path. It is also possible to access the path browser from the **File** menu-bar, instead.

Exercise 2.
Type `path` to check which directories are placed on your path. Add you personal directory to the path (assuming that you created your personal directory for working with MATLAB). ■

1.5 Workspace issues

If you work in the Command Window, MATLAB memorizes all commands that you entered and all variables that you created. These commands and variables are said to reside in the MATLAB *workspace*. They might be
6 easily recalled when needed, e.g. to recall previous commands, the $\boxed{\uparrow}$-key is used. Variables can be verified
24,25 with the commands `who`, which gives a list of variables present in the workspace, and `whos`, which includes also information on name, number of allocated bytes and class of variables. For example, assuming that you performed all commands from section **1.1**, after typing `who` you should get the following information:

```
>> who
Your variables are:
a         ans       b         x
```

The command `clear <name>` deletes the variable `<name>` from the MATLAB workspace, `clear` or `clear all` removes all variables. This is useful when starting a new exercise. For example:

```
>> clear a x
>> who
Your variables are:
ans       b
```

1.6 Saving and loading data

The easiest way to save or load MATLAB variables is by using (clicking) the **File** menu-bar, and then selecting
26 the **Save Workspace as...** or **Load Workspace...** items respectively. Also MATLAB commands exist which save data to files and which load data from files.

The command `save` allows for saving your workspace variables either into a binary file or an ASCII file (check **Preliminaries** on binary and ASCII files). Binary files automatically get the `'.mat'` extension, which is not true for ASCII files. However, it is recommended to add a `'.txt'` or `.dat` extension.

Exercise 3.
Learn how to use the `save` command by exercising:

```
>> s1 = sin(pi/4);
>> c1 = cos(pi/4); c2 = cos(pi/2);
>> str = 'hello world';         % this is a string
>> save                         % saves all variables in binary format to matlab.mat
>> save data                    % saves all variables in binary format to data.mat
>> save numdata s1, c1          % saves numeric variables s1 and c1 to numdata.mat
>> save strdata str             % saves a string variable str to strdata.mat
>> save allcos.dat c* -ascii    % saves c1,c2 in 8-digit ascii format to allcos.dat
```

■

The `load` command allows for loading variables into the workspace. It uses the same syntax as `save`.

Exercise 4.
Assuming that you have done the previous exercise, try to load variables from the created files. Before each `load` command, clear the workspace and after loading check which variables are present in the workspace (use `who`).

```
>> load                         % loads all variables from the file matlab.mat
>> load data s1 c1              % loads only specified variables from the file data.mat
>> load str data                % loads all variables from the file strdata.mat
```

It is also possible to read ASCII files that contain rows of space separated values. Such a file may contain comments that begin with a percent character. The resulting data is placed into a variable with **the same** name as the ASCII file (without the extension). Check, for example:

```
>> load allcos.dat              % loads data from allcos.dat into variable allcos
>> who                          % lists variables present in the workspace now
```

■

2 Basic syntax and variables

2.1 MATLAB as a calculator

There are three kinds of numbers used in MATLAB: integers, real numbers and complex numbers. In addition,
4-7 MATLAB has representations of the non-numbers: `Inf`, for positive infinity, generated e.g. by `1/0`, and `NaN`,
14-17 Not-a-Number, obtained as a result of the mathematically undefined operations such as `0/0` or ∞ - ∞.
27

You have already got some experience with MATLAB and you know that it can be used as a calculator. To do that you can, for example, simply type:

```
>> (23*17)/7
```

The result will be:

```
ans =
   55.8571
```

MATLAB has six basic arithmetic operations, such as: `+`, `-`, `*`, `/` or `\` (right and left divisions) and `^` (power). Note that the two division operators are different:

```
>> 19/3                          % mathematically: 19/3
ans =
    6.3333
>> 19\3, 3/19                    % mathematically: 3/19
ans =
    0.1579
ans =
    0.1579
```

Basic built-in functions, trigonometric, exponential, etc, are available for a user. Try `help elfun` to get the list of elementary functions.

Exercise 5.
Evaluate the following expressions by hand and use MATLAB to check the answers. Note the difference between the left and right divisors. Use `help` to learn more on commands rounding numbers, such as: `round`, `floor`, `ceil`, etc.

- `2/2 * 3`
- `8 * 5\4`
- `8 * (5\4)`
- `7 - 5 * 4\9`
- `6 - 2/5 + 7^2 - 1`
- `10/2\5 - 3 + 2 * 4`
- `3^2/4`
- `3^2^3`
- `2 + round (6/9 + 3 * 2)/2`
- `2 + floor (6/9 + 3 * 2)/2`
- `2 + ceil (6/9 + 3 * 2)/2`
- `x = pi/3, x = x - 1, x = x + 5, x = abs(x)/x`

■

Exercise 6.
Define the format in MATLAB such that empty lines are suppressed and the output is given with 15 digits. Calculate:

```
>> pi
>> sin(pi)
```

Note that the answer is not exactly `0`. Use the command `format` to put MATLAB in its standard-format. ■

INTERMEZZO

2.2 An introduction to floating-point numbers

286 In a computer, numbers can be represented only in a *discrete* form. It means that numbers are stored within a limited range and with a finite precision. Integers can be represented *exactly* with the base of 2 (read **Preliminaries** on bits and the binary system). The typical size of an integer is 16 bits, so the largest positive integer, which can be stored, is $2^{16} = 65536$. If negative integers are permitted, then 16 bits allow for representing integers between -32768 and 32767. Within this range, operations defined on the set of integers can be performed exactly.

However, this is not valid for other real numbers. In practice, computers are integer machines and are capable of representing real numbers only by using complicated codes. The most popular code is the *floating point* standard. The term floating point is derived from the fact that there is no fixed number of digits before and after the decimal point, meaning that the decimal point can float. Note that most floating-point numbers that a computer can represent are just approximations. Therefore, care should be taken that these approximations lead to reasonable results. If a programmer is not careful, small discrepancies in the approximations can cause meaningless results. Note the difference between e.g. the integer arithmetic and floating-point arithmetic:

Integer arithmetic:

```
2 + 4 =  6
3 * 4 = 12
25/11 =  2
```

Floating-point arithmetic

```
      18/7 =  2.5714
2.5714 * 7 = 17.9998
   10000/3 = 3.3333e+03
```

When describing floating-point numbers, precision refers to the number of bits used for the fractional part. The larger the precision, the more exact fractional quantities can be represented. Floating-point numbers are often classified as single precision or double precision. A double-precision number uses twice as many bits as a single-precision value, so it can represent fractional values much better. However, the precision itself is not double. The extra bits are also used to increase the range of magnitudes that can be represented.

408-410 MATLAB relies on a computer's floating point arithmetic. You could have noticed that in the last exercise since the value of $\sin(\pi)$ was almost zero, and not completely zero. It came from the fact that both the value of π is represented with a finite precision and the sin function is also approximated.

The fundamental type in MATLAB is `double`, which stands for a representation with a double precision. It uses 64 bits. The single precision obtained by using the `single` type offers 32 bits. Since most numeric operations require high accuracy the `double` type is used by default. This means, that when the user is inputting integer values in MATLAB (for instance, `k = 4`), the data is still stored in `double` format.

The relative accuracy might be defined as the smallest positive number ϵ that added to 1, creates the result larger than 1, i.e. $1+\epsilon > 1$. It means that in floating-point arithmetic, for positive values smaller than ϵ, the result equals to 1 (in exact arithmetic, of course, the result is always larger than 1). In MATLAB, ϵ is stored in the built-in variable `eps` $\approx$ 2.2204e-16. This means that the relative accuracy of individual arithmetic operations is about 15 digits.

END INTERMEZZO

2.3 Assignments and variables

Working with complex numbers is easily done with MATLAB.

Exercise 7.
Choose two complex numbers, for example `-3 + 2i` and `5 - 7i`. Add, subtract, multiply, and divide these two numbers. ■

During this exercise, the complex numbers had to be typed four times. To reduce this, assign each number to a variable. For the previous exercise, this results in:

4-6
10-12

```
>> z  = -3 + 2*i; w = 5 - 7*i;
>> y1 = z + w; y2 = z - w;
>> y3 = z * w;
>> y4 = z / w; y5 = w \ z;
```

Formally, there is no need to declare (i.e. define the name, size and the type of) a new variable in MATLAB. A variable is simply created by an assignment (e.g. `z = -3 + 2*i`), i.e. values are assigned to variables. Each newly created numerical variable is *always* of the `double` type, i.e. real numbers are approximated with the highest possible precision. You can change this type by converting it into e.g. the `single` type[1]. In some cases, when huge matrices should be handled and precision is not very important, this might be a way to proceed. Also, when only integers are taken into consideration, it might be useful to convert the `double` representations into e.g. `int32`[1] integer type. Note that integer numbers are represented exactly, no matter which numeric type is used, as long as the number can be represented in the number of bits used in the numeric type.

Bear in mind that *undefined* values cannot be assigned to variables. So, the following is not possible:

```
>> clear x;                     % to make sure that x does not exist
>> f = x^2 + 4 * sin(x)
```

It becomes possible by:

```
>> x = pi / 3;  f = x^2 + 4 * sin(x)
```

Variable name begins with a letter, followed by letters, numbers or underscores. MATLAB recognizes only first 31 characters of the name.

Exercise 8.
Here are some examples of different types of MATLAB variables. You do not need to understand them all now, since you will learn more about them during the course. Create them manually in MATLAB:

[1]a variable `a` is converted into a different type by performing e.g. `a = single(a)`, `a = int32(a)` etc.

Variable name	Value/meaning
`ans`	the default variable name used for storing the last result
`pi`	$\pi = 3.14159...$
`eps`	the smallest positive number that added to 1, creates a result larger than 1
`inf`	representation for positive infinity, e.g. `1/0`
`nan` or `NaN`	representation for `not-a-number`, e.g. `0/0`
`i` or `j`	$i = j = \sqrt{-1}$
`nargin/nargout`	number of function input/output arguments used
`realmin/realmax`	the smallest/largest usable positive real number

Table 2: Built-in variables in MATLAB.

```
>> this_is_my_very_simple_variable_today = 5  % check what happens; the name is very long
>> 2t = 8                                     % what is the problem with this command?
>> M  = [1 2; 3 4; 5 6]                       % a matrix
>> c  = 'E'                                   % a character
>> str = 'Hello world'                        % a string
>> m   = ['J','o','h','n']                    % try to guess what it is
```

Check the types by using the command `whos`. Use `clear <name>` to remove a variable from the workspace. ■

As you already know, MATLAB variables can be created by an assignment. There is also a number of built-in variables, e.g. `pi`, `eps` or `i`, summarized in Table 2. In addition to creating variables by assigning values to
7,45,84 them, another possibility is to copy one variable, e.g. `b` into another, e.g. `a`. In this way, the variable `a` is automatically created (if `a` already existed, its previous value is lost):

```
>> b = 10.5;
>> a = b;
```

A variable can be also created as a result of the evaluated expression:

```
>> a = 10.5;  c = a^2 + sin(pi*a)/4;
```

or by loading data from text or `'*.mat'` files.

If `min` is the name of a function (see `help min`), then `a` defined, e.g. as:

```
>> b = 5; c = 7;
>> a = min (b,c);              % create a as the minimum of b and c
```

will call that function, with the values `b` and `c` as parameters. The result of this function (its return value) will be written (assigned) into `a`. So, variables can be created as results of the execution of built-in or user-defined functions (you will learn more how to built own functions in section **7.2**).

Important: do not use variable names which are defined as function names (for instance `mean` or `error`)[2]! If you are going to use a suspicious variable name, use `help <name>` to find out if the function already exists.

3 Mathematics with vectors and matrices

The basic element of MATLAB is a matrix (or an array). Special cases are:

- a 1×1-matrix: a scalar or a single number;
- a matrix existing only of one row or one column: a vector.

Note that MATLAB may behave differently depending on the input, whether it is a number, a vector or a 2D matrix.

3.1 Vectors

Row vectors are lists of numbers separated either by commas or by spaces. They are examples of simple *arrays*.
41-49 First element has index 1. The number of entries is known as the *length* of the vector (the command `length` exists as well). Their entities are referred to as *elements* or *components*. The entries must be enclosed in `[ ]`:

[2]There is always one exception of the rule: variable `i` is often used as counter in a loop, while it is also used as $i = \sqrt{-1}$.

```
>> v = [-1 sin(3) 7]
v =
   -1.0000    0.1411    7.0000
>> length(v)
ans =
     3
```

A number of operations can be done on vectors. A vector can be multiplied by a scalar, or added/subtracted to/from another vector with *the same* length, or a number can be added/subtracted to/from a vector. All these operations are carried out element-by-element. Vectors can be also built from the already existing ones.

```
>> v = [-1 2 7]; w = [2 3 4];
>> z = v + w                    % an element-by-element sum
z =
     1     5    11
>> vv = v + 2                   % add 2 to all elements of vector v
vv =
     1     4     9
>> t = [2*v, -w]
ans =
    -2     4    14    -2    -3    -4
```

Also, a particular value can be changed or displayed:

```
>> v(2) = -1                    % change the 2nd element of v
v =
    -1    -1     7
>> w(2)                         % display the 2nd element of w
ans =
     3
```

3.1.1 Colon notation and extracting parts of a vector

A colon notation is an important shortcut, used when producing row vectors (see Table 3 and `help colon`):

```
>> 2:5
ans =
     2     3     4     5
>> -2:3
ans =
    -2    -1     0     1     2     3
```

In general, `first:step:last` produces a vector of entities with the value `first`, incrementing by the `step` until it reaches `last`:

```
>> 0.2:0.5:2.4
ans =
    0.2000    0.7000    1.2000    1.7000    2.2000
>> -3:3:10
ans =
    -3     0     3     6     9
>> 1.5:-0.5:-0.5                % negative step is also possible
ans =
    1.5000    1.0000    0.5000         0   -0.5000
```

Parts of vectors can be extracted by using a colon notation:

```
>> r = [-1:2:6, 2, 3, -2]       % -1:2:6  =>  -1  1  3  5
r =
    -1     1     3     5     2     3    -2
```

```
>> r(3:6)                        % get elements of r which are on the positions from 3 to 6
ans =
     3     5     2     3
>> r(1:2:5)                      % get elements of r which are on the positions 1, 3 and 5
ans =
    -1     3     2
>> r(5:-1:2)                     % what will you get here?
```

3.1.2 Column vectors and transposing

To create *column* vectors, you should separate entries by by new lines or by a semicolon ';':

```
>> z = [1
        7
        7];
z =
     1
     7
     7
>> u = [-1; 3; 5]
u =
    -1
     3
     5
```

The same operations as on row vectors can be perform on column vectors. However, you cannot for example add a column vector to a row vector. To do that, you need an operation called *transposing*, which converts a column vector into a row vector and vice versa:

```
>> u'                            % u is a column vector and u' is a row vector
ans =
    -1     3     5
>> v = [-1 2 7];                 % v is a row vector
>> u + v                         % you cannot add a column vector u to a row vector v
??? Error using ==> +
Matrix dimensions must agree.
>> u' + v
ans =
    -2     5    12
>> u + v'
ans =
    -2
     5
    12
```

If `z` is a complex vector, then `z'` gives the conjugate transpose of `z`, e.g.:

```
>> z = [1+2i, -1+i]
z =
   1.0000 + 2.0000i  -1.0000 + 1.0000i
>> z'                            % this is the conjugate transpose
ans =
   1.0000 - 2.0000i
  -1.0000 - 1.0000i
>> z.'                           % this is the traditional transpose
ans =
   1.0000 + 2.0000i
  -1.0000 + 1.0000i
```

3.1.3 Product, divisions and powers of vectors

You can now compute the inner product between two vectors x and y of the same length, $\mathrm{x}^T \mathrm{y} = \sum_i \mathrm{x}_i \mathrm{y}_i$, in a simple way:

```
>> u = [-1; 3; 5]          % a column vector
>> v = [-1; 2; 7]          % a column vector
>> u * v                   % you cannot multiply a column vector by a column vector
??? Error using ==> *
Inner matrix dimensions must agree.
>> u' * v                  % this is the inner product
ans =
     42
```

An another way to compute the inner product is by the use of the dot product, i.e. `.*`, which performs alement-wise multiplication. For two vectors x and y, of the same length, it is defined as a vector $[\mathrm{x}_1\mathrm{y}_1, \mathrm{x}_2\mathrm{y}_2, \ldots, \mathrm{x}_n\mathrm{y}_n]$, thus, the corresponding elements of two vectors are multiplied. For instance:

```
>> u .* v                  % this is an element-by-element multiplication
     1
     6
    35
>> sum(u.*v)               % this is an another way to compute the inner product
ans =
    42
>> z = [4 3 1];            % z is a row vector
>> sum(u'.*z)              % this is the inner product
ans =
    10
>> u'*z'                   % since z is a row vector, u'*z' is the inner product
ans =
    10
```

You can now tabulate easily values of a function for a given list of arguments. For instance:

```
>> x = 1:0.5:4;
>> y = sqrt(x) .* cos(x)
y =
    0.5403    0.0866   -0.5885   -1.2667   -1.7147   -1.7520   -1.3073
```

Mathematically, it is not defined how to divide one vector by another. However, in MATLAB, the operator `./` is defined to perform an element-by-element division. It is, therefore, defined for vectors of the same size and type:

```
>> x = 2:2:10
x =
     2     4     6     8    10
>> y = 6:10
y =
     6     7     8     9    10
>> x./y
ans =
    0.3333    0.5714    0.7500    0.8889    1.0000
>> z = -1:3
z =
    -1     0     1     2     3
>> x./z                    % division 4/0, resulting in Inf
Warning: Divide by zero.
ans =
   -2.0000       Inf    6.0000    4.0000    3.3333
>> z./z                    % division 0/0, resulting in NaN
Warning: Divide by zero.
```

```
ans =
     1   NaN     1     1     1
```

The operator `./` can be also used to divide a scalar by a vector:

```
>> x=1:5; 2/x                % this is not possible
??? Error using ==> /
Matrix dimensions must agree.
>> 2./x                      % but this is!
ans =
    2.0000    1.0000    0.6667    0.5000    0.4000
```

Exercise 9.
Become acquainted with operating on row and column vectors. Perform, for instance:

- Create a vector consisting of the even numbers between 21 and 47.
- Let x = [4 5 9 6].
  - Subtract 3 from each element.
  - Add 11 to the odd-index elements.
  - Compute the square root of each element.
  - Raise to the power 3 each element.
- Create a vector `x` with the elements:
  - `2, 4, 6, 8, ...`
  - `9, 7, 5, 3, 1, -1, -3, -5`
  - `1, 1/2, 1/3, 1/4, 1/5,...`
  - `0, 1/2, 2/3, 3/4, 4/5,...`
- Create a vector `x` with the elements: $\mathtt{x_n} = \frac{(-1)^n}{2n-1}$ for $n = 1, 2, 3, ...$ Find the sum of the 100-element vector.
- Given a vector `t`, write down the MATLAB expressions that will compute:
  - $\mathtt{ln}\,(2 + \mathtt{t} + \mathtt{t}^2)$
  - $\mathtt{cos}\,(\mathtt{t})^2 - \mathtt{sin}\,(\mathtt{t})^2$
  - $\mathtt{e}^{\mathtt{t}}\,(1 + \mathtt{cos}\,(3\mathtt{t}))$
  - $\mathtt{tan}^{-1}(\mathtt{t})$

  Test them for t = 1 : 0.2 : 2.
- Given x = [2 1 3 7 9 4 6], explain what the following commands do (note that `x(end)` points to the last element of `x`):
  - `x(3)`
  - `x(1:7)`
  - `x(1:end)`
  - `x(1:end-1)`
  - `x(2:2:6)`
  - `x(6:-2:1)`
  - `x(end-2:-3:2)`
  - `sum(x)`
  - `mean(x)`
  - `min(x)`
- Let x = [1 + 3i, 2 − 2i] be a complex vector. Check the following expressions:
  - $\mathtt{x}'$
  - $\mathtt{x}.'$
  - $\mathtt{x} * \mathtt{x}'$
  - $\mathtt{x} * \mathtt{x}.'$

■

Exercise 10.
Use the knowledge on computing the inner product to find:

1. the Euclidean length of the vector x = [2 1 3 7 9 4 6].
2. the angle between two column vectors, which is defined as $\cos\alpha = \frac{\mathtt{x}^T\mathtt{y}}{||\mathtt{x}||\,||\mathtt{y}||}$. Compute an angle between the vectors:
   - x = [3 2 1] and y = [1 2 3]
   - x = 1 : 5 and y = 6 : 10

■

Command	Result
`A(i,j)`	A_{ij}
`A(:,j)`	j-th column of A
`A(i,:)`	i-th row of A
`A(k:l,m:n)`	$(l-k+1) \times (n-m+1)$ matrix with elements A_{ij} with $k \leq i \leq l$, $m \leq j \leq n$
`v(i:j)'`	'vector-part' $(v_i, v_{i+1}, \ldots, v_j)$ of vector v

Table 3: Manipulation of (groups of) matrix elements.

