Pictures of non-locality in quantum mechanics[1]

Aleks Kissinger
Oxford University Department of Computer Science

May 21, 2014

[1] Joint work with Bob Coecke (Oxford), Ross Duncan (ULB), and Quanlong Wang (Beijing)

A magic particle machine

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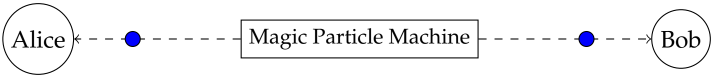

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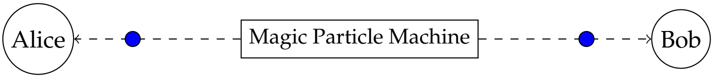

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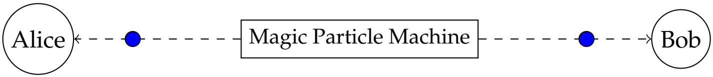

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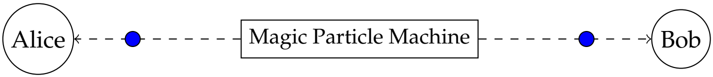

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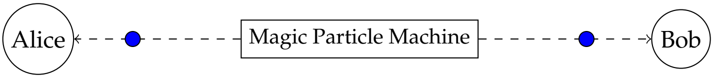

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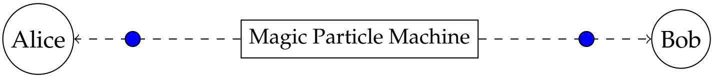

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- ...and the same happens when they both measure Y.
- ...but when they measure different things their outcomes are totally uncorrelated.
- Seems to be some kind of non-local behaviour here. Spooky action at a distance?

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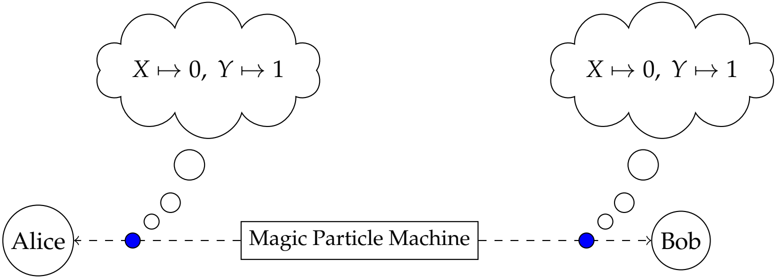

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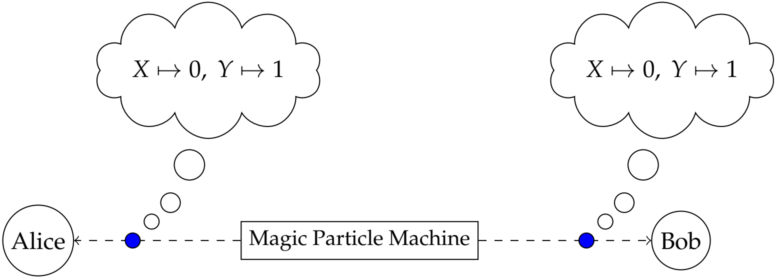

- If it only chooses from pairs of particles that agree on the *hidden variables* X and Y, the outcomes will appear correlated.

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- In 1990, Mermin described a situation where LHV models could be ruled out *possibilistically*.

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- S.C. observables used in the Mermin argument (Pauli-Z and Pauli-X) are represented by $\mathbb{Z}_2$. This is applied to derive a contradiction.

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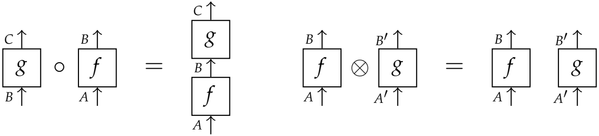

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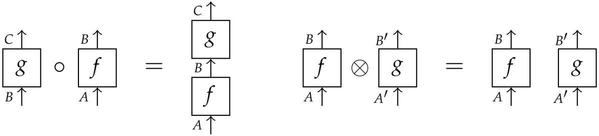

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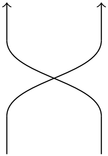

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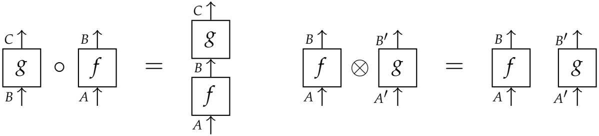

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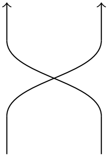
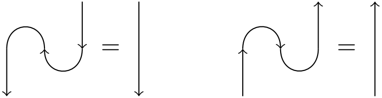


- Compact closure:



Pure quantum mechanics

- Quantum state: vectors $|\psi\rangle \in \mathcal{H}$

  Dirac notation: Column vectors are written as "kets" $|\psi\rangle \in \mathcal{H}$, and row vectors are written as "bras": $|\psi\rangle^\dagger = \langle\psi| \in \mathcal{H}^*$. Composing, they form "bra-kets", which is just the inner product: $\langle\psi|\phi\rangle$.

