Formal Reasoning 2016
Test Block 5: Modal Logic
(19/12/16)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score fifteen points. Good luck!

1. Draw a tree according to the structure of the modal formula:

$$\neg(\Diamond a \to \Box\Diamond a) \wedge (\Box a \to \Box\Box a)$$

2. In this exercise we use the dictionary:

R	It rains
W	I get wet

Give an English sentence that approximates the meaning of the formula from doxastic logic

$$\Box(R \to W) \to \Box(\Box R \to \Box W)$$

as well as possible.

3. Give a Kripke model in which the formula

$$\Box(R \to W) \to \Box(\Box R \to \Box W)$$

is false, and explain your answer.

4. Give an LTL formula that states that first after some non-zero amount of time a will be true, then at some point in time after that b will be true, and finally some time after that c will be true.

5. (a) Axiom scheme 4 is:

$$\Box f \to \Box\Box f$$

Are all instances of axiom scheme 4 true in all LTL models?

If this is the case, explain why. If this is not the case, then give an instance of the scheme as well as an LTL model that together form a counterexample, and explain why.

(b) Axiom scheme 5 is:

$$\Diamond f \to \Box\Diamond f$$

Are all instances of axiom scheme 5 true in all LTL models?

If this is the case, explain why. If this is not the case, then give an instance of the scheme as well as an LTL model that together form a counterexample, and explain why.