Command	Result
`n = rank(A)`	n becomes the rank of matrix A
`x = det(A)`	x becomes the determinant of matrix A
`x = size(A)`	x becomes a row-vector with 2 elements: the number of rows and columns of A
`x = trace(A)`	x becomes the trace (sum of diagonal elements) of matrix A
`x = norm(v)`	x becomes the Euclidean length of vector v
`C = A + B`	sum of two matrices
`C = A - B`	subtraction of two matrices
`C = A * B`	multiplication of two matrices
`C = A .* B`	'element-by-element' multiplication (A and B are of equal size)
`C = A ^ k`	power of a matrix ($k \in Z$; can also be used for A^{-1})
`C = A .^ k`	'element-by-element' power of a matrix
`C = A'`	the transposed of a matrix; A^T
`C = A ./ B`	'element-by-element' division (A and B are of equal size)
`X = A \ B`	finds the solution in the least squares sense to the system of equations $AX = B$
`X = B / A`	finds the solution of $XA = B$, analogous to the previous command
`C = inv(A)`	C becomes the inverse of A
`C = null(A)`	C is an orthonormal basis for the null space of A obtained from the singular value decomposition
`C = orth(A)`	C is an orthonormal basis for the range of A
`C = rref(A)`	C is the reduced row echelon form of A
`L = eig(A)`	L is a vector containing the (possibly complex) eigenvalues of a square matrix A
`[X,D] = eig(A)`	produces a diagonal matrix D of eigenvalues and a full matrix X whose columns are the corresponding eigenvectors of A
`S = svd(A)`	S is a vector containing the singular values of A
`[U,S,V] = svd(A)`	S is a diagonal matrix with nonnegative diagonal elements in decreasing order; columns of U and V are the accompanying singular vectors
`x = linspace(a,b,n)`	generates a vector x of n equally spaced points between a and b
`x = logspace(a,b,n)`	generates a vector x starting at 10^a and ended at 10^b containing n values
`A = eye(n)`	A is an $n \times n$ identity matrix
`A = zeros(n,m)`	A is an $n \times m$ matrix with zeros (default $m = n$)
`A = ones(n,m)`	A is an $n \times m$ matrix with ones (default $m = n$)
`A = diag(v)`	gives a diagonal matrix with the elements $v_1, v_2, \ldots, v_n$ on the diagonal
`X = tril(A)`	X is lower triangular part of A
`X = triu(A)`	X is upper triangular part of A
`A = rand(n,m)`	A is an $n \times m$ matrix with elements uniformly distributed between 0 and 1
`A = randn(n,m)`	ditto - with elements standard normal distributed
`v = max(A)`	v is a vector with the maximum value of the elements in each column of A or v is the maximum of all elements if A is a vector
`v = min(A)`	ditto - with minimum
`v = sum(A)`	ditto - with sum

Table 4: Frequently used matrix operations and functions.

3.2 Matrices

Row and column vectors are special types of matrices. An $n \times k$ matrix is a rectangular array of numbers having n rows and k columns. Defining a matrix in MATLAB is similar to defining a vector. The generalization
49-61 is straightforward, if you see that a matrix consists of row vectors (or column vectors). Commas or spaces are
75-86 used to separate elements in a row, and semicolons are used to separate individual rows. For example, the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is defined as:

```
>> A = [1 2 3; 4 5 6; 7 8 9]    % row by row input
A =
   1   2   3
   4   5   6
   7   8   9
```

Another examples are, for instance:

```
>> A2 = [1:4; -1:2:5]
A2 =
     1     2     3     4
    -1     1     3     5
>> A3 = [1  3
        -4 7]
A3 =
     1     3
    -4     7
```

From that point of view, a row vector is a $1 \times k$ matrix and a column vector is an $n \times 1$ matrix. Transposing a vector changes it from a row to a column or the other way around. This idea can be extended to a matrix, where transposing interchanges rows with the corresponding columns, as in the example:

```
>> A2
A2 =
     1     2     3     4
    -1     1     3     5
>> A2'                          % transpose of A2
ans =
     1    -1
     2     1
     3     3
     4     5
>> size(A2)                     % returns the size (dimensions) of A2: 2 rows, 4 columns
ans =
     2     4
>> size(A2')
ans =
     4     2
```

3.2.1 Special matrices

There is a number of built-in matrices of size specified by the user (see Table 4). A few examples are given below:

```
>> E = []                       % an empty matrix of 0-by-0 elements!
E =
     []
>> size(E)
ans =
     0     0
>> I = eye(3);                  % the 3-by-3 identity matrix
I =
```

```
     1     0     0
     0     1     0
     0     0     1
>> x = [2; -1; 7]; I*x         % I is such that for any 3-by-1 x holds I*x = x
ans =
     2
    -1
     7
>> r = [1 3 -2]; R = diag(r)   % create a diagonal matrix with r on the diagonal
R =
     1     0      0
     0     3      0
     0     0     -2
>> A = [1 2 3; 4 5 6; 7 8 9];
>> diag(A)                     % extracts the diagonal entries of A
ans =
     1
     5
     9
>> B = ones(3,2)
B =
     1     1
     1     1
     1     1
>> C = zeros (size(C'))        % a matrix of all zeros of the size given by C'
C =
     0     0     0
     0     0     0
>> D = rand(2,3)               % a matrix of random numbers; you will get a different one!
D =
    0.0227    0.9101    0.9222
    0.0299    0.0640    0.3309
>> v = linspace(1,2,4)         % a vector is also an example of a matrix
v =
    1.0000    1.3333    1.6667    2.0000
```

3.2.2 Building matrices and extracting parts of matrices

It is often needed to build a larger matrix from the smaller ones:

```
>> x = [4; -1], y = [-1 3]
x =
     4
    -1
y =
    -1     3
>> X = [x y']                  % X consists of the columns x and y'
X =
     4    -1
    -1     3
>> T = [ -1 3 4; 4 5 6]; t = 1:3;
>> T = [T; t]                  % add to T a new row, namely the row vector t
T =
    -1     3     4
     4     5     6
     1     2     3
>> G  = [1 5; 4 5; 0 2];       % G is a matrix of the 3-by-2 size; check size(G)
>> T2 = [T G]                  % concatenate two matrices
T2 =
```

```
    -1     3     4     1     5
     4     5     6     4     5
     1     2     3     0     2
>> T3 = [T; G ones(3,1)]       % G is 3-by-2, T is 3-by-3
T3 =
    -1     3     4
     4     5     6
     1     2     3
     1     5     1
     4     5     1
     0     2     1
>> T3 = [T; G'];               % this is also possible; what do you get here?
>> [G' diag(5:6); ones(3,2) T] % you can concatenate many matrices
ans =
     1     4     0     5     0
     5     5     2     0     6
     1     1    -1     3     4
     1     1     4     5     6
     1     1     1     2     3
```

A part can be extract from a matrix in a similar way as from a vector. Each element in the matrix is indexed by a row and a column to which it belongs. Mathematically, the element from the i-th row and the j-th column of the matrix A is denoted by A_{ij}; MATLAB provides the `A(i,j)` notation.

```
>> A = [1:3; 4:6; 7:9]
A =
     1     2     3
     4     5     6
     7     8     9
>> A(1,2), A(2,3), A(3,1)
ans =
     2
ans =
     6
ans =
     7
>> A(4,3)                      % this is not possible: A is a 3-by-3 matrix!
???  Index exceeds matrix dimensions.
>> A(2,3) = A(2,3) + 2*A(1,1)  % change the value of A(2,3)
A =
     1     2     3
     4     5     8
     7     8     9
```

It is easy to extend a matrix automatically. For the matrix A it can be done e.g. as follows:

```
>> A(5,2) = 5                  % assign 5 to the position (5,2); the uninitialized
A =                            % elements become zeros
     1     2     3
     4     5     8
     7     8     9
     0     0     0
     0     5     0
```

If needed, the other zero elements of the matrix `A` can be also defined, by e.g.:

```
>> A(4,:) = [2, 1, 2];         % assign vector [2, 1, 2] to the 4th row of A
>> A(5,[1,3]) = [4, 4];        % assign: A(5,1) = 4 and A(5,3) = 4
>> A                           % how does the matrix A look like now?
```

Different parts of the matrix A can be now extracted:

```
>> A(3,:)                        % extract the 3rd row of A
ans =
     7     8     9
>> A(:,2)                        % extract the 2nd column of A
ans =
     2
     5
     8
     1
     5
>> A(1:2,:)                      % extract the rows 1st and 2nd of A
ans =
     1     2     3
     4     5     8
>> A([2,5],1:2)                  % extract a part of A
ans =
     4     5
     4     5
```

As you have seen in the examples above, it is possible to manipulate (groups of) matrix-elements. The commands are shortly explained in Table 3.

The concept of an empty matrix [] is also very useful in MATLAB. For instance, a few columns or rows can be removed from a matrix by assigning an empty matrix to it. Try for example:

```
>> C = [1 2 3 4; 5 6 7 8; 1 1 1 1];
>> D = C; D(:,2) = []            % now a copy of C is in D; remove the 2nd column of D
>> C ([1,3],:) = []              % remove the rows 1 and 3 from C
```

Exercise 11.
Clear all variables. Define the matrix: `A = [1:4; 5:8; 1 1 1 1]`. Predict and check the result of:

- `x = A(:,3)`
- `B = A(1:3,2:2)`
- `A(1,1) = 9 + A(2,3)`
- `A(2:3,1:3) = [0 0 0;0 0 0]`
- `A(2:3,1:2) = [1 1;3 3]`
- `y = A(3:3,1:4)`
- `A = [A;2 1 7 7;7 7 4 5]`
- `C = A([1,3],2)`
- `D = A([2,3,5],[1,3,4])`
- `D(2,:) = []`

■

Exercise 12.
Define the matrices: `T = [ 3 4; 1 8; -4 3]; A = [diag(-1:2:3) T; -4 4 1 2 1]`. Perform the following operations on the matrix A:

- extract a vector consisting of the 2nd and 4th elements of the 3rd row
- find the minimum of the 3rd column
- find the maximum of the 2nd row
- compute the sum of the 2nd column
- compute the mean of the 1st and 4th rows
- extract the submatrix consisting of the 1st and 3rd rows and all columns
- extract the submatrix consisting of the 1st and 2nd rows and the 3rd, 4th and 5th columns
- compute the total sum of the 1st and 2nd rows
- add 3 to all elements of the 2nd and 3rd columns

■

Exercise 13.
Let `A = [2 4 1; 6 7 2; 3 5 9]`. Provide the commands which:

- assign the first row of `A` to a vector `x`;
- assign the last 2 rows of `A` to a vector `y`;
- add up the columns of `A`;
- add up the rows of `A`;
- compute the standard error of the mean of each column of `A` (i.e. the standard deviation divided by the square root of the number of elements used to compute the mean).

■

☞ **Exercise 14.**
Let `A = [2 7 9 7; 3 1 5 6; 8 1 2 5]`. Explain the results or perform the following commands:

- `A'`
- `A(1,:)'`
- `A(:,[14])`
- `A([23],[31])`
- `reshape(A,2,6)`
- `A(:)`
- `flipud(A)`
- `fliplr(A)`
- `[A A(end,:)]`
- `[A; A(1:2,:)]`
- `sum(A)`
- `sum(A')`
- `mean(A)`
- `mean(A')`
- `sum(A,2)`
- `mean(A,2)`
- `min(A)`
- `max(A')`
- `min(A(:,4))`
- `[min(A)' max(A)']`
- `max(min(A))`
- `[[A; sum(A)] [sum(A,2); sum(A(:))]]`
- assign the even-numbered columns of `A` to an array `B`
- assign the odd-numbered rows to an array `C`
- convert `A` into a 4-by-3 array
- compute the reciprocal of each element of `A`
- compute the square-root of each element of `A`
- remove the second column of `A`
- add a row of all 1's at the beginning and at the end
- swap the 2nd row and the last row

■

3.2.3 Operations on matrices

Table 4 (page 15) shows some frequently used matrix operations and functions. The important ones are dot operations on matrices, matrix-vector products and matrix-matrix products. In the class of the dot operations, there are dot product, dot division and dot power. Those operations work as for vectors: they address matrices in the element-by-element way, therefore they can be performed on matrices of the same sizes. They also allow for scalar-matrix operations. For the dot product or division, corresponding elements are multiplied together or divided one by another. A few examples of basic operations are provided below:

```
>> B = [1 -1 3; 4 0 7]
B =
     1    -1     3
     4     0     7
>> B2 = [1 2; 5 1; 5 6];
>> B  = B + B2'                % add two matrices; why B2' is needed instead of B2?
B =
     2     4     8
     6     1    13
>> B-2                         % subtract 2 from all elements of B
ans =
     0     2     6
     4    -1    11
>> ans = B./4                  % divide all elements of the matrix B by 4
ans =
    0.5000    1.0000    2.0000
    1.5000    0.2500    3.2500
>> 4/B                         % this is not possible
??? Error using ==> /
Matrix dimensions must agree.

>> 4./B                        % this is possible; equivalent to: 4.*ones(size(B)) ./ B
ans =
    2.0000    1.0000    0.5000
    0.6667    4.0000    0.3077
>> C = [1 -1 4; 7 0 -1];
>> B .* C                      % multiply element-by-element
ans =
     2    -4    32
    42     0   -13
>> ans.^3 - 2                  % do for all elements: raise to the power 3 and subtract 2
ans =
         6    -66    32766
    74086     -2    -2199
```

```
>> ans ./ B.^2                  % element-by-element division
ans =
    0.7500   -1.0312   63.9961
  342.9907   -2.0000   -1.0009
>> r = [1 3 -2]; r * B2       % this is a legal operation: r is a 1-by-3 matrix and B2 is
ans =                         % 3-by-2 matrix;  B2 * r is an illegal operation
     6    -7
```

Concerning matrix-vector and matrix-matrix products, two things should be reminded. First, an $n \times k$ matrix A (having n rows and k columns) can be multiply by a $k \times 1$ (column) vector x, resulting in a column $n \times 1$ vector y, i.e.: $A\dot{x} = y$ such that $y_i = \sum_{p=1}^{k} A_{ip}\, x_p$. You can also multiply a row $1 \times n$ vector by A, resulting in a row vector $1 \times k$. Second, an $n \times k$ matrix A can be multiply by a matrix B, only if B has k rows, i.e. B is $k \times m$ (m is arbitrary). As a result, you get $n \times m$ matrix C, such that $A\dot{B} = C$ and $C_{ij} = \sum_{p=1}^{k} A_{ip}\, B_{pj}$.

```
>> b = [1 3 -2];
>> B = [1 -1 3; 4 0 7]
B =
     1    -1     3
     4     0     7
>> b * B                      % not possible: b is 1-by-3 and B is 2-by-3
??? Error using ==> *
Inner matrix dimensions must agree.

>> b * B'                     % this is possible: a row vector multiplied by a matrix
ans =
    -8   -10
>> B' *ones(2,1)
ans =
     5
    -1
    10
>> C = [3 1; 1 -3];
>> C * B
ans =
     7    -3    16
   -11    -1   -18
>> C.^3                       % this is element-by-element power
ans =
    27     1
     1   -27
>> C^3                        % this is equivalent to C*C*C
ans =
    30    10
    10   -30
>> ones(3,4)./4 * diag(1:4)
ans =
    0.2500    0.5000    0.7500    1.0000
    0.2500    0.5000    0.7500    1.0000
    0.2500    0.5000    0.7500    1.0000
```

Exercise 15.
Perform all operations from Table 4, using chosen matrices `A` and `B`, vector `v` and scalars `k`, `a`, `b`, `n`, and `m`. ■

Exercise 16.
Let `A` be a square matrix.

1. Create a matrix `B`, whose elements are the same as those of `A` except the entries on the main diagonal. The diagonal of `B` should consist of 1s.
2. Create a tridiagonal matrix `T`, whose three diagonal are taken from the matrix `A`. *Hint:* you may use the commands `triu` and `tril`.

■

Symbol	Color	Symbol	Line style
`r`	red	`.`, `o`	point, circle
`g`	green	`*`	star
`b`	blue	`x`, `+`	x-mark, plus
`y`	yellow	`-`	solid line
`m`	magenta	`--`	dash line
`c`	cyan	`:`	dot line
`k`	black	`-.`	dash-dot line

Table 5: Plot colors and styles.