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- Measurement is the Born rule: The probability of getting the i-th outcome depends on "how close" $|\psi\rangle$ is to $|z_i\rangle$:

$$\text{Prob}(i, |\psi\rangle) = |\langle z_i|\psi\rangle|^2 = \langle z_i|\psi\rangle\langle\psi|z_i\rangle$$

Mixed quantum mechanics

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- Evolution: certain kind of (higher order) linear operator $\Phi : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H}')$

From quantum mechanics to categorical quantum mechanics

We will now apply two slogans from categorical quantum mechanics:

1. Topology of diagrams can be exploited to make life easier.
2. The must important thing about classical data is what you can do with it.

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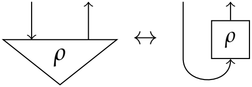

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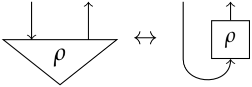
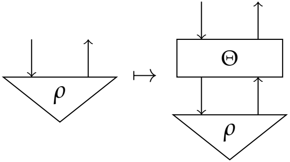


- So, higher-order operations $\Phi : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H}')$ can be represented as first-order maps:



Slogan 2: Classical data

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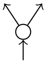
(i) copied: 

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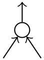
(iii) compared: 

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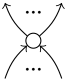
...or any combination of (i-iv): 

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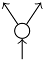
(i) copied: 

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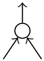
(iii) compared: 

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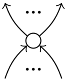
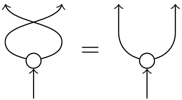
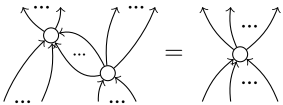
...or any combination of (i-iv): 

- We call the general thing a "spider". Spiders are commutative, and adjacent spiders merge:





Spiders and Observables

- Fix some orthonormal basis $\{|z_i\rangle\}$, then we can define a spider with m in-edges and n out-edges is defined as a linear map:

$$\mathrm{sp}_{m,n} :: \underbrace{|z_i\rangle \otimes \ldots \otimes |z_i\rangle}_{m} \mapsto \underbrace{|z_i\rangle \otimes \ldots \otimes |z_i\rangle}_{n}$$

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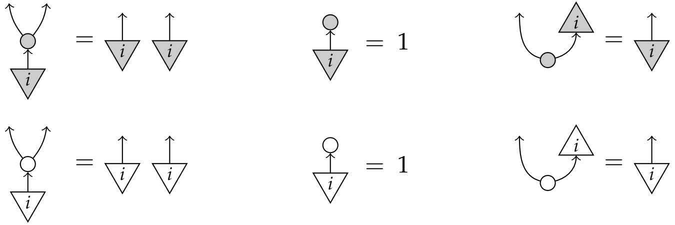

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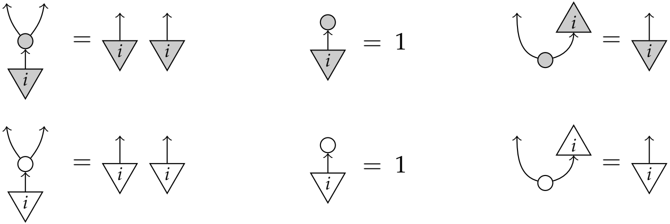


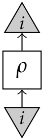
- So we have three equivalent pictures of classical data:

$$\text{quantum observables} \leftrightarrow \text{ONBs} \leftrightarrow \text{families of spiders}$$

The Born Rule and Born Vectors

- For an observable X defined by  in **FHilb**, the Born rule says the probability of getting the i-th outcome when measuring X is:

$$\mathrm{Prob}(i, \rho) = \langle i|\, \rho \,|i\rangle =$$



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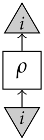
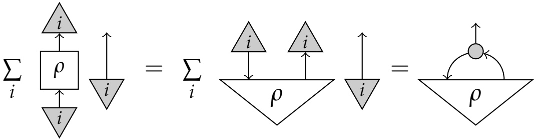


- We can encode the probability distribution over measurement outcomes as a vector written in the X basis:



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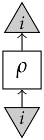

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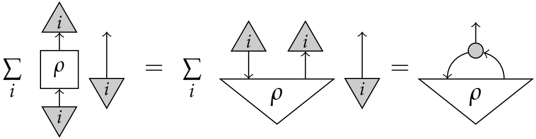
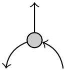


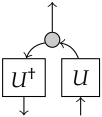
- We call any map $|\Gamma) : I \to A$ obtained as above as a Born vector, with respect to X.

Measurements

$m_{\circ}$:=



- Any measurement can be represented by first performing a unitary, then $m_{\circ}$:



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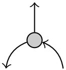
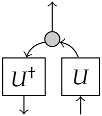
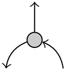
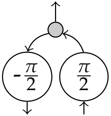
$m_{\circ}$:= 

- Any measurement can be represented by first performing a unitary, then $m_{\circ}$:



- We focus on two measurements in particular for the concrete case. For  corresponding to the Pauli-Z and  the (strongly complementary) Pauli-X observables:

