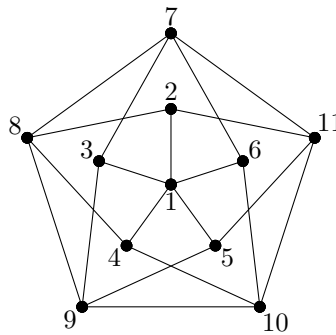# Formal Reasoning 2015
## Test 4: Discrete Mathematics
### (02/12/15)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. The test is *closed book* and you are not allowed to use a calculator. Good luck!

1. This is the so-called Grötzsch-graph:



   (a) Does this graph have an Euler circuit? Explain your answer. (10 points)

   (b) Does this graph have a Hamilton circuit? Explain your answer. (10 points)

   (c) Does this graph have an isomorphism onto itself where not every vertex is mapped onto itself? Explain your answer. (10 points)

   (d) Give the chromatic number of this graph and give a corresponding coloring. You don't have to explain why this is indeed the chromatic number, as long as it is correct. (15 points)

2. (a) Give $(n+1)^3$ using Newton's Binomial Theorem and indicate in Pascal's Triangle where its coefficients can be found. (10 points)

   (b) We define the following recurrence relation: (10 points)

   $$a_0 = 0$$
   $$a_{n+1} = a_n + 3n^2 + 3n + 1 \qquad \text{for } n \geq 0$$

   Use this recurrence relation to compute $a_2$ and show how you found this answer.

   (c) Prove for the sequence from the previous question that $a_n = n^3$ for all $n \geq 0$. (15 points)

3. Sinterklaas[1] has a bag with six different presents for three well behaving children. In how many ways can he distribute the gifts over the children, if he wants that each child gets two presents in his or her shoe? (10 points)

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[1]Sinterklaas or Sint-Nicolaas is a mythical figure with legendary, historical and folkloric origins based on Saint Nicholas. Sinterklaas is celebrated annually with the giving of gifts on 5 December, the night before Saint Nicholas Day in the Northern Netherlands and on the morning of 6 December, Saint Nicholas Day itself, in the (Roman Catholic) southern provinces, Belgium, Luxembourg and Northern France (French Flanders, Lorraine and Artois). He is the primary source of the popular Christmas icon of Santa Claus.

   According to the tradition children will place their shoe near the fire place and if they have behaved well they will receive some gifts in it.