Exercise 17.
Given the vectors `x` = [1 3 7], `y` = [2 4 2] and the matrices `A` = [3 1 6; 5 2 7], `B` = [1 4; 7 8; 2 2], determine which of the following statements will correctly execute (and if not, try to understand why) and provide the result:

- `x + y`
- `x + A`
- `x' + y`
- `A - [x' y']`
- `[x; y] + A`
- `[x; y']`
- `[x; y]`
- `A - 3`
- `A + B`
- `B' + A`
- `B * A`
- `A .* B`
- `A' .* B`
- `2 * B`
- `2 .* B`
- `B./x'`
- `B./[x' x']`
- `2/A`
- `ones(1,3) * A`
- `ones(1,3) * B`

■

Exercise 18.
76-79 Let A be a random 3×3 matrix, and b a random 3×1 vector. Given that $Ax = b$, then $x = A^{-1}b$. Find x (there exist at least two possible commands to do this; look at Table 4, page 15). Having found `x`, check whether $Ax - b$ is close to a zero vector. ■

Exercise 19.
Let A be a random 5×5 matrix, and b a random 5×1 vector. Given that $Ax = b$, try to find `x`. Explain what is the diference between the operators \, / and the command `inv`. ■

Exercise 20.
Let `A = ones(6) + eye(6)`. Normalize columns of the matrix `A` so that all columns of the resulting matrix, say `B`, have the Euclidean norm (length) equal to 1. Then, find the angles between consecutive columns of the matrix `B`. ■

Exercise 21.
Find two 2×2 matrices A and B which fulfill that `A.*B` $\neq$ `A*B`. Then, try to find *all* 2×2 matrices A and B for which the following holds: `A.*B` = `A*B` (make use one or more from the following operations: `/`, \, or the command `inv`). ■

4 Visualization

MATLAB can be used to visualize the results of an experiment. Therefore, you should define variables, each of them containing all values of one parameter to plot.

4.1 Simple plots

184 With the command `plot` a graphical display can be made. `plot(y)` draws the points $[1, y(1)]$, $[2, y(2)]$, …, $[n, y(n)]$ and connects the points with a straight line. `plot(x,y)` does the same for $[x(1), y(1)]$, $[x(2), y(2)]$, …, $[x(n), y(n)]$. `x` and `y` have to be row- or column vectors of equal length.

The commands `loglog`, `semilogx` and `semilogy` are similar to `plot`, except that they use logarithmic axes.

Exercise 22.
Type the following commands after predicting the result:

```
>> x = 0:10;
>> y = 2.^x;                % this is the same as y = [1 2 4 8 16 32 64 128 256 512 1024]
>> plot(x,y)                % to get a graphic representation
>> semilogy(x,y)            % to get logarithmic axes
```

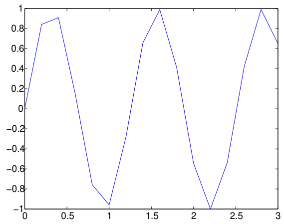
Command	Result
`grid on/off`	adds a grid to the plot at the tick marks or removes it
`axis([xmin xmax ymin ymax])`	sets the minimum and maximum values of the axes
`box off/on`	removes the axes box or shows it
`xlabel('text')`	plots the label text on the x axis
`ylabel('text')`	plots the label text on the y axis
`title('text')`	plots a title above the graph
`text(x,y,'text')`	adds text at the point (x,y)
`gtext('text')`	adds text at a manually (with a mouse) indicated point
`legend('fun1','fun2')`	plots a legend box (move it with your mouse) to name your functions
`legend off`	deletes the legend box
`clf`	clear the current figure
`subplot`	create a subplot in the current figure

Table 6: Plot-manipulations.

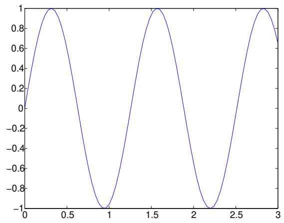
As you can see, the same figure is used for both plot commands. The previous function is removed as soon as the next is displayed. The command `figure` gives you an extra figure. Repeat the previous commands, but generate a new figure before plotting the second function, so that you can see both functions in separate windows. You can also switch back to a figure using `figure(n)`, where `n` is its number. ■

To plot a graph of a function, it is important to sample the function sufficiently well. Compare the following examples:

```
>> n = 5;
>> x = 0:1/n:3;       % coarse sampling
>> y = sin(5*x);
>> plot(x,y)
```



```
>> n = 25;
>> x = 0:1/n:3;       % good sampling
>> y = sin(5*x);
>> plot(x,y)
```



The solid line is used by `plot` by default. It is possible to change the style and the color, e.g.:

```
>> x = 0:04:3; y = sin(5*x);
>> plot(x,y,'r--')
```

produces the dashed red line. The third argument of `plot` specifies the color (optional) and the line style. Table 5 shows a few possibilities, `help plot` shows all. To add a title, grid and to label the axes, one uses:

```
>> title('Function y = sin(5*x)');
>> xlabel('x-axis');
>> ylabel('y-axis');
>> grid                          % remove grid by calling grid off
```

Exercise 23.
Make a plot connecting the coordinates: (2, 6), (2.5, 18), (5, 17.5), (4.2, 12.5) and (2,12) by a line. ■

Exercise 24.
Plot the function $y = \sin(x) + x - x\cos(x)$ in two separate figures for the intervals: $0 < x < 30$ and $-100 < x < 100$. Add a title and axes description. ■

4.2 Several functions in one figure

When more than one function is plotted in one figure, it is useful to use different symbols and colors.

184-188 There are different ways to draw more than one function on the same figure. The first one is with the command
197-201 `hold on`. After this command all functions will be plotted in the same figure until the command `hold off` is used. An example:

```
>> x1 = 1:.1:3.1; y1 = sin(x1);
>> plot(x1,y1,'md');
>> x2 = 1:.3:3.1; y2 = sin(-x2+pi/3);
>> hold on
>> plot(x2,y2,'k*-.')
>> plot(x1,y1,'m-')
>> hold off
```

A second method to display more than one function in one figure is to plot several functions at the same time. The next commands will have the same results as the commands in the previous example:

```
>> x1 = 1:.1:3.1; y1 = sin(x1);
>> x2 = 1:.3:3.1; y2 = sin(-x2+pi/3);
>> plot(x1, y1,'md', x2, y2, 'k*-.', x1, y1, 'm-')
```

To make the axes better fitting the curves, perform:

```
>> axis([1,3.1,-1,1])
```

The same can be achieved by `axis tight`. It might be also useful to exercise with `axis` options (see `help`), e.g. `axis on/off`, `axis equal`, `axis image` or `axis normal`. A descriptive legend can be included with the command `legend`, e.g.:

```
>> legend ('sin(x)', 'sin(-x+pi/3)');
```

It is also possible to produce a few subplots in one figure window. With the command `subplot`, the window can be horizontally and vertically divided into $p \times r$ subfigures, which are counted from 1 to pr, row-wise, starting from the top left. The commands: `plot`, `title`, `grid` etc work only in the current subfigure.

```
>> x  = 1:.1:4;
>> y1 = sin(3*x);
>> y2 = cos(5*x);
>> y3 = sin(3*x).*cos(5*x);
>> subplot(1,3,1);  plot(x,y1,'m-');  title('sin(3*x)')
>> subplot(1,3,2);  plot(x,y2,'g-');  title('cos(5*x)')
>> subplot(1,3,3);  plot(x,y3,'k-');  title('sin(3*x) * cos(5*x)')
```

Exercise 25.
Plot the functions $f(x) = x$, $g(x) = x^3$, $h(x) = e^x$ and $z(x) = e^{x^2}$ over the interval $[0, 4]$ on the normal scale and on the log-log scale. Use an appropriate sampling to get smooth curves. Describe your plots by using the functions: `xlabel`, `ylabel`, `title` and `legend`. ■

Exercise 26.
Make a plot of the function: $f(x) = 1/sin(x)$ over the interval $[0.01, 0.1]$. How do you create `x` so that the plot looks good? ■

Exercise 27.
Plot a circle with the radius $r = 2$, knowing that the parametric equation of a circle is $[x(t), y(t)] = [r\,cos(t), r\,sin(t)]$ for $t = [0, 2\pi]$. ■

OPTIONAL

4.3 Other 2D plotting features

In the introduction of this section, some commands similar to `plot`, `loglog`, `semilogx` and `semilogy` were
201-211 mentioned. There are, however, more ways to display data. MATLAB has a number of functions designed for plotting specialized 2D graphs, e.g.: `fill`, `polar`, `bar`, `barh`, `pie`, `hist`, `errorbar` or `stem`. In the example below, `fill` is used to create a polygon:

```
>> N = 5; k = -N:N;
>> x = sin(k*pi/N);
>> y = cos(k*pi/N);                   % x and y - vertices of the polygon to be filled
>> fill(x,y,'g')
>> axis square
>> text(-0.45,0,'I am a green polygon')
```

Exercise 28.
To get an impression of other visualizations, type the following commands and describe the result (note that the command `figure` creates a new figure window):

```
>> figure                              % bar plot of a bell shaped curve
>> x = -2.9:0.2:2.9;
>> bar(x,exp(-x.*x));
>> figure                              % stairstep plot of a sine wave
>> x = 0:0.25:10;
>> stairs(x,sin(x));
>> figure                              % errorbar plot
>> x = -2:0.1:2;
>> y = erf(x);                         % error function; check help if you are interested
>> e = rand(size(x)) / 10;
>> errorbar (x,y,e);
>> figure
>> r = rand(5,3);
>> subplot(1,2,1);  bar(r,'grouped')   % bar plot
>> subplot(1,2,2);  bar(r,'stacked')
>> figure
>> x = randn(200,1);                   % normally distributed random numbers
>> hist(x,15)                          % histogram with 15 bins
```
■

END OPTIONAL

4.4 Printing

189-196 Before printing a figure you made, you might want to add some information, such as a title, or change somewhat the lay-out. Table 6 shows some of the commands that can be used.

Exercise 29.
Plot the functions $y1 = \sin(4x)$, $y2 = x\,\cos(x)$, $y3 = (x+1)^{-1}\sqrt{x}$ for $x = 0 : 0.25 : 10$; and a single point $(x,y) = (4,5)$ in one figure with different colors and styles. Make a legend containing the three functions. Make labels for both axes and a title. Add a text to the single point saying: single point. Change the minimum and maximum values of the axes such that function y3 can be looked at in more detail. ■

When the displayed figure is to your liking, you can print it to paper. The easiest way to do this is to click on **File** in the menu-bar and to choose **Print**. If in the print window you click `OK`, your figure will be printed on the printer indicated in the print window.

There is also a `print` command, which can be used to send a figure to a printer or output it to a file. You can optionally specify a print device (i.e. an output format such as *tiff* or *postscript*) and options that control various characteristics of the printed file (i.e., which Figure to print etc). You can also print to a file if you specify the file name. If you do not provide an extension, `print` adds one. Since they are many parameters they will not be explained here (check `help print` to learn more). Instead, try to understand the examples:

```
>> print -dwinc                 % print the current Figure to the current printer in color
>> print -f1 -deps myfile.eps   % print Figure no.1 to the file myfile.eps in black
>> print -f1 -depsc myfilec.eps % print Figure no.1 to the file myfilecol.eps in color
>> print -dtiff myfile1.tiff    % print the current Figure to the file myfile1.tiff
>> print -dpsc myfile1c.ps      % print the current Figure to the file myfile1.ps in color
>> print -f2 -djpeg myfile2     % print Figure no.2 to the file myfile2.jpg
```

Exercise 30.
Practise with printing (especially to a file) the figures from previous exercises. ■

4.5 3D line plots

214 The command `plot3` to plot lines in 3D is equivalent to the command `plot` in 2D. The format is the same as for `plot`, it is, however, extended by an extra coordinate. An example is plotting the curve r defined parametrically as $r(t) = [t\ \sin(t), t\ \cos(t), t]$ over the interval $[-10\,\pi, 10\,\pi]$.

```
>> t = linspace(-10*pi,10*pi,200);
>> plot3(t.*sin(t), t.*cos(t), t, 'md-');   % plot the curve in magenta
>> title('Curve r(t) = [t sin(t), t cos(t), t]');
>> xlabel('x-axis'); ylabel('y-axis');  zlabel('z-axis');
>> grid
```

Exercise 31.
Make a 3D smooth plot of the curve defined parametrically as: $[x(t), y(t), z(t)] = [sin(t), cos(t), sin^2(t)]$ for $t = [0, 2\pi]$. Plot the curve in green, with the points marked by circles. Add a title, description of axes and the grid. You can rotate the image by clicking **Tools** at the Figure window and choosing the **Rotate 3D** option or by typing `rotate3D` at the prompt. Then by clicking at the image and dragging your mouse you can rotate the axes. Exercise with this option. ■

4.6 Plotting surfaces

216-220 MATLAB provides a number of commands to plot 3D data. A surface is defined by a function $f(x, y)$, where for each pair of (x, y), the height z is computed as $z = f(x, y)$. To plot a surface, a rectangular domain of the (x, y)-plane should be sampled. The mesh (or grid) is constructed by the use of the command `meshgrid` as follows:

```
>> [X, Y] = meshgrid (-1:.5:1, 0:.5:2)
X =
   -1.0000   -0.5000         0    0.5000    1.0000
   -1.0000   -0.5000         0    0.5000    1.0000
   -1.0000   -0.5000         0    0.5000    1.0000
   -1.0000   -0.5000         0    0.5000    1.0000
   -1.0000   -0.5000         0    0.5000    1.0000
Y =
         0         0         0         0         0
    0.5000    0.5000    0.5000    0.5000    0.5000
    1.0000    1.0000    1.0000    1.0000    1.0000
    1.5000    1.5000    1.5000    1.5000    1.5000
    2.0000    2.0000    2.0000    2.0000    2.0000
```

The domain $[-1, 1] \times [0, 2]$ is now sampled with 0.5 in both directions and it is described by points $[X(i, j), Y(i, j)]$. To plot a smooth surface, the chosen domain should be sampled in a more dense way. To plot a surface, the command `mesh` or `surf` can be used:

```
>> [X,Y] = meshgrid(-1:.05:1, 0:.05:2);
>> Z = sin(5*X) .* cos(2*Y);
>> mesh(X,Y,Z);
>> title ('Function z = sin(5x) * cos(2y)')
```

You can also try `waterfall` instead of `mesh`.

Exercise 32.
Produce a nice graph which demonstrates as clearly as possible the behavior of the function $f(x, y) = \frac{x\,y^2}{x^2+y^4}$ near the point $(0, 0)$. Note that the sampling around this points should be dense enough. ■

Exercise 33.
Plot a sphere, which is parametrically defined as $[x(t, s), y(t, s), z(t, s)] = [\cos(t)\,cos(s),\ \cos(t)\ \sin(s),\ sin(t)]$ for $t, s = [0, 2\,\pi]$ (use `surf`). Make first equal axes, then remove them. Use `shading interp` to remove black lines (use `shading faceted` to restore the original picture). ■

Exercise 34.
Plot the parametric function of r and θ: $[x(r, \theta), y(r, \theta), z(r, \theta)] = [r\ \cos(\theta),\ r\ \sin(\theta),\ \sin(6\,cos(r) - n\theta)]$. Choose n to be constant. Observe, how the graph changes depending on different n. ■

The MATLAB function `peaks` is a function of two variables, obtained by translating and scaling Gaussian distributions. Perform, for instance:

```
>> [X,Y,Z] = peaks;       % create values for plotting the function
>> surf(X,Y,Z);           % plot the surface
>> figure
>> contour (X,Y,Z,30);    % draw the contour lines in 2D
>> colorbar               % adds a bar with colors corresponding to the z-axis
>> title('2D-contour of PEAKS');
>> figure
>> contour3(X,Y,Z,30);    % draw the contour lines in 3D
>> title('3D-contour of PEAKS');
>> pcolor(X,Y,Z);         % z-values are mapped to the colors and presented as
                          % a 'checkboard' plot; similar to contour
```

231-236 Use `close all` to close all figures and start a new task (or use `close 1` to close Figure no.1 etc). Use `colormap` to define different colors for plotting.

To locate e.g. the minimum value of the `peaks` function on the grid, you can proceed as follows:

```
>> [mm,I] = min(Z);       % a row vector of the min. elements from each column
>>                        % I is a vector of corresponding
>> [Zmin, j] = min (mm);  % Zmin is the minimum value, j is the index
                          % Zmin is the value of  Z(I(j),j)
>> xpos = X(I(j),j);      %
>> ypos = Y(I(j),j);      % position of the minimum value
>> contour (X,Y,Z,25);
>> xlabel('x-axis'); ylabel('y-axis');
>> hold on
>> plot(xpos(1),ypos,'*');
>> text(xpos(1)+0.1,ypos,'Minimum');
>> hold off
```

It is also possible to combine two or more plots into one figure. For instance:

```
>> surf(peaks(25)+6);     % move the z-values with the vector [0,0,6]
>> hold on
>> pcolor(peaks(25));
```

Exercise 35.
Plot the surface $f(x,y) = x\,y\,e^{-x^2-y^2}$ over the domain $[-2,2] \times [-2,2]$. Find the values and the locations of the minima and maxima of this function. ■

OPTIONAL

4.7 Animations

A sequence of graphs can be put in motion in MATLAB(the version should be at least 5.0), i.e. you can make a movie using MATLAB graphics tools. To learn how to create a movie, analyze first the script below which shows the plots of $f(x) = sin(nx)$ over the interval $[0, 2\pi]$ and $= 1, \ldots, 5$:

```
N = 5;
M = moviein(N);
x = linspace (0,2*pi);
for n=1:N
  plot (x,cos(n*x),'r-');
  xlabel('x-axis')
  if n > 1,
    ss = strcat('cos(',num2str(n),'x)');
  else
    ss = 'cos(x)';
  end
  ylabel(ss)
  title('Cosine functions cos(nx)','FontSize',12)
  axis tight
```

```
  grid
  M(:,n) = getframe;
  pause(1.8)
end
movie(M)                        % this plays a quick movie
```

Here, a `for`-loop construction has been used to create the movie frames. You will learn more on loops in section **5.3**. Also the command `strcat` has been used to concatenate strings. Use `help` to understand or learn more from section **8.1**.

Play this movie to get acquainted. Five frames are first displayed and at the end, the same frames are played again faster. Command `moviein`, with an integral parameter, tells MATLAB that a movie consisting of `N` frames is going to be created. Consecutive frames are generated inside the loop. Via the command `getframe` each frame is stored in the column of the matrix m. The command `movie(M)` plays the movie just created and saved in columns of the matrix `M`. Note that to create a movie requires quite some memory. It might be useful to clear `M` from the workspace later on.

Exercise 36.
Write a script that makes a movie consisting of 5 frames of the surface $f(x,y) = sin(nx)\,sin(ky)$ over the domain $[0, 2\pi] \times [0, 2\pi]$ and $n = 1 : 5$. Add a title, description of axes and shading. ■

END OPTIONAL

5 Control flow

A control flow structure is a block of commands that allows conditional code execution and making loops.

5.1 Logical and relational operators

To use control flow commands, it is necessary to perform operations that result in logical values: TRUE or
88-91 FALSE. In MATLAB the result of a logical operation is `1` if it is true and `0` if it is false. Table 7 shows the relational and logical operations[3]. Another way to get to know more about them is to type `help relop`. The relational operators `<`, `<=`, `>`, `>=`, `==` and `~=` can be used to compare two arrays of the same size or an array to a scalar. The logical operators `&`, `|` and `~` allow for the logical combination or negation of relational operators. In addition, three functions are also available: `xor`, `any` and `all` (use `help` to find out more).

<table>
<tr><th>Command</th><th>Result</th></tr>
<tr><td><code>a = (b > c)</code></td><td><code>a</code> is 1 if <code>b</code> is larger than <code>c</code>. Similar are: <code><</code>, <code>>=</code> and <code><=</code></td></tr>
<tr><td><code>a = (b == c)</code></td><td><code>a</code> is 1 if <code>b</code> is equal to <code>c</code></td></tr>
<tr><td><code>a = (b ~= c)</code></td><td><code>a</code> is 1 if <code>b</code> is not equal <code>c</code></td></tr>
<tr><td><code>a = ~b</code></td><td>logical complement: <code>a</code> is 1 if <code>b</code> is 0</td></tr>
<tr><td><code>a = (b & c)</code></td><td>logical AND: <code>a</code> is 1 if <code>b</code> = TRUE AND <code>c</code> = TRUE</td></tr>
<tr><td><code>a = (b | c)</code></td><td>logical OR: <code>a</code> is 1 if <code>b</code> = TRUE OR <code>c</code> = TRUE</td></tr>
</table>

Table 7: Relational and logical operations.

Important: The logical `&` and `|` have the equal precedence in MATLAB, which means that those operators associate from left to right. A common situation is:

```
>> b = 10;
>> 1 | b > 0  & 0
ans =
     0
>> (1 | b > 0) & 0         % this indicates the same as above
ans =
     0
>> 1 | (b > 0  & 0)
ans =
     1
```

[3]"equal to" in the table on page 88 of "The Student Edition of Matlab" must be "==" instead of "="

This shows that you should *always* use brackets to indicate in which way the operators should be evaluated.

The introduction of the logical data type has forced some changes in the use of non-logical 0-1 vectors as indices for subscripting. You can see the differences by executing the following commands that attempt to extract the elements of y that correspond to either the odd or even elements of `x`:

- `y(rem(x,2))` vs. `y(logical(rem(x,2)))`   `% odd elements`
- `y(~rem(x,2))` vs. `y(~logical(rem(x,2)))`   `% even elements`

Exercise 37.
Exercise with logical and relational operators:

1. Predict and check the result of each of the operations of Table 7 for `b = 0` and `c = -1`.
2. Predict and check the result of each logical operator for `b = [2 31 -40 0]` and `c = 0`.
3. Define two random vectors (`randn(1,7)`) and perform all logical operations, including `xor`, `any` and `all`.

■

Exercise 38.
Exercise with logical and relational operators:

1. Let `x = [1 5 2 8 9 0 1]` and `y = [5 2 2 6 0 0 2]`. Execute and explain the results of the following commands:
   - `x > y`
   - `y < x`
   - `x == y`
   - `x <= y`
   - `y >= x`
   - `x | y`
   - `x & (~y)`
   - `(x > y) | (y < x)`
   - `(x > y) & (y < x)`
2. Let `x = 1 : 10` and `y = [3 5 6 1 8 2 9 4 0 7]`. The exercises here show the techniques of logical-indexing. Execute and interpret the results of the following commands:
   - `(x > 3) & (x < 8)`
   - `x(x > 5)`
   - `y(x <= 4)`
   - `x((x < 2) | (x >= 8))`
   - `y((x < 2) | (x >= 8))`
   - `x(y < 0)`

■

Exercise 39.
Let `x = [3 16 9 12 -1 0 -12 9 6 1]`. Provide the command(s) that will:

- set the positive values of `x` to zero;
- set values that are multiples of 3 to 3 (make use of `rem`);
- multiply the even values of `x` by 5;
- extract the values of `x` that are greater than 10 into a vector called `y`;
- set the values in `x` that are less than the mean to 0;
- set the values in `x` that are above the mean to their difference from the mean.

■

Exercise 40.
Execute the following commands and try to understand how `z` is defined.

```
>> hold on
>> x = -3:0.05:3;  y = sin(3*x);
>> subplot(1,2,1); plot(x,y); axis tight
>> z = (y < 0.5) .* y;
>> subplot(1,2,2); plot(x,y,'r:'); plot(x,z,'r'); axis tight
>> hold off
```

■

Before moving on, check whether you now understand the following relations:

```
>> a = randperm(10);   % random permutation
>> b = 1:10;
>> b - (a <= 7)        % subtracts from b a 0-1 vector, taking 1 for a <= 7 and 0 otherwise
>> (a >= 2) & (a < 4)  % returns ones at positions where 2 <= a < 4
>> ~(b > 4)            % returns ones at positions where b <= 4
>> (a == b) | b == 3   % returns ones at positions where a is equal to b or b is equal to 3
>> any(a > 5)          % returns 1 when ANY of the a elements are larger than 5
>> any(b < 5 & a > 8)  % returns 1 when there in the evaluated expression
>>                     % (b < 5 & a > 8) appears at least one 1
>> all(b > 2)          % returns 1 when ALL b elements are larger than 2
```

5.1.1 The command `find`

You can extract all elements from the vector or the matrix satisfying a given condition, e.g. equal to 1 or
61-62 larger than 5, by using logical addressing. The same result can be obtained via the command `find`, which return the positions (indices) of such elements. For instance:

```
>> x = [1 1 3 4 1];
>> i = (x == 1)
i =
     1     1     0     0     1
>> y = x(i)
y =
     1     1     1
>> j = find(x == 1)    % j holds indices of those elements satisfying x == 1
j =
     1     2     5
>> z = x(j)
z =
     1     1     1
```

An another example is:

```
>> x = -1:0.05:1;
>> y = sin(x) .* sin(3*pi*x);
>> plot (x,y, '-'); hold on
>> k = find (y <= -0.1)
k =
     9    10    11    12    13    29    30    31    32    33
>> plot (x(k), y(k), 'ro');
>> r = find (x > 0.5 & y > 0)
r =
    35    36    37    38    39    40    41
>> plot (x(r), y(r), 'r*');
```

`find` operates in a similar way on matrices:

```
>> A = [1 3 -3 -5; -1 2 -1 0; 3 -7 2 7];
>> k = find (A >= 2.5)
k =
     3
     4
    12
>> A(k)
ans =
     3
     3
     7
```

In this way, `finds` reshapes first the matrix `A` into a column vector, i.e. it operates on `A(:)`, i.e. all columns all concatenated one after another. Therefore, `k` is a list of indices of elements larger than or equal to 2.5 and `A(k)` gives the values of the selected elements. Also the row and column indices can be returned, as shown below:

```
>> [I,J] = find (A >= 2.5)
I =
     3
     1
     3
J =
     1
     2
     4
>> [A(I(1),J(1)), A(I(2),J(2)), A(I(3),J(3))]   % lists the values
ans =
     3     3     7
```

Exercise 41.
Let $\mathtt{A} = \mathtt{ceil}(5 * \mathtt{randn}(6,6))$. Perform the following:

- find the indices and list all elements of `A` which are smaller than `-3`;
- find the indices and list all elements of `A` which are smaller than `5` and larger than `-1`;
- remove those columns of `A` which contain at least one `0` element.

Exercise with both: logical indexing and the command `find`. ■

5.2 Conditional code execution

117-119 Selection control structures, `if`-blocks, are used to decide which instruction to execute next depending whether *expression* is TRUE or not. The general description is given below. In the examples below the command `disp` is frequently used. This command displays on the screen the text between the quotes.

- `if ... end`

Syntax

```
if logical_expression
   statement1
   statement2
   ....
end
```

Example

```
if (a > 0)
  b = a;
  disp ('a is positive');
end
```

- `if ... else ... end`

Syntax

```
if logical_expression
   block of statements
   evaluated if TRUE
else
   block of statements
   evaluated if FALSE
end
```

Example

```
if (temperature > 100)
  disp ('Above boiling.');
  toohigh = 1;
else
  disp ('Temperature is OK.');
  toohigh = 0;
end
```

- `if ... elseif ... else ... end`

Syntax

```
if logical_expression1
   block of statements evaluated
   if logical_expression1 is TRUE
elseif logical_expression2
   block of statements evaluated
   if logical_expression2 is TRUE
else
   block of statements evaluated
   if no other expression is TRUE
end
```

Example

```
if (height > 190)
  disp ('very tall');
elseif (height > 170)
  disp ('tall');
elseif (height < 150)
  disp ('small');
else
  disp ('average');
end
```

Important: To do the exercises below, a *script m-file* can be of use. It is an external file that contains a sequence of MATLAB commands. What you need to do is to open an editor, enter all commands needed for the solution of a task, save it with the extension `'.m'` (e.g. `mytask.m`) and then run it from the Command Window, by typing `mytask`. To open the MATLAB editor, go to the **File** menu-bar, choose the **Open** option and then **m-file**. A new window will appear where you can write your scripts and save them on the disk. All commands in the script will be executed in MATLAB. You will learn more on script m-files in section **7.1**. m-script file must be saved in one of the directories in MATLAB's path.

An example: A script `short.m` can be created from the two lines given below and run by calling `short`.

```
x = 0:0.5:4;
plot(x,sin(x),'*-');
```

Exercise 42.
In each of the following questions, evaluate the given code fragments. Investigate each of the fragments for the various starting values given on the right. Use MATLAB to check your answers (be careful, since those fragments are not always the right MATLAB expressions):

1.
```
if n > 1                a) n =  7   m = ?
  m = n + 2             b) n =  0   m = ?
else                    c) n = -7   m = ?
  m = n - 2
end
```

2.
```
if s <= 1               a) s =   1  t = ?
  t = 2z                b) s =   7  t = ?
elseif s < 10           c) s =  57  t = ?
  t = 9 - z             d) s = 300  t = ?
elseif s < 100
  t = sqrt(s)
else
  t = s
end
```

3.
```
if t >= 24              a) t = 50   h = ?
  z = 3t + 1            b) t = 19   h = ?
elseif t < 9            c) t = -6   h = ?
  z = t^2/3 - 2t        d) t =  0   h = ?
else
  z = -t
end
```

4.
```
if 0 < x < 7            a) x = -1   y = ?
  y = 4x                b) x =  5   y = ?
elseif 7 < x < 55       c) x = 30   y = ?
  y = -10x              d) x = 56   y = ?
else
  y = 333
end
```

■

Exercise 43.
Create a script that asks for a number N and computes the drag coefficient C, depending on the Reynold's number N (make use of the `if ... elseif ...` construction). The command `input` might be useful here
32-33 (use `help` if needed).

$$C = \begin{cases} 0, & N \leq 0 \\ 24/N, & N \in (0,\, 0.1] \\ 24/N\,(1 + 0.14\,N^{0.7}), & N \in (0.1,\, 1e3] \\ 0.43, & N \in (1e3,\, 5e5] \\ 0.19 - 8e4/N, & N > 5e5 \end{cases}$$

Check whether your script works correctly. Compute the drag coefficient for e.g. N = -3e3, 0.01, 56, 1e3, 3e6 (remember that e.g. 3e3 is a MATLAB notation of $3 * 10^3$). ■

Exercise 44.
Write a script that asks for an integer and checks whether it can be divided by 2 or 3. Consider all possibilities, such as: divisible by both 2 and 3, divisible by 2 and not by 3 etc (use the command `rem`). ■

Another selection structure is `switch`, which switches between several cases depending on an expression, which
119-120 is either a scalar or a string.

Syntax

```
switch expression
  case choice1
     block of commands1
  case {choice2a, choice2b,...}
     block of commands2
      ...
  otherwise
     block of commands
end
```

Example

```
method = 2;
switch method
  case {1,2}
     disp('Method is linear.');
  case 3
     disp('Method is cubic.');
  case 4
     disp('Method is nearest.');
  otherwise
     disp('Unknown method.');
  end
```

The statements following the first `case` where the expression matches the choice are executed. This construction can be very handy to avoid long `if .. elseif ... else ... end` constructions. The expression can be a scalar or a string. A scalar expression matches a choice if `expression == choice`. A string expression matches a choice if `strcmp(expression, choice)` returns 1 (is true) (`strcmp` compares two strings).
Important: Note that the `switch`-construction only allows the execution of one group of commands.

Exercise 45.
Assume that the months are represented by numbers from 1 to 12. Write a script that asks you to provide a month and returns the number of days in that particular month. Alternatively, write a script that asks you to provide a month name (e.g. 'June') instead of a number. Use the `switch`-construction. ■

5.3 Loops

Iteration control structures, *loops*, are used to repeat a block of statements until some condition is met. Two
114-117 types of loops exist:

- the `for` loop that repeats a group of statements a *fixed* number of times;

Syntax

```
for index = first:step:last
  block of statements
end
```

Example

```
sumx = 0;
for i=1:length(x)
   sumx = sumx + x(i);
end
```

You can specify any `step`, including a negative value. The `index` of the `for`-loop can be also a vector. See some examples of possible variations:

Example 1

```
for i=1:2:n
  ...
end
```

Example 2

```
for i=n:-1:3
  ....
end
```

Example 3

```
for x=0:0.5:4
  disp(x^2);
end
```

Example 4

```
for x=[25 9 81]
  disp(sqrt(x));
end
```

- `while` loop, which evaluates a group of commands as long as *expression* is TRUE.

Syntax

```
while expression
  statement1
  statement2
  statement3
  ...
end
```

Example

```
N = 100;
iter = 1;
msum = 0;
while iter <= N
  msum = msum + iter;
  iter = iter + 1;
end;
```

A simple example how to use the loop construct can be to draw graphs of $f(x) = cos(n\,x)$ for $n = 1, \ldots, 9$ in different subplots. Execute the following script:

```
figure
hold on
x = linspace(0,2*pi);
for n=1:9
  subplot(3,3,n);
  y = cos(n*x);
  plot(x,y);
  axis tight
end
```

Given two vectors `x` and `y`, an example use of the loop construction is to create a matrix `A` whose elements are defined, e.g. as $\mathtt{A}_{\mathtt{ij}} = \mathtt{x}_\mathtt{i}\,\mathtt{y}_\mathtt{j}$. Enter the following commands to a script:

```
n = length(x);
m = length(y);
for i=1:n
   for j=1:m
     A(i,j) = x(i) * y(j);
   end
 end
```

and create `A` for `x` = [`1 2 -1 5 -7 2 4`] and `y` = [`3 1 -5 7`]. Note that `A` is of size n-by-m. The same problem can be solved by using the `while`-loop, as follows:

```
n = length(x);
m = length(y);
i = 1; j = 1;                % initialize i and j
while i <= n
  while j <= m
    A(i,j) = x(i) * y(j);
    j = j+1;                 % increment j; it does not happen automatically
  end
  i = i+1;                   % increment i
end
```

Exercise 46.
Determine the sum of the first 50 squared numbers with a control loop. ■

Exercise 47.
Write a script to find the largest value n such that the sum: $\sqrt{1^3} + \sqrt{2^3} + \ldots + \sqrt{n^3}$ is less than 1000. ■

Exercise 48.
Use a loop construction to carry out the computations. Write short scripts.

1. Given the vector `x` = [`1 8 3 9 0 1`], create a short set of commands that will:
   - add up the values of the elements (check with `sum`);
   - computes the running sum (for element j, the running sum is the sum of the elements from 1 to j; check with `cumsum`);
   - computes the sine of the given x-values (should be a vector).

2. Given `x` = [`4 1 6 -1 -2 2`] and `y` = [`6 2 -7 1 5 -1`], compute matrices whose elements are created according to the following formulas:
   - $\mathtt{a}_{\mathtt{ij}} = \mathtt{y}_\mathtt{i}/\mathtt{x}_\mathtt{j}$;
   - $\mathtt{b}_\mathtt{i} = \mathtt{x}_\mathtt{i}\,\mathtt{y}_\mathtt{i}$ and add up the elements of `c`;
   - $\mathtt{c}_{\mathtt{ij}} = \mathtt{x}_\mathtt{i}/(2 + \mathtt{x}_\mathtt{i} + \mathtt{y}_\mathtt{j})$;
   - $\mathtt{d}_{\mathtt{ij}} = 1/\max(\mathtt{x}_\mathtt{i}, \mathtt{y}_\mathtt{j})$.

3. Write a script that transposes a matrix `A`. Check its correctness with the MATLAB operation: `A'`.

4. Create an m-by-n array of random numbers (use `rand`). Move through the array, element by element, and set any value that is less than 0.5 to 0 and any value that is greater than (or equal to) 0.5 to 1.

5. Write a script that will use the random-number generator `rand` to determine:

- the number of random numbers it takes to add up to 10 (or more);
- the number of random numbers it takes before a number between 0.8 and 0.85 occurs;
- the number of random numbers it takes before the mean of those numbers is within 0.01 and 0.5.

It will be worthwhile to run your script several times because you are dealing with random numbers. Can you predict any of the results that are described above?

■

Exercise 49.
Write a script that asks for a temperature in degrees Celsius `tc` and computes the equivalent temperature in degrees Fahrenheit `tf` (use the formula $\mathtt{tf} = 9/5 * \mathtt{tc} + 32$). The script should keep running until no number is provided to convert. The functions `input` and `isempty` (use `help` to learn more) should be useful here. ■

5.4 Evaluation of logical and relational expressions in the control flow structures

The relational and logical expressions may become more complicated. It is not difficult to operate on them if you understand how they are evaluated. To explain more details, let us consider the following example:

```
if (~isempty(data)) & (max(data) < 5)
 ....
end
```

This construction of the `if`-block is necessary to avoid comparison if `data` happens to be an empty matrix. In such a case you cannot evaluate the right logical expression and MATLAB gives an error. The & operator returns 1 only if both expressions: `~isempty (data)` and `max(data) < 5` are true, and 0 otherwise. When `data` is an empty matrix, the next expression is not evaluated since the whole &-expression is already known to be false. The second expression is checked only if `data` is a non-empty matrix. Remember to put logical expression units between brackets to avoid wrong evaluations!

Important: The fact that computers make use of floating-point arithmetic means that often you should be careful when comparing two floating-point numbers just by:

```
if (x == y)
  ....
end
```

(Of course, such a construction is allowed e.g. when you know that `x` and `y` represent integers.) Instead of the above construction, you may try using this:

```
if (abs (x - y) < tolerance)         % e.g. tolerance = 1e-10
  ....
end
```

Exercise 50.
Consider the following example:

```
max_iter  = 50;
tolerance = 1e-4;
iter = 0;
xold = 0.1;
x    = 1;
while (abs (x - xold) > tolerance) & (iter < max_iter)
  xold = x;
  x    = cos(xold);
  iter = iter + 1;
end
```

This short program tries to solve the equation $\cos(x) = x$ (`x` is the solution found). Make the script `solve_cos` from the presented code. Note that the `while`-loop is repeated as long as both conditions are true. If either the condition `(|x - xold| <= tolerance)` or `(iter >= max_iter)` is fulfilled then the loop is quitted. Run the script `solve_cos` for different `tolerance` parameters, e.g.: 1e-3, 1e-6, 1e-8, 1e-10 etc. Use `format long` to check more precisely how much the found `x` is really different from `cos(x)`. For each `tolerance` value check the number of performed iterations (`iter` value). ■

Exercise 51.
Create the script `solve_cos2`, which is equal to the one given, replacing the `while`-loop condition by:

```
while (abs (x - xold) > tolerance) | (iter < max_iter)
```

Try to understand the difference and confirm your expectations by running solve_cos2. What happens to iter? ■

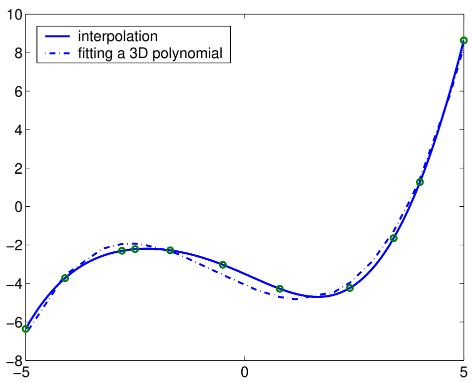
6 Numerical analysis

Numerical analysis can be used whenever it is impossible or very difficult to determine the analytical solution. MATLAB can be used to find, for example, the minimum, maximum or integral of a certain function.

There are two ways to describe data with an analytical function:

- **curve fitting or regression**; finding a smooth curve that best fits (approximates) the data according to a criterion (e.g. best least square fit). The curve does not have to go through any of the data points.
- **interpolation**; the data are assumed to be correct, desired in a way to describe what happens between the data points. In other words, given a finite set of points, the goal of interpolation is to find a function from a given class of functions (e.g. polynomials) which passes through the specified points.

The figure below illustrates the difference between regression (curve fitting) and interpolation:



In this section we will briefly discuss some issues on curve fitting, interpolation, function evlaution, integration and differentiation. It is also possible to solve differential equations which are shortly treated in "The Student
179-182 Edition of Matlab"book.

6.1 Curve fitting

MATLAB interprets the best fit of a curve by the 'least squares curve fitting'. The curve used is restricted
153-156 to polynomials. With the command `polyfit` any polynomial can be fitted to the data. `polyfit (x, y, n)` finds the coefficients of a polynomial of degree n that fits the data (finds the linear relation between x and y). Let's start with finding a linear regression of some data:

```
>> x = 0:10;
>> y = [-.10 .24 1.02 1.58 2.84 2.76 2.99 4.05 4.83 5.22 7.51]
>> p = polyfit (x, y, 1)          % find the fitting polynomial of order 1
   p = 0.6772   -0.3914
```

The output of `polyfit` is a row vector of the polynomial coefficients, the solution of this example is therefore $y = 0.6772x - 0.3914$.

Exercise 52.
Execute the above example. Plot the data and the fitted curve in a figure. Determine the 2nd and 9th order polynomial of these data as well, and display the solution in a figure.
Hint: `x1 = linspace(xmin,xmax,n);` results in a vector with n element evenly distributed from `xmin` till `xmax`. `z = polyval(p,x1);` calculates the values of the polynomial for every element of `x1`. ■

Exercise 53.
Define x and y as follows:

```
>> x = -5:5;
>> y = 0.2*x.^3 - x + 2;             % a 3rd order polynomial
>> y = y + randn(11,1);              % add some noise to y
```

Determine the 3rd order polynomial fitting y (note that because of added noise the coefficients are different that originally chosen). Try some other polynomials as well. Plot the results in a figure. ■

6.2 Interpolation

157-161 The simplest way to examine an interpolation is by displaying the function plots with the command `plot`. The neighboring data points are connected with straight lines. This is called a linear interpolation. When more data-points are taken into account the effect of interpolation becomes less visible. Analyze:

```
>> x1 = linspace(0,2*pi,2);
>> x2 = linspace(0,2*pi,4);
>> x3 = linspace(0,2*pi,16);
>> x4 = linspace(0,2*pi,256);
>> plot(x1,sin(x1),x2,sin(x2),x3,sin(x3),x4,sin(x4))
>> legend('2 points','4 points','16 points','256 points')
```

157-165 There exist also MATLAB functions which interpolate between points, e.g. `interp1` or `spline`, as a 1D interpolation and `interp2`, as a 2D interpolation. Perform the following commands:

```
>> N = 50;
>> x = linspace(0,5,N);
>> y = sin(x) .*sin(6*x);
>> subplot(2,1,1); plot(x,y);
>> hold on
>> p = randperm(N);
>> pp = p(1:round(N/2));             % choose randomly N/2 indices of points from [0,5]
>> pp = sort(pp);                    % sort the indices
>> xx = x(pp);                       % select the points
>> yy = y(pp);
>> plot(xx,yy,'ro-')                 % this is a 'coarse' version
>> yn = interp1(xx,yy,x,'nearest'); % nearest neighbor interpolation on all x points
>> plot(x,yn,'g')
>> axis tight
>> legend('Original','Crude version','Nearest neighbor interpolation')
>>
>> subplot(2,1,2); plot(xx,yy,'ro-');
>> hold on
>> yc = interp1(xx,yy,x,'linear');  % linear interpolation on all x points
>> plot(x,yc,'g')
>> ys = spline(xx,yy,x);             % spline interpolation
>> plot(x,ys,'k')
>> axis tight
>> legend('Crude version','Linear interpolation','Spline interpolation')
```

You can also see a coarse approximation of the function `peaks`:

```
>> [X,Y,Z] = peaks(10);
>> [Xi,Yi] = meshgrid(-3:.25:3,-3:.25:3);
>> Zi = interp2(X,Y,Z,Xi,Yi);
>> mesh(Xi,Yi,Zi);
```

6.3 Evaluation of a function

168-173 As said before, the command `plot` plots a function by connecting defined data points. Sometimes, however, a function is constant and not very interesting over some range and then acts unexpectedly wildly over another. With `plot` this function might be misinterpreted. The command `fplot` is then useful. An example:

Command	Result
fplot('f',[min_x max_x])	plots the function f on [min_x, max_x]
fmin('f',min_x,max_x)	returns the x-value for which the function f reaches minimum on [min_x, max_x]
fzero('f',x0)	returns the x-value for which the function f is zero in the neighborhood of x0

Table 8: Evaluation of a function.

```
>> x = 0:pi/8:2*pi;
>> y = sin(8*x);
>> plot(x,y,'b')
>> hold on
>> fplot('sin(8*z)',[0 2*pi],'r')
>> title('sin(8*z)')
>> hold off
```

Execution of these commands results in a figure with an almost straight blue line and a red sine. `fplot` can also be used to plot a defined function:

```
>> f = 'sin(8*z)';
>> fplot(f,[0 2*pi],'r')
>> title(f)
```

6.3.1 Inline functions

It might be useful to define a function that will be only used during the current MATLAB session. MATLAB offers a command `inline` to define the so-called inline functions in the Command Window, e.g.:

```
>> f = inline('cos(x).*sin(2*x)')
f =
     Inline function:
     f(x) = cos(x).*sin(2*x)
>> g = inline('sqrt(x.^2+y.^2)','x','y')
g =
     Inline function:
     g(x,y) = sqrt(x.^2+y.^2)
```

You can evaluate this function in a usual way:

```
>> f(-2)
ans =
   -0.3149
>> g(3,4)
ans =
     5
>> A = [1 2; 3 4];
>> B = [2 3; 4 5];
>> g(A,B)                               % the function also works with arrays
ans =
    2.2361    3.6056
    5.0000    6.4031
```

Let $f(x) = \frac{1}{(x-0.1)^2+0.1} + \frac{1}{(x-1)^2+0.1}$. The point at which f takes its minimum is found using `fmin` (or `fminbnd` in MATLAB version 6.0). By default, the relative error is of 1e-4. It is possible, however, to obtain a higher accuracy, by specifying an extra parameter while calling `fmin`.

```
>> format long                       % you need this format to see the change in accuracy
>> f = inline ('1./((x-0.1).^2 + 0.1) + 1./((x-1).^2 + 0.1)');
>> fplot(f,[0 2]);
>> xm1 = fmin(f,0.3,1); fm1 = f(xm1);
>> xm2 = fmin(f,0.3,1,[0,1e-8]); fm2 = f(xm2);
>> [xm1, xm2]                        % compare the two answers
```

Important: Note that `format` does not change the accuracy of the numbers or calculations; it just changes the numbers of digits displayed on the screen.

Exercise 54.
Use `fmin` to find the maximum of $f(x) = \frac{1}{(x-0.1)^2+0.1} + \frac{1}{(x-1)^2+0.1}$ in the interval $[0, 0.5]$. Choose an error tolerance such that the maximum is correct to $\pm 10^{-6}$. If x_m denotes the computed solution, check the answer by evaluating f at $x_m + 10^{-6}$ and $x_m - 10^{-6}$. The values should be smaller at both these neighboring points. *Hint:* there is no function `fmax`. Instead, find minimum of $-f(x)$. ■

Exercise 55.
Determine the maximum of the function $f(x) = x\ \cos(x)$ over the interval $10 < x < 15$. ■

Exercise 56.
Find the zero, i.e x_0 of the functions $f(x) = \cos(x)$ and $g(x) = \sin(2x)$ around the point 2. Use a command from Table 8. Check that the values are approximately zero for x_0. Check that f has different signs at values $x_0 - 10^{-4}$ and $x_0 + 10^{-4}$. ■

6.4 Integration and differentiation

The integral –or the surface underneath a 2D function– can be determined with the command `trapz`. `trapz`
173-182 does this by measuring the surface between the x-axis and the -by a straight line connected- data points. The accuracy of this method depends on the distance between the data points:

```
>> x = 0:0.5:10; y = 0.5 * sqrt(x) + x .* sin(x);
>> integral1 = trapz(x,y)
integral1 =
   18.1655

>> x = 0:0.05:10; y = 0.5 * sqrt(x) + x .* sin(x);
>> integral2 = trapz(x,y)
integral2 =
   18.3846
```

A more accurate result can be obtained by using the command `quad` or `quadl`, which also numerically evaluate an integral of the function f specified by a string or by an inline definition. Let $f = \frac{1}{(x-0.1)^2+0.1} + \frac{1}{(x-1)^2+0.1}$.

```
>> f = '1./((x-0.1).^2 + 0.1) + 1./((x-1).^2 + 0.1)';
>> integral1 = quad(f,0,2)
>> integral2 = quadl(f,0,2)
```

You can also add an extra parameter to `quad` and `quadl`, specifying the accuracy.

Exercise 57.
Find the integral of the function $f(x) = e^{-x^2/2}$ over the interval $[-3, 3]$. Exercise with different MATLAB commands and different accuracy. ■

Differentiation is done by determining the slope in each data point. Numerically this is done by first fitting a polynomial, followed by differentiating this function:

```
>> x = 0:10;
>> y = [-.10 .24 1.02 -1.58 2.84 2.76 7.75 .35 4.83 5.22 7.51];
>> p = polyfit(x,y,5)
p =
    0.0026   -0.0554    0.3634   -0.6888    0.2747    0.0877
>> dp = polyder(p)
dp =
    0.0132   -0.2216    1.0903   -1.3776    0.2747
>> x1 = linspace(0,10,200);
>> z  = polyval(p,x1);
>> dz = polyval(dp,x1);
>> plot(x,y,'g.-',x1,z,'b',x1,dz,'r:')
>> legend('data points','curve fit','derivative')
```

Exercise 58.
Determine the integral and derivative of the 4th polynomial fitted through `(x,y)` with `x = 4:13` and `y = [12.84 12.00 7.24 -0.96 -11.16 -20.96 -27.00 -24.96 -9.56 25.44]`. ■

6.5 Numerical computations and the control flow structures

Take a look at the problem of a Taylor expansion. The series will be determined numerically until the additional terms are smaller than a defined precision. Look at the Taylor expansion of $\frac{1}{1-x} = 1+x+x^2+\ldots$ with $x = 0.42$:

```
s  = 0; x = 1;
x0 = 0.42;
while (x > 1e-6)
  s = s + x;
  x = x * x0;
end
```

or

```
s = 0; x = 1;
x0 = 0.42;
while (x > (1e-6)*s)
  s = s + x;
  x = x * x0;
end
```

The approximation of $\frac{1}{1-x}$ is returned in `s` (first letter of sum). In the first case (on the left), the iteration process is stopped as soon as the next term `t` is smaller than an absolute value of 1e-6. In the other case, this value is relative to the outcome.

Exercise 59.
Find the Taylor expansion of $\frac{1}{1-x}$ and $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ with the absolute precision of 1e-5 for $x = 0.4$ and $x = -0.4$. ■

It is also possible to determine the Taylor expansion of $\frac{1}{1-x} = 1 + x + x^2 + \ldots$ with 30 terms ($x = 0.42$):

```
s  = 1; x = 1;
x0 = 0.42;
n  = 30;
for k = 1:n
  x = x * x0;
  s = s + x;
end
```

or

```
s  = 1; x = 1; x0 = 0.42;
n1 = 1; n2 = 30;
step = 1;
for k = n1:step:n2
    x = x * x0;
    s = s + x;
end
```

showing that the index, `k`, can be indicated with a starting point, step size and end point.

7 Script and function m-files

7.1 Script m-files

MATLAB commands can be entered at the MATLAB prompt. When a problem is more complicated this becomes
29-33 inefficient. A solution is using *script m-files*. They are useful when the number of commands increases or when you want to change values of some variables and re-evaluate them quickly. Formally, a script is an external file that contains a sequence of MATLAB commands (statements). However, it is not a function, since there are no input/output parameters and the script variables remain in the workspace. So, **when you run a script, the commands in it are executed as if they have been entered through the keyboard.**

Exercise 60.
Create yourself a script: open the MATLAB editor (go to the **File** menu-bar, choose the **Open** option and then **m-file**; the MATLAB Editor Window will appear), enter the lines listed below and save as `sinplot.m`:

```
x = 0:0.2:6;
y = sin(x);
plot(x,y);
title('Plot of y = sin(x)');
```

and then run it by:

```
>> sinplot
```

The `sinplot` script affects the workspace. Check:

```
>> clear                   % all variables are removed from the workspace
>> who                     % no variables present
>> sinplot
>> who
Your variables are:
x         y
```

■

These generic names, x and y, may be easily used in further computations and this can cause *side effects*. Side effects occur in general when a set of commands change variables other than the input arguments. Since scripts create and change variables in the workspace (without warning), a bug, hard to track down, may easily appear. So, it is important to remember that **the commands within a script have access to all variables in the workspace and all variables created in this script become a part of the workspace.** Therefore, it is better to use function m-files (see section **7.2**), when there is a specific problem to be solved.

Exercise 61.
Create a script which:

1. plots the function $y = 2^{\cos(x)}$ at the range $[-10, 10]$. Choose a sufficient length for x in order to obtain a smooth plot.
2. plots the functions $y = x^2$ and $x = y^2$ in the same figure. Add a title, legend and axes descriptions.

■

OPTIONAL

The diary command can be also useful when creating scripts. diary on begins saving the text that is entered and displayed in the Command Window into a file called *diary*. Another name can be used alternatively, when it is provided instead of on and after the diary. diary off stops the saving procedure.

Exercise 62.
Check yourself the following:

```
>> diary myplot.m
>> x = 0:0.2:6;
>> y = sin(x);
>> z = x * y;
??? Error using ==> *
Inner matrix dimensions must agree.
>> z = x .* y;     % correction
>> plot(x,z);
>> title('Plot of y = x * sin(x)');
>> diary off
```

Note, that before running myplot, its contents must be edited by removing the >> signs and the error note. ■

END OPTIONAL

Exercise 63.
Write the script cubic_roots that for given a, b and c returns the roots of the cubic equation: $a\,x^2 + b\,x + c = 0$.
■

Exercise 64.
Write a script that plots the graph of equilateral triangle with two vertices $[a, a]$ and $[b, a]$. Find the third vertex. Use fill to paint the triangle. ■

7.2 Function m-file

Functions m-files are true subprograms, since they take input arguments and/or return output parameters.
126-134 They can call other functions, as well. Variables defined and used inside a function, different from the input/output arguments, are invisible to other functions and the command environment. The general syntax of a function is presented below:

```
function [outputArgs] = function_name (inputArgs)
```

`outputArgs` are enclosed in `[ ]`:

– a comma-separated list of variable names;

– `[ ]` is optional when only one argument is present;

– functions without `outputArgs` are legal[4].

`inputArgs` are enclosed in `( )`:

– a comma-separated list of variable names;

– functions without `inputArgs` are legal.

MATLAB provides a structure for creating your own functions. The first line of the file should be a definition of a new function (also called a header). After that, a continuous sequence of comment lines should appear. Their goal is to explain what the function does, especially when this is not trivial. Not only a general description, but also the expected input parameters, returned output parameters and synopsis should appear there. The comment lines (counted up to the first non-comment line) are important since they are displayed in response to the `help` command. Finally, the remainder of the function is called *the body*. Function m-files terminate execution and return when they reached the end of the file or, alternatively, when the command `return` is encountered. As an example, the function `average` is defined as follows:

output argument

function name

input argument

the first line must be the function definition

```
function avr = average (x)
%AVERAGE  computes the average value of a vector x
%         and returns it in avr

% Notes: an example of a function

n   = length(x);
avr = sum(x)/n;
return;
```

comment

function body

a blank line within the comment; Notes information will NOT appear when you ask: `help average`

Important: The name of the function and the name of the file stored on disk should be *identical*. In our case, the function should be stored in a file called `average.m`.

Exercise 65.
Create the function `average` and store it on disk as `average.m`. Remember about the comment lines. Check its usability by calling `help average`. Run `average.m` by typing `avr1 = average(1:10);` ■

Here is an another example of a function:

```
function [avr,sd] = stat(x)
%STAT Simple statistics.
%     Computes the average value and the standard deviation of a vector x.
n   = length(x);
avr = sum(x)/n;
sd  = sqrt(sum((x - avr).^2)/n);
return;
```

Warning: The functions `mean` and `std` already exist in MATLAB. As long as a function name is used as variable name, MATLAB can not perform the function. Many other, easily appealing names, such as `sum` or `prod` are reserved by MATLAB functions, so be careful when choosing your names (see section **12.3**).

The `return` statement can be used to force an early return. An exemplary use of the `return` is given below:

```
function d = determinant(A)
%DETERMINANT  Computes the determinant of a matrix A
[m,n] = size(A);
if (m ~= n)
  disp ('Error. Matrix should be square.');
  return;
else
  d = det(A);                      % standard Matlab function
end
return;
```

[4]in other programming languages, functions without output arguments are called *procedures*

Analyze the use of the `return` command in the function `checkarg`, which is presented in section **7.2.1** as well.

When controlling the proper use of parameters, the function `error` may become useful. It displays an error
129 message, aborts function execution, and returns to the command environment. Here is an example:

```
if (a >= 1)
  error ('a must be smaller than 1');
end
```

Change some scripts that you created into functions, e.g. create the function `drag`, computing the drag coefficient (see section **5.2**), or `solve_cos` (see section **5.4**) or `cubic_roots` (see section **7.1**).

Exercise 66.
Write the function `[elems, mns] = nonzero(A)` that takes as the input argument a matrix `A` and returns all nonzero elements of `A` in the column vector `elems`. The output parameter `mns` holds values of the means of all columns of `A`. ■

Exercise 67.
Create the function `[A,B,C] = sides(a,b,c)` that takes three positive numbers `a`, `b` and `c`. If they are sides of a triangle, then the function returns its angles `A`, `B` and `C`, measured in degrees. Display an error when necessary. ■

Exercise 68.
The area of a triangle with sides of length a, b, and c is given by: $ar = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where $s = (a+b+c)/2$. Write a function that accepts a, b and c as inputs and returns the value ar as output. Note that the sides should be non-negative and should fulfill the triangle inequality. Make use of the `error` command. ■

Exercise 69.
Create the function `randint` that randomly generates a matrix of integer numbers (use the command `rand`). These integers should come from the interval $[a, b]$. Exercise in documenting the function. Use the following function header:

```
function r = randint(m,n,a,b)             % a m-by-n matrix
```

If this seems too difficult, start first with the fixed interval, e.g. $[a, b] = [0, 5]$ (and remove `a` and `b` from the function definition) and then try to make it work for an arbitrary interval. ■

OPTIONAL

A function m-file may contain more than one function. The function appearing first in the m-file shares the file name, as mentioned before. Other functions are *subfunctions* and can be called from the primary function only. So, they cannot be accessed from outside the file, in which they are defined (neither from the Command Window nor via other m-files). A subfunction is created by defining a new function with the `function` statement after the body of the preceding function. The use of subfunctions is recommended to keep the function readable when it becomes too long and too complicated. For example, `average` is now a subfunction within the file `stat.m`:

```
function [a,sd] = stat(x)
%STAT Simple statistics.
%     Computes the average value and the standard deviation of a vector x.
n  = length(x);
a  = average(x,n);
sd = sqrt(sum((x - avr).^2)/n);
return;

function a = average (x,n)
%AVERAGE  subfunction
a = sum(x)/n;
return;
```

END OPTIONAL

7.2.1 Special function variables

Each function has two internal variables: `nargin` - the number of function input arguments that were used to call the function and `nargout` - the number of output arguments. Analyze the following function:

```
function [out1,out2] = checkarg (in1,in2,in3)
%CHECKARG Demo on using the nargin and nargout variables.
if (nargin == 0)
   disp('no input arguments');
   return;
elseif (nargin == 1)
   s = in1;
   p = in1;
   disp('1 input argument');
elseif (nargin == 2)
   s = in1+in2;
   p = in1*in2;
   disp('2 input arguments');
elseif (nargin == 3)
   s = in1+in2+in3;
   p = in1*in2*in3;
   disp('3 input arguments');
else
   error('Too many inputs.');
end

if (nargout == 0)
  return;
elseif (nargout == 1)
  out1 = s;
else
  out1 = s;
  out2 = p;
end
```

Exercise 70.
Construct the function `checkarg`, call it with different number of input and output arguments and try to understand its behavior. For example:

```
>> checkarg
>> s = checkarg(-6)
>> s = checkarg(23,7)
>> [s,p] = checkarg(3,4,5)
```

■

7.2.2 Local and global variables

Each m-file function has access to a part of memory separate from MATLAB 's workspace. This is called the *function workspace*. This means that each m-file function has its own **local** variables, which are separate from those of other function and from the workspace. To understand it better analyze the following diagram:

MATLAB **workspace**		**myfun.m**

`>> a = -1;`
`>> b = 20;`
`>> c = myfun (a,b)`
	(a, b) ⟶ (x, y)
	c ⟵ z
		`function z = myfun (x,y)`
		`...`
		`z = x + cos(x-y);`
		`return;`

In the MATLAB workspace, variables `a`, `b` and `c` are available. Variables `x`, `y` and `z` are visible only in the function `myfun`. However, if several functions and/or the workspace, **all** declare a particular variable as `global`, then they all share this variable (see `help global`). Any assignment to that variable is available to all other functions and/or the workspace. However, you should be careful when using global variables. It is very easy to get confused and end up with serious errors.

OPTIONAL

7.2.3 Indirect function evaluation

Using indirect function evaluation makes programming even more general, since functions can become input arguments. The crucial MATLAB command here is `feval`, an abbreviation of **f**unction **eval**uation. The `feval` command allows execution of a function specified by a string. The general definition is as follows:

```
[y1,..,yn] = feval (F,x1,...,xn),
```

where `F` is a name of a function defined in MATLAB, `x1,...,xn` are input arguments and `y1,...,yn` are possible output parameters. Consider an example:

```
>> x = pi; y = cos(x);
>> z = feval('cos',x);
```

The last command is also equivalent to the following two expressions:

```
>> F = 'cos';
>> z = feval(F,x)
```

Indirect function evaluation is a nice tool to build a program with a function given as an argument.

☞ **Exercise 71.**
Create the function `funplot` and try to understand how it works:

```
function funplot (F, xstart, xend, col);
%FUNPLOT  makes a plot of the function F at the interval [xstart, xend].
%         The plot should be made in one of the standard Matlab colors, so
%         'col' is one of the following value: 'b','k','m','g','w','y' or 'r'.
% default values:
%       [xstart,xend] = [0,10]
%       col = 'b'

% Note: illustrates the use of feval command

if (nargin == 0)
  error ('No function is provided.');
end
if (nargin < 2)
  xstart = 0;
  xend   = 10;
end
if (nargin == 2)
  error ('Wrong number of arguments. You should provide xstart and xend.');
end
if (nargin < 4)
  col = 'b';
end

if (xstart == xend),
   error ('The [xstart, xend] should be a non-zero range.');
elseif (xstart > xend),
   exchange = xend;
   xend     = xstart;
   xstart   = exchange;
end

switch col
  case {'b','k','m','g','w','y','r'}
    ;                                   % do nothing; the right color choice
  otherwise
    error ('Wrong col value provided.')
end

x = linspace(xstart, xend);
```

```
y = feval(F,x);
plot (x,y,col);
description = ['Plot of ', F];
title (description);
return;
```

Note the use of comments, the `nargin` variable and the `switch`-construction. Call `funplot` for different built-in functions, like `sin`, `exp`, etc. Test it for your own functions as well. Write for example a function `myfun` that computes $\sin(x\,\cos(x))$ or $\log(\,|x\,\sin(x)|\,)$. Explain why it would be wrong to use the fragment given below instead of its equivalent part in `funplot`.

```
if nargin < 2
   xstart = 0;
   xend   = 10;
elseif nargin < 3
   error ('Wrong number of arguments. You should provide xstart and xend.');
elseif nargin < 4
   col = 'b';
end
```

■

END OPTIONAL

7.3 Scripts vs. functions

The most important difference between a script and a function is that *all* script's parameters and variables are externally accessible (i.e. in the workspace), where function variables are not. Therefore, a script is a good tool for documenting work, designing experiments and testing. In general, create a function to solve a given problem for arbitrary parameters. Use a script to run functions for specific parameters required by the assignment.

☞ **Exercise 72.**
Create the function `binom` that computes the value of the binomial symbol $\binom{n}{k}$. Make the function header: `function b = binom (n,k)`. Note that in order to write this function, you will have to create the `factorial` function, which computes the value of $n! = 1 * 2 * ... * n$. This may become a separate function (enclosed in a new file) or a subfunction in the `binom.m` file. Try to implement both cases if you got acquainted with the notion of a subfunction. Having created the `binom` function, write a script that displays on screen all the binomial symbols for $n = 8$ and $k = 1, 2, ..., 8$ or write a script that displays on screen the following 'triangle' (use the `fprintf` command; try `help fprintf` or see section **8.2**):

$$\begin{array}{llll}
\binom{1}{1} & & & \\
\binom{2}{1} & \binom{2}{2} & & \\
\binom{3}{1} & \binom{3}{2} & \binom{3}{3} & \\
\binom{4}{1} & \binom{4}{2} & \binom{4}{3} & \binom{4}{4}
\end{array}$$

■

☞ **Exercise 73.**
Solve the following exercises by using either a script or a function. The nature of the input/output and display options is left to you. Problems are presented with the increasing difficulty; start simple and add complexity. If possible, try to solve all of them.

1. Write a function which computes the cumulative product of a vector elements. The cumulative product up to x_j - the j-th element of the vector x is defined by $C_j = \prod_{k=1}^{j} x_k$ for $j = 1 : \mathtt{length}(\mathtt{x})$. Check your results with the built-in function `cumprod`.
2. Write MATLAB function `m = wmean (x,w)` computing the weighted arithmetic mean, given the vector `x` and a vector of nonnegative weights `w`, such that $\sum_{i=1}^{n} w_i > 0$, i.e. $\frac{\sum_{i=1}^{n} w_i\, x_i}{\sum_{i=1}^{n} w_i}$. Add error messages to terminate execution of the function in the case when:
   – `x` and `w` are of different lengths,
   – at least one element of `w` s negative,
   – sum of all weights is equal to zero.

3. Compute the value of π using the following series:

$$\frac{\pi^2 - 8}{16} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2\,(2n+1)^2}$$

   How many terms are needed to obtain an accuracy of 1e-12? How accurate is the sum of 100 terms?

4. Write a program that approximates π by computing the sum:

$$\frac{\pi}{4} \approx \sum_{n=0}^{m} \frac{(-1)^n}{2n+1}$$

   The more terms in summation the larger the accuracy (although this is not an efficient formula, since you add and subtract numbers). How many terms are needed to approximate π with 5 decimals? Use the sum to approximate π using 10, 100, 1e3, 1e4, 5e4, 1e5, 5e5 and 1e6 terms. For each of these numbers compute the approximation error. Plot the error as a function of the term numbers used in a summation.

5. The Fibonacci numbers are computed according to the following relation:

$$F_n = F_{n-1} + F_{n-2}, \quad \text{with} \quad F_0 = F_1 = 1$$

   - Compute the first 10 Fibonacci numbers.
   - For the first 50 Fibonacci numbers, compute the ratio $\frac{F_n}{F_{n-1}}$. It is claimed that this ratio approaches the value of the golden mean $\frac{1+\sqrt{5}}{2}$. What do your results show?

6. Consider a problem of computing the n-th Fibonacci number. Find three different ways to implement this and construct three different functions, say `fib1`, `fib2` and `fib3`. Measure the execution time of each function (use the commands `tic` and `toc`) for, say, $n = 20, 40$ or 100.

7. The Legendre polynomials $P_n(x)$ are defined by the following recurrence relation:

$$(n+1)\,P_{n+1}(x) - (2n+1)\,x\,P_n(x) + n\,P_{n-1}(x) = 0$$

   with $P_0(x) = 1$, $P_1(x) = x$ and $P_2(x) = (3x^2 - 1)/2$. Compute the next 3 Legendre polynomials and plot all 6 over the interval $[-1, 1]$.

8. Write a script that asks for an integer n (or a function that has n as the input argument) and then computes the following: while the value of n is greater than 1, replace the integer with $(n/2)$ if the integer is even. Otherwise, replace the integer with $(3n + 1)$. Make provision to count the number of values in (or the length of) the sequence that results.

   Example calculation: If $n = 10$, the sequence of integers is $5, 16, 8, 4, 2, 1$, so the length is 6.

   Make a plot of the length of the sequence that occurs as a function of the integers from 2 to 30. For example, when $n = 10$, the length is 6 while for $n = 15$, the length is 17. Is there any pattern? Try larger numbers to see if any pattern occurs. Is there any integer for which the sequence does not terminate?

9. Provide all prime numbers smaller then the given N.

8 Text

8.1 Character strings

94-99 Although MATLAB mainly works with numbers, it might occasionally be necessary to operate on text. Text is saved as a character string. The string is a vector of ASCII values which are displayed as their character string representation. Since text is a vector of characters, it can be addressed in the same way as any vector. An example:

```
>> t = 'This is a character string'
t =
This is a character string
>> size(t)
```

```
ans =
     1   27
>> whos
  Name      Size         Bytes  Class
  t         1x27            54  char array
>> t(10:19)
ans =
 character
>> t([2,3,10,17])
ans =
hi t
```

To see the underlying ASCII representation, the string has to be converted using the command `double` or `abs`:

```
>> double(t(1:12))
ans =
    84   104   105   115    32   105   115    32    97    32    99   104
```

The function `char` provides the reverse transformation:

```
>> t([16:17])
ans =
ct
>> t([16:17])+3              % to perform a mathematical operation ASCII codes are used
ans =
   102   119
>> t([16:17])-3              % ASCII codes again
ans =
    96   113
>> char(t([16:17])-2)        % transform ASCII codes to characters
ans =
ar
```

Exercise 74.
Perform the described commands. Use string `t` to form a new string `u` that contains the word "character" only. Convert string `u` to form the same word spelled backwards, i.e. `u1 = retcarahc`. ■

The tables on page 97 and 98 of "The Student Edition of Matlab" book give an overview of string conversions and functions. With e.g. `findstr` a string can be searched for a character or the start of a combination of characters. Such an example with a few other examples on the use of different string functions is given below:

```
>> findstr(t, 'c')          % finds a character 'c' in string t; positions are returned
ans =
    11 16
>> findstr(t, 'racter')     % finds the beginning of a string 'racter' in t
ans =
    14
>> findstr(t,u)              % finds string u in string t; returns the position
ans =
    11
>> strcat(u,u1)              % concatenates multiple strings: u and u1
ans =
    characterretcarahc
>> strcmp(u,u1)              % comparison of two strings;
ans =                        % 1 for identical strings; 0 for different strings
     0
>> q = num2str(34.35)        % converts a number into a string
q =
34.35
>> z = str2num('7.6')        % converts a string into a number
z =
```

```
    7.6
>> whos q z                 % q is now a string, i.e. a character array; z is a number
  Name      Size         Bytes  Class
  q         1x5             10  char array
  z         1x1              8  double array
>> t = str2num('1 -7  2')   % converts a string into a vector of number
t =
     1    -7     2
>> t = str2num('1 - 7  2')  % here spaces around the sign - or + are important!
t =                         % performs: [1-7, 2]
    -6     2
>> A = round(4*rand(3,3))+0.5;
>> ss = num2str(A)          % since A is random, you will get different numbers here
ss =
-3.5        -3.5         6.5
-2.5        -1.5         0.5
 5.5        -1.5        -3.5
>> whos ss
  Name      Size         Bytes  Class
  ss        3x28           168  char array
>> ss(2,1), ss(3,15:28)    % ss is a char array; you can address it in a standard way
ans =
-
ans =
.5        -3.5
>> ss(1:2,1:3)
ans =
-3.
-2.
```

Exercise 75.
Perform the commands shown in the example above. Define another string, e.g. `s = 'Nothing wastes more energy than worrying'` and exercise with `findstr`. ■

Exercise 76.
Become familiar with `num2str` and `str2num` commands. Check e.g. what happens with `ss = num2str([1 2 3; 4 5 6])` or `q = str2num('1 3.25; 5 5.5')`. Analyze also the commands `str2double` and `int2str` (use `help`). ■

8.2 Text input and output

The `input` command can be used to prompt (ask) the user for numeric or string input:

31-33

```
>> myname = input('Enter your name: ','s');
>> age = input('Enter your age: ');
```

You have learned in section **7.2** that inputs can be passed on to functions. This is a recommended approach, since using the `input` command for more complex programs makes automatic computation impossible.

There are two common text output functions: `disp` and `fprintf`. The `disp` function only displays the value of one parameter, either a numerical array or a string (the recognition is done automatically). For example:

```
>> disp('This is a statement.')      % a string
This is a statement.
>> disp(rand(3))                     % a matrix
    0.2221    0.0129    0.8519
    0.4885    0.0538    0.5039
    0.2290    0.3949    0.4239
```

The `fprintf` function (familiar to C programmers) is useful for writing data to a file (see section **11.1**) or
97 printing on screen when precise formatting is important. Try, for instance (an explanation will follow):

```
>> x = 2;
```

```
>> fprintf('Square root of %g is %8.6f.\n', x, sqrt(x));
Square root of 2 is 1.414214.
>> str = 'beginning';
>> fprintf('Every %s is difficult.\n',str);
Every beginning is difficult.
```

Formally, `fprintf` converts, formats and prints its arguments to a file or displays them on screen according to a format specification. For displaying on screen, the following syntax is used:

```
fprintf (format,a,...)
```

The `format` string contains ordinary characters, which are copied to the output, and *conversion specifications*, each of which converts and prints the next successive argument to `fprintf`. Each conversion specification is introduced by the character '%' and ended by a conversion character. Between the '%' and the conversion character there may appear:

- a minus sign; controlling the alignment within the field defined.
- a digit string specifying a minimum field length; The converted number will be printed in a field at least this wide, and wider if necessary.
- a period which separates the field width from the next digit string;
- a digit string - the precision which specifies the maximum number of characters to be printed from a string or the number of digits printed after decimal point of a `single` or `double` type.

Format specification is given below in terms of the conversion characters and their meanings:

	The argument
`d`	is converted into decimal notation;
`u`	is converted into unsigned decimal notation;
`c`	is taken to be a single character;
`s`	is a string;
`e`	is taken to be a `single` or `double` and converted into decimal notation of the form: `[-]m.nnnnnnE[±]xx`, where the length of `n`'s is specified by the precision;
`f`	is taken to be a `single` or `double` and converted into decimal notation of the form: `[-]mmm.nnnnn`, where the length of `n`'s is specified by the precision;
`g`	is specified by `e` or `f`, whichever is shorter; non-significant zeros are not printed;

The special formats `\n, \r, \t, \b` can be used to produce next line (i.e. makes sure that the next command or output will be written on the next line), carriage return, tab, and backspace respectively. Use `\\` to produce a backslash character and `%%` to produce the percent character.

Analyze the following example:

```
>> fprintf('look at %20.6e!\n', 1000*sqrt(2))
look at          1.414214e+3!
>> fprintf('look at %-20.6f!', 1000*sqrt(2))
look at 1414.213562         ! >>
```

For both commands, the minimum field length is 20, the number of digits after the decimal point is 6. In the first case the value of $1000 * \sqrt{2}$ is padded on the right, in the second, because of the `-` on the left. The difference in the presentation of the value of $1000 * \sqrt{2}$ is caused by the conversion characters e and f.

Exercise 77.
Try to exercise to understand how to use the `input`, `disp` and `fprintf` commands. For instance, try to read a vector with real numbers using `input`. Then, try to display this vector, both by calling `disp` and formatting an output by `fprintf`. Make a few variations. Try the same with a string. ■

Exercise 78.
Study the following examples in order to become familiar with the `fprintf` possibilities and exercise yourself to understand how to use these specifications:

```
>> str = 'life is beautiful';
>> fprintf('My sentence is: %s\n',str);          % note the \n format
My sentence is: life is beautiful
>> fprintf('My sentence is: %30s\n',str);
```

```
My sentence is:              life is beautiful
>> fprintf('My sentence is: %30.10s\n',str);
My sentence is:                       life is be
>> fprintf('My sentence is: %-20.10s\n',str);
My sentence is: life is be
>>
>> name = 'John';
>> age = 30;
>> salary = 6130.50;
>> fprintf('My name is %4s. I am %2d. My salary is f %7.2f.\n',name, age, salary);
>> My name is John. I am 30. My salary is f 6130.50.
>>
>> x = [0, 0.5, 1];
>> y = [x; exp(x)];
>> fprintf('%6.2f  %12.8f\n',y);
  0.00    1.00000000
  0.50    1.64872127
  1.00    2.71828183
>> fprintf('%6.1e  %12.4e\n',y);
0.0e+00    1.0000e+00
5.0e-01    1.6487e+00
1.0e+00    2.7183e+00
>>
>> x = 1:3:7;
>> y = [x; sin(x)];
>> fprintf('%2d  %10.4g\n',y);
 1      0.8415
 4     -0.7568
 7       0.657
```

■

Warning: `fprintf` uses its first argument to decide how many arguments follow and what their types are. If you provide a wrong type or there are not enough arguments, you will get nonsense for an answers. So, be careful with formatting your output. Look, e.g., what happens in the following case (`age` is not provided):

```
>> fprintf('My name is %4s. I am %2d. My salary is f %7.2f.\n',name,salary);
My name is John. I am 6.130500e+03. My salary is f >>
```

Remark: The function `sprintf` is related to `fprintf`, but writes to a string instead. Analyze the example:

```
>> str = sprintf('My name is %4s. I am %2d. My salary is f %7.2f.\n',name,age,salary)
str =
My name is John. I am 30. My salary is f 6500.50.
```

☞ **Exercise 79.**
Define a string `s = 'How much wood could a wood-chuck chuck if a wood-chuck could chuck wood?'`. Exercise with `findstr`, i.e. find all appearances of the substrings `'wood'`, `'o'`, `'uc'` or `'could'`. Try to build `ss` by concatenation of seperate words. Try to do this in a few ways, e.g. make use of `strcat`, `disp`, `sprintf` or `fprintf`. ■

☞ **Exercise 80.**

- Write a script/function that converts a Roman numeral to its decimal equivalent. There are two distinct situations:

  The 'old' style where the order of the symbols does not matter. In this case, IX and XI both mean 10 + 1 or 11. You should be able to handle the following conversion table:

Roman	Decimal
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

The 'new' style where the order of the symbols does matter. For example, IX is 9 (10 - 1), XC is 90 (100 - 10). The conversion table given above still holds and you may assume for this case that the only instances of order you will encounter are: IV (4), IX (9), XL (40), XC (90), CD (400) and CM (900). The function `input` will be useful here. *Hint:* try the case of the 'old' style first.

- Write a function that will do the inverse of the previous problem - convert a decimal number into a Roman number.

■

Exercise 81.
Write a coder (and a decoder) based on a Caesar encoding scheme. This scheme is based on a fixed shift of all letters of the alphabet to the right, e.g., say with a shift 1, 'abcde' becomes 'bcdef' and 'alphabet' becomes 'bmqibcfu'. Of course, 'z' becomes then 'a'. You can imagine a shift with 2, 3, etc, but the shift 26 gives again the same alphabet. Write two functions `coder` and `decoder` whose one of the input arguments is shift. The second input argument should be a string or a text file where the message is written. In case of coding, the string or the text file should contain the original message, and in case of decoding, you should provide an encoded string or file. ■

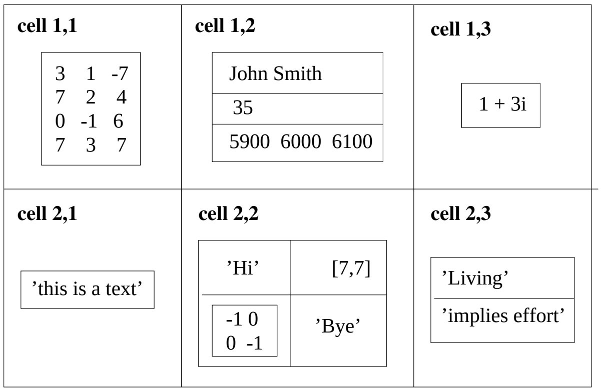
OPTIONAL

9 Cell arrays and structures

9.1 Cell arrays

239-245
99-102

Cell arrays are arrays whose elements are *cells*. Each cell can contain any data, including numeric arrays, strings, cell arrays etc. For instance, one cell can contain a string, another a numeric array etc. Below, there is a schematic representation of an exemplar cell array:



Cell arrays can be built up by assigning data to each cell. The cell contents are accessed by brackets {}. For example:

```
>> A(1,1) = {[3 1 -7;7 2 4;0 -1 6;7 3 7]};
>> A(2,1) = {'this is a text'};
>> B(1,1) = {'John Smith'}; B(2,1) = {35}; B(3,1) = {[5900 6000 6100]}
>> A(1,2) = {B};     % cell array B becomes one of the cells of A
>> C(1,1) = {'Hi'}; C(2,1) = {[-1 0;0 -1]}; C(1,2) = {[7,7]}; C(2,2) = {'Bye'};
>> A(2,2) = {C};     % cell array C becomes one of the cells of A
>> A                 % A represents now the first 2 columns of the exemplar cell array
   A =
        [4x3 double]    {3x1 cell}
    'this is a text'    {2x2 cell}
>> A(2,2)            % access the cell but not its contents
ans =
    {2x2 cell}
>> A{2,2}            % use {} to display the contents
ans =
    'Hi'            [1x2 double]
    [2x2 double]    'Bye'
```

```
>> A{2,2}{2,1}       % take the (2,1) element of the cell A{2,2}
 ans =
    -1     0
     0    -1
```

241 There are also two useful functions with meaningful names: `celldisp` and `cellplot`. Use `help` to learn more.

The common application of cell arrays is the creation of text arrays. Consider the following example:

```
>> M = {'January';'February';'March';'April';'May';'June';'July';'August';
'September';'October';'November';'December'};
>> fprintf ('It is %s.\n', M{9});
It is September.
```

Exercise 82.
Exercise with the concept of a cell array, first by typing the examples presented above. Next, create a cell array `W` with the names of week days, and using the command `fprintf`, display on screen the current date with the day of the week. The goal of this exercise is also to use `fprintf` with a format for a day name and a date, in the spirit of the above example. ■

9.2 Structures

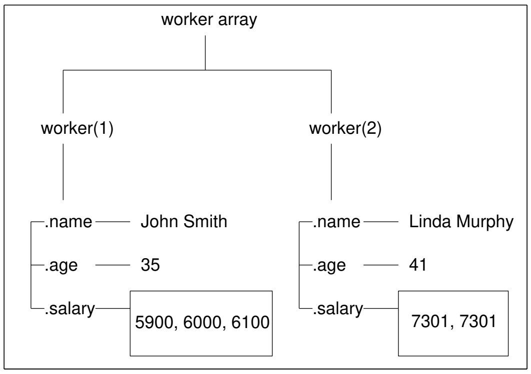
245-249 Structures are MATLAB arrays with data objects composed of *fields*. Each field contains one item of information. For example, one field might include a string representing a name, another a scalar representing age or an array of the last few salaries. Structures are especially useful for creating and handling a database. One of the possible ways to create a structure is by assigning data to individual fields. Imagine that you want to construct a database with some information on workers of a company:

```
>> worker.name = 'John Smith';
>> worker.age = 35;
>> worker.salary = [5900, 6000, 6100];
>> worker =
      name: 'John Smith'
       age: 35
    salary: [5900 6000 6100]
```

In this way, a 1×1 structure array `worker` with three fields: `name`, `age` and `salary` is constructed. To expand the structure, add a subscript after the structure name:

```
>> worker(2).name = 'Linda Murphy';  % after this, a 2nd subarray is created
>> worker(2).age = 41;
>> worker(2).salary = [7301, 7301];  % field sizes do not need to match!
>> worker
1x2 struct array with fields:
    name
    age
    salary
```

Since the structure has now size 1×2, MATLAB does not display contents of all fields. The data is now organized as follows:



Structures can also be build by using the `struct` function, for example:

```
>> employee=struct( 'name','John Smith','age',35,'salary',[5900, 6000, 6100]);
```

To access an entire field, include a field name after a period. To access a subarray, follow the standard way of using subscripts:

```
>> worker(1).age
   35
>> worker(2).salary(2)
       7301
>> worker(2)
      name: 'Linda Murphy'
       age: 41
    salary: [7301 7301]
```

An alternative way is to use the `getfield` function:

```
>> getfield(worker,{2},'salary')
        7301        7301
```

There exist also the function `setfield`, which allows assigning values . New fields can be added or deleted
248 from every structure. To delete a field, use the command `rmfield`. Analyze the following example:

```
>> worker2 = worker;
>> worker2 = rmfield (worker, 'age');
>> worker2(2).street = 'Bakerstreet 5';
>> worker2(2)
      name: 'Linda Murphy'
    salary: [7301 7301]
    street: 'Bakerstreet 5'
>> worker2(1)
    name: 'John Smith'
    salary: [5900 6000 6100]    % in all other substructures address field is empty
    street: []                  % you should assign new values for each substructure
```

Operating on fields and field elements is done in the same way as on any other array. Consider the following example, where the average salary for each worker is computed.

```
avr_salary(1) = mean (worker(1).salary);
avr_salary(2) = mean (worker(2).salary);
```

Remark: structures as a concept are common organizations of data in other programming languages. They are created in different ways, but the intention remains the same.

Exercise 83.
Construct a structure `friend`, with the following fields: `name`, `address`, `age`, `birthday`. Insert a few friends with related information. Remove e.g. the field `age` and add the field `phone`. ■

General remark: *Classes* and *objects* allow for adding new data types and new operations to MATLAB. For instance, the class of a variable describes the structure of the variables and the operations permitted as well as functions to be used. An object is an instance of a particular classs. The use of classes and objects is the base of *object-oriented programming*, which is also possible in MATLAB.

END OPTIONAL

10 Optimizing the performance of MATLAB code

MATLAB is a matrix language, so it is designed for matrix operations. For best performance you should take advantage of this fact.

10.1 Vectorization - speed-up of computations

43-62 Vectorization is simply the use of compact expressions that operate on all elements of a vector without explicitly executing a loop. MATLAB is optimized such that vector or matrix operations are much more efficient than loops. Most built-in functions support vectorized operations. So, try to replace loops with vector operations, when possible. For example, instead of using:

```
for i = 1:10
  t(i) = 2*i;
  y(i) = sin (t(i));
end
```

try this:

```
t = 2:2:20;
y = sin (t);
```

Copying or transforming matrices can be vectorized, as well. Check the equivalence between the scalar version:

```
n = 10;
A = rand(n,n);
B = ones(n,n);
for k=1:n
  B(2,k) = A(2,k);           % A and B have the same size
end
```

and the vectorized code of the last loop:

```
B(2,:) = A(2,:);
```

or between the following loop:

```
for k=1:n
  B(k,1) = A(k,k);
end
```

and the vectorized expression:

```
B(1:n,1) = diag(A(1:n,1:n));
```

Logical operations and proper use of the colon notation make the programs work faster. However, some experience is needed to do this correctly. Therefore, the advice is: **start first with the scalar code and then vectorize it where possible, by removing loops.**

Exercise 84.
Look at your script or function m-files already written, especially your loop exercises. Try to vectorize their codes when possible. ■

Exercise 85.
Try to vectorize the following codes:

```
% code 1
n = 20;
m = 10;
A = rand(n,m);
for i=1:n
  for k=1:m
     if (A(i,j) > 0.5)
       A(i,j) = 1;
     else
       A(i,j) = 0;
     end
  end
end
```

```
% code 2
n = 20;
m = 10;
A = randn(n,m);
x = randn(n,1);
p = zeros(1,m);
for i=1:m
  p(i) = sum (x .* A(:,i));    % what is p here?
end
```

■

10.2 Array preallocation

In 'real' programming languages memory allocation is necessary. In MATLAB, it is not needed, but it improves the execution speed and besides, practicing it, is a good habit. Preallocate the arrays, which store output results. This prevents MATLAB from resizing an array each time you enlarge it. Preallocation also helps to reduce memory fragmentation if you work with large matrices. During the MATLAB session memory can be fragmented. As a result, there may be plenty of free memory, but not enough continuous space to store a large variable.

Exercise 86.
Create two scripts with the listings given below, run them and compare the results, i.e. measured times of their performances. The pair of commands `tic` and `toc` is used to measure the execution time of the operations:

```
% script 1
clear s x;
tic;
x = -250:0.1:250;

for i=1:length(x)
  if (x(i) > 0)
    s(i) = sqrt(x(i));
  else
    s(i) = 0;
  end
end
toc;
```

```
% script 2
clear s x;
tic;
x = -250:0.1:250;
s = zeros (size(x));     % preallocate memory
for i=1:length(x)
  if (x(i) > 0)
    s(i) = sqrt(x(i));
  end


end
toc;
```

Remember that scripts work in the workspace, interfering with the existing variables. Therefore, for a fair comparison the variables `s` and `x` should be removed before running each script. Having done this, try to
107;128 vectorize the code. ■

Exercise 87.
Consider the following sequence of commands:

```
>> d = pi/50;
>> n = 2 + pi/d;
>> m = n/2;
>> for j = 1:n
>>   x(j) = (i+3)*d;
>>   y(j) = cos(2*x(i));
>> end
>> for j = n+1:m
>>   x(j) = (i-1)*d;
>>   y(j) = cos(4*x(i));
>> end
```

Improve the performance of the code above. Pre-allocate memory for the vectors `x` and `y`. Vectorize the calculation of these vectors. ■

10.3 MATLAB'S tricks and tips

Many of the MATLAB's tricks use the fact that there are two ways of addressing matrix elements using a vector as an index:

1. If `x` and `y` are vectors, then `x(y)` is the vector `[x(y(1)), x(y(2)), ..., x(y(n))]`, where `n=length(y)`. For instance:

```
>> x = [3 -1 4 2 7 2 3 5 4];
>> y = 1:2:9; x(y)
ans =
   3     4     7     3     4
>> y = [5 5 5]; x(y)
ans =
     7     7     7
```

```
>> y = [1 5 1 1 7]; x(y)
ans =
     3     7     3     3     3
```

2. If x and y are vectors of the same size and y only consists of 0s and 1s then MATLAB interprets y via logical-indexing. As a result, the elements of x are returned whose position corresponds to the location of a 1 in x. For instance:

```
>> x = [3 -1 4 2 7 2 3 5 4];
>> y = x < 4, x(y)
y =
     1     1     0     1     0     1     1     0     0
ans =
     3    -1     2     2     3
>> y = (x == 2) | (x > 6), x(y)
y =
     0     0     0     1     1     1     0     0     0
ans =
     2     7     2
```

The examples before should serve you to optimize your MATLAB routines. Use `help` when necessary to learn more about commands used and test them with small matrices. When more solutions are given, try to use `tic` and `toc` to measure the performance for large matrices to decide which solution is faster.

- Create a row (column) vector of n uniformly spaced elements:

```
>> a  = -1; b = 1; n = 50;
>> x1 = a:2/(n-1):b;            % a row vector
>> y1 = (a:2/(n-1):b)';         % a column vector
>> x2 = linspace(a,b,n);        % a row vector
>> y2 = linspace(a,b,n)';       % a column vector
```

- Shift k (k should be positive) elements of a vector:

```
>> x = [3 -1 4 2 7 2 3 5 4];
>> x([end  1:end-1]);           % shift right (or down for columns) 1 element
>> k = 5;
>> x([end-k+1:end  1:end-k]);  % shift right (or down for columns) k elements
>> x([2: end  1]);              % shift left  (or up for columns) 1 element
>> x([k+1:end  1:k]);           % shift left  (or up for columns) k elements
```

- Initialize a vector with a constant.

```
>> n = 10000;
>> x = 5 * ones(n,1);          % 1st solution
>> y = repmat(5,n,1);          % 2nd solution - should be faster
>> z = zeros(n,1); z(:)=5;     % 3rd solution - should be the fastest
```

- Create an $n \times n$ matrix of 3's.

```
>> n = 1000;
>> A = 3 * ones(n,n);        % 1st solution
>> B = repmat(3,n,n);        % 2nd solution - should be much, much faster!
```

- Create a matrix consisting of a row vector duplicated m times:

```
>> m = 10;
>> x = 1:5;
>> A = ones(m,1) * x;        % 1st solution
>> B = x(ones(m,1),:);       % 2nd solution - should be faster
```

  Create a matrix consisting of a column vector duplicated n times:

```
>> n = 5;
>> x = (1:5)';
>> A = x * ones(1,n);        % 1st solution
>> B = x(:,ones(n,1));       % 2nd solution - should be faster
```

- Given a vector x, create a vector y in which each element is replicated n times:

```
>> n = 5;
>> x = [2 1 4];
>> y = x(ones(1,n),:); y = y(:)';
>> x = [2 1 4]';
>> y = x(:,ones(1,n))'; y = y(:);
```

- Reverse a vector:

```
>> x = [3 -1 4 2 7 2 3 5 4];
>> n =length(x);
>> x(n:-1:1)                % 1st solution
>> fliplr(x)                % 2nd solution - should be faster
```

Reverse one column of a matrix:

```
>> A = round (5*rand(4,5));
>> c = 2;
>> A(:,c) = flipud (A(:,c));
```

- Interchange rows or columns of a matrix:

```
>> A  = round (5*rand(4,5));
>> i1 = 1; i2 = 4;
>> A([i1,i2],:) = A([i2,i1],:);     % swap rows
>> A(:,[i1,i2]) = A(:,[i2,i1]);     % swap columns
```

- Making a row from a matrix A is done by A(:).
- Find out those elements which are shared by two matrices (or vectors):

```
>> A = round (5*rand(4,5));
>> B = round (6*rand(3,6));
>> intersect (A(:),B(:))            % different sizes of A and B are permitted
```

- Combine two vectors into one, removing repetitive elements:

```
>> x = 1:10;
>> y = [1 5 4 1 7 -1 2 2 6];
>> union (x,y)
```

- Find unique elements in a vector:

```
>> x = [1 5 4 1 7 -1 2 2 6 1 1];
>> unique (x)
```

- Keep only the diagonal elements of the matrix multiplication, i.e. vectorize the loop (both A and B are of n-by-m matrices):

```
z = zeros(n,1);
for i=1:n
  z(i) = A(i,:) * B(i,:)';
end
```

The solutions:

```
>> z = diag(A * B');               % 1st solution
>> z = sum (A .* B, 2);            % 2nd solution solution - should be faster
```

- Scale all columns of the matrix A by a column x:

```
>> A = round (7*rand(4,5));
>> [n,m] = size(A);
>> x = (1:n)';                     % x is n-by-1
>> B = A ./ x(:,ones(m,1));
```

Scale all rows of the matrix A by a row vector x:

```
>> A = round (7*rand(4,5));
>> [n,m] = size(A);
>> x = 1:m;                            % x is 1-by-m
>> B = A ./ x(ones(n,1),:);
```

Other useful tips are:

- Use the `ginput` command to to input data with a mouse. Try, for instance:

```
>> x = 0; y = 0;
>> while ~isempty(x)
    [x1,y1] = ginput(1);
    plot([x x1],[y y1],'b.-');
    hold on
    x = x1; y = y1;
   end
```

- If you want to find out what takes so long in you MATLAB code, use the command `profile`, which 'helps you debug and optimize M-files by tracking their execution time. For each function, the profiler records information about execution time, number of calls, parent functions, child functions, code line hit count, and code line execution time.' Try, e.g.:

```
>> profile on -detail builtin
>> rgb = imread('ngc6543a.jpg');
>> profile report
```

11 File input/output operations

404-406 MATLAB file input and output (I/O) functions read and write arbitrary binary and formatted text files. This enables you to read data collected in other formats and to save data for other programs, as well. Before reading or writing a file you must open it with the `fopen` command:

```
>> fid = fopen (file_name, permission);
```

The `permission` string specifies the type of access you want to have:

- `'r'` - for reading only
- `'w'` - for writing only
- `'a'` - for appending only
- `'r+'` - both for reading and writing

Here is an example:

```
>> fid = fopen ('results.txt','w')    % tries to open the file results.txt for writing
```

The `fopen` statement returns an integer *file identifier*, which is a handle to the file (used later for addressing and accessing your file). When `fopen` fails (e.g. by trying to open a non-existing file), the file identifier becomes -1. It is also possible to get an error message, which is returned as the second optional output argument.

It is a good habit to test the file identifier each time when you open a file, especially for reading. Below, the example is given, when the user provides a string until it is a name of a readable file:

```
fid = 0;
while fid < 1
  fname = input ('Open file: ', 's');
  [fid, message] = fopen (fname, 'r');
  if (fid == -1)
    disp (message);
  end
end
```

Exercise 88.
Create a script with the code given above and check its behavior when you give a name of a non-existing file (e.g. `noname.txt`) and a readable file (e.g. one of your functions). ■

When you finish working on a file, use `fclose` to close it up. MATLAB automatically closes all open files when you exit it. However, you should close your file when you finished using it:

```
fid = fopen ('results.txt', 'w');
...
fclose(fid);
```

11.1 Text files

The `fprintf` command converts data to character strings and displays it on screen or writes it to a file. The general syntax is:

```
fprintf (fid,format,a,...)
```

For more detailed description, see section **8.2**. Consider the following example:

```
>> x = 0:0.1:1;
>> y = [x; exp(x)];
>> fid = fopen ('exptab.txt','w');
>> fprintf(fid, 'Exponential function\n');
>> fprintf(fid, '%6.2f  %12.8f\n',y);
>> fclose(fid);
```

Exercise 89.
Prepare a script that creates the `sintab.txt` file, containing a short table of the sinus function. ■

The `fscanf` command is used to read a formatted text file. The general function definition is:

```
 [A,count] = fscanf (fid, format, size)
```

This function reads text from a file specified by file identifier `fid`, converts it according to the given `format` (the same rules apply as in case of the `fprintf` command) and returns it in a matrix A. `count` is an optional output argument standing for the number of elements successfully read. The optional argument `size` says how many elements should be read from the file. If it is not given, then the entire file is considered. The following specifications can be used:

- `n` - read at most `n` elements into a column vector;
- `inf` - read at most to the end of the file;
- [m, n] - read at most `m n` elements filling at least an `m`-by-`n` matrix, in column order; n can be inf.

Here is an example:

```
>> a = fscanf (fid, '%5d', 25);        % read 25 integers into a vector a
>> A = fscanf (fid, '%5d', [5 5]);     % read 25 integers into a 5 x 5 matrix A
```

MATLAB can also read lines from a formatted text and store it in strings. Two functions can be used: `fgets` and `fgetl`. The only difference is that `fgetl` copies the newline character while `fgets` does not.

Exercise 90.
Create the following script and try to understand how it works (use the `help` command to learn more on the `feof` function):

```
fid  = fopen ('exptab.txt','r');
title = fgetl (fid);
k = 0;
while ~feof(fid)                        % as long as end-of-file is not reached do ...
  k = k+1;
  line = fgetl (fid);                   % get a line from the exponential table
  tab(k,:) = str2num (line);            % convert the line into a vector from tab matrix
end
fclose(fid);
```

Look at the matrix `tab`. How does it differ from the originally created matrix? ■

Reading lines from a formatted text file may be especially useful when you want to modify an existing file. Then, you may read a specified number of lines of a file and add something at the found spot.

Exercise 91.
Create a script that reads the `exptab.txt` file and at the end of the file adds new exponential values, say, for $x = 1.1 : 0.1 : 3$. Note that you should open the `exptab.txt` file both for reading and writing. ■

OPTIONAL

Exercise 92.
Write the function `countlet` that: opens a file specified by a name, reads this file line by line and counts the number of appearance of the specified letter (so, there are two input arguments: the name and the letter). Remember to return an error message when there is a problem with opening the file. Create a text file `test.txt` with some text, e.g. by writing random words or retyping a few sentences of a book. Test your function by calling:

```
>> c = countlet ('test.txt', 'd');
>> c = countlet ('countlet.m', 'a');          % YES, you can do this!
```

■ END OPTIONAL

11.2 Binary files

There are two important variables to write and read from a binary file: `fread` and `fwrite`. The definition of the function `fwrite` is given below:

```
count = fwrite (fid, A, precision)
```

This command writes the elements of matrix `A` to the provided file, while converting values to the specified precision. The data is written in column order. `count` is the number of successfully written elements.

The definition of the function `fread` is given below:

```
[A, count] = fread (fid, size, precision)
```

This reads binary data from the specified file and writes it into matrix `A`. `count` is an optional parameter, which returns the number of elements successfully read. The `size` argument is optional and its specification is the same as in case of the `fscanf` function (see section **11.1**). The `precision` argument controls the number of bits read for each value and their interpretation as character, integer or floating-point values, e.g. `'uchar'`, `'int16'` or `'single'` (learn more from `help`). By default, numeric values are returned in `double` precision.

OPTIONAL

Exercise 93.
Exercise with the following scripts:

```
% Write the 5-by-5 magic square into binary file
M = magic(5);                          % creates a magic square
fid1  = fopen ('magic5.bin','w');
count = fwrite (fid1, M, 'int16');     % count should be 25
fclose(fid1);
```

Note that a 50-byte binary file should appear (25 integer numbers, each stored in 2 bytes).

```
% Read the 5-by-5 magic square into binary file
fid2  = fopen ('magic5.bin','rb');
[A, count] = fread (fid2, [5,5], 'int16');      % count should be 25
fclose(fid2);
```

Check what will happen if you skip the specification of either size or type, e.g.

```
[B, count] = fread (fid2);
```

or

```
[B, count] = fread (fid2, [5,5]);
```

Note that each time you should open and close the magic5.bin file. ■

A particular location within a file is specified by a *file position indicator*. This indicator is used to determine where in the file the next read or write operation will begin. The MATLAB functions operating on the file position indicator are summarized below:

Function	Purpose
feof	determines if file position indicator reached the end-of-file
fseek	sets file position indicator to the specified byte with respect to the given origin
ftell	returns the location of file position indicator
frewind	resets file position indicator to beginning of file

To understand how fseek and ftell work, consider this script (you can use on-line help to learn more about these function's specifications):

```
a = 1:5;
fid = fopen ('five.bin','w');
count = fwrite (fid, a, 'single');
fclose(fid);
```

Five integers are written to the file five.bin with a single precision, so each number uses 4 bytes. Try to understand what is happening here:

```
>> fid = fopen ('five.bin','r');      % open for reading
>> status = fseek (fid, 12, 'bof');   % move the file position indicator forward 12 bytes
>>                                    % from the beginning of file 'bof'; status is 0 when
>>                                    % the operation was successful
>> four = fread (fid, 1, 'single');   % read one element at the current position, starting
>>                                    % from byte 13 so it should be the number 4
>> pos = ftell (fid);                 % check the position; the number 4 is read, so it
>>                                    % should be 16 now
>> status = fseek (fid, -8, 'cof');   % move the file position indicator backward 8 bytes
>>                                    % from the current position 'cof'
>> pos = ftell (fid);                 % check the position; it should be 8
>> three = fread (fid, 1, 'single');  % read one element, so it should be 3
>> fclose(fid);
```

END OPTIONAL

12 Writing and debugging MATLAB programs

The recommendations in this section are general for programming in any language. Learning them now will turn out to be beneficial in the future or while learning real programming languages like *C*, where structured programming is indispensable.

12.1 Structural programming

Never write all code at once; program in small steps and make sure that each of these small steps works as expected, before proceeding to the next one. Each step should be devoted to only one task. Do not solve too many tasks in one module, because you may easily make a mess. This is called a *structured* or *modular* way of programming. Formally, modularity is the hierarchical organization of a system or a task into self-contained subtasks and subsystems, each having a prescribed input-output communication. It is an essential feature of a well designed program. The benefit of structural programming are: easier error detection and correction, modifiability, extensibility and portability.

Below the general approach to a program development is presented:

1. **Specification.**
   Read and understand the problem. The computer cannot do anything itself: you have to tell it how to operate. Before using the computer, some level of preparation and thought is required. Some basic questions to be asked are:

- What are the parameters/inputs for this problem?
- What are the results/outputs for this problem?
- What form should the inputs/outputs be provided in?
- What sort of algorithms is needed to find the outputs from the inputs?

2. **Design.**
Split your problem into a number of smaller and easier tasks. Decide how to implement them. Start with a schematic implementation to solve your problem, e.g. create function headers or script descriptions (decide about the input and output arguments). To do this, you may use, for example, a top-down approach. You start at the most general level, where your first functions are defined. Each function may be again composed of a number of functions (subfunctions). While 'going down' your functions become more precise and more detailed.

As an example, imagine that you have to compare the results of the given problem for two different datasets, stored in the files `data1.dat` and `data2.dat`. Schematically, such a top-down approach could be designed as:

⋄ This is the top (the most general) level. A script `solve_it` is created:

```
[d1, d2] = read_data ('data1.dat', 'data2.dat');
[res1, err1] = solve_problem (d1);
[res2, err2] = solve_problem (d2);
compare_results (res1, res2, err1, err2);
```

⋄ This is the second level. The following functions `read_data`, `solve_problem` and `compare_results` belong here. Each of them has to be defined in a separate file:

- ```
  function [d1, d2] = read_data (fname1, fname2)
  % Here should be some description.
  %
  fid1 = fopen (fname1,'w');
  ....                           % check whether the file fname1 exists
  fclose(fid1);
  fid2 = fopen (fname2,'w');
  ....                           % check whether the file fname2 exists
  fclose(fid2);
  ....
  d1 = ...
  d2 = ...
  return;
  ```
- ```
  function [res, err] = solve_problem (d)
  % Here should be some (possibly detailed) description.
  %
  ....
  res = ...                      % the data d is used to compute res
  err = compute_error (res);
  return;
  ```
- ```
  function compare_results (res1, res2, err1, err2)
  % Some description.
  tol = 1e-6;
  ....
  if abs (err1 - err2) > tol
    fprintf ('The difference is significant.')
  else
    fprintf ('The difference is NOT significant.')
  end;
  return;
  ```

⋄ In this example, this is the last level. The function `solve_problem` uses the function: `compute_error`, which has to be defined:

```
function err = compute_error (res)
% Here should be some (possibly detailed) description.
```
```

```
%
....
err = ....                         % the variable res is used to compute err
return;
```

3. **Coding.**
   Implement the algorithms sequentially (one by one). Turning your algorithm into an efficient code is not a one-shot process. You will have to try, make errors, correct them and even modify the algorithm. So, **be patient**. While implementing, make sure that all your outputs are computed at some point. Remember about the comments and the style requirements (see section **12.3**).

4. **Running and debugging** (see also section **12.2**).
   Bugs will often exist in a newly written program. Never, ever, believe or assume that the code you just created, works. **Always check the correctness of each function or script: Twice.** You may add some extra lines to your code which will present the intermediate results (screen displays, plots, writes to files) to help you controlling what is going on. Those lines can be removed later.

5. **Testing and Verification.**
   After the debugging process, the testing stage starts. Prepare a number of tests to verify whether your program does what it is expected to do. Remember that good tests are those for which the answers are known. Your program should produce correct results for normal test conditions as well as boundary conditions.

6. **Maintenance.**
   In solving your task, new ideas or problems may appear. Some can be interesting and creative and some can help you to understand the original problem better; you may see an extent to your problem or a way to incorporate new things. If you have a well-designed problem, you will be able to easily modify it after some time. Take a responsibility to improve your solutions or correct your errors when found later.

12.2 Debugging

Debugging is the process by which you isolate and fix any problem with your code. Two kinds of errors may occur: *syntax error* and *runtime error*. Syntax errors can usually be easily corrected by MATLAB error messages. Runtime errors are algorithmic in nature and they occur when e.g. you perform a calculation incorrectly. They are usually difficult to track down, but they are apparent when you notice unexpected results.

Debugging is an inevitable process. The best way to reduce the possibility of making a runtime error is *defensive programming*:

- Do not assume that input is correct, simply check.
- Where reasonable and possible, provide a default option or value.
- Provide diagnostic error messages.
- Optionally print intermediate results to check the correctness of your code.

Defensive programming is a part of the early debugging process. Another important part is modularity, breaking large task into small subtasks, which allows for developing tests for each of them more easily. You should always remember to run the tests again after the changes have been made. To make this easy, provide extra print statements that can be turned on or off.

MATLAB version 5.*x* provides an interactive debugger. It allows you to set and clear *breakpoints*, specific lines in an m-file at which the execution halts. It also allows you to change the workspace and execute the lines in an m-file one by one. The MATLAB m-file editor also has a debugger. The debugging process can be also done from the command line. To use the debugging facility to find out what is happening, you start with the `dbstop` command. This command provides a number of options for stopping execution of a function. A particularly useful option is:

```
dbstop if error
```

This stops any function causing an error. Then just run the MATLAB function. Execution will stop at the point where the error occurs, and you will get the MATLAB prompt back so that you can examine variables or step through execution from that point. The command `dbstep` allows you to step through execution one line at a time. You can continue execution with the `dbcont`. To exit debug mode, type `dbquit`. For more information, use `help` for the following topics: `dbstop`, `dbclear`, `dbcont`, `dbstep`, `dbtype`, `dbup` and `dbquit`.

12.3 Recommended programming style

Programming style is a set of conventions that programmers follow to standardize their code to some degree and to make the overall program easier to read and to debug. This will also allow you to quickly understand what you did in your program when you look at it weeks or months from now. The style conventions are for the reader only, but **you** will become that reader one day.

Below some style requirements and style guidelines are presented. These are recommendations, and some personal variations in style are acceptable, but you should not ignore them. It is important to organize your programs properly since it will improve the readability, make the debugging task easier and save time of the potential readers.

1. You should **always** comment difficult parts of the program! But ... do not explain the obvious.
2. Comments describing tricky parts of the code, assumptions, or design decisions are suggested to be placed above the part of the code you are attempting to document. Try to avoid big blocks of comments except for the description of the m-file header.
3. Indent a few spaces (preferably 2 or 3) before lines of the code and comments inside the control flow structures. The layout should reflect the program 'flow'. Here is an example:

```
x = 0:0.1:500;
for i=1:length(x)
  if x(i) > 0
    s(i) = sqrt(x(i));
  else
    s(i) = 0;
  end
end
```

4. Avoid the use of magic numbers; use a *constant* variable instead. When you need to change the number, you will have to do it only once, rather than searching all over your code. An example:

```
% A BAD code that uses
% magic numbers
r = rand(1,50);
for i = 1:50
   data(i) = i * r(i);
end
y = sum(data)/50;
disp(['Number of points is 50.']);
```

```
% This is the way it SHOULD be

n = 50;  % number of points
r = rand(1,n);
data = (1:n) .* r;

avr = sum(data)/n;
disp(['Number of points is ',int2str(n)]);
```

5. Avoid the use of more than *one* code statement per line in your script or function m-files.
6. No line of code should exceed 80 characters (it is a rare case when this is not possible).
7. Avoid declaring global variables. You will hardly ever encounter a circumstance under which you will really need them. Global variables can get you into trouble without your noticing it!
8. Variables should have meaningful names. You may also use the standard notation, e.g. `x`, `y` are real-valued, `i`, `j`, `k` are indices or `n` is an integer. This will reduce the number of comments and make your code easier to read. However, here are some pitfalls when choosing variable names:
   - A meaningful variable name is good, but when it gets longer than 15 characters, it tends to obscure rather than improve the code readability.
   - Be careful with names since there might be a conflict with MATLAB'S built-in functions, or reserved names like *mean*, *end*, *sum* etc (check in index or ask `which <name>` in MATLAB - if you get the response `<name> not found` it means that you can safely use it).
   - 417-429 Avoid names that look similar or differ only slightly from each other.
9. Use white spaces; both horizontally and vertically, since it will greatly improve the readability of your program. Blank lines should separate larger blocks of the code.
10. Test your program before submitting it. Do not just assume it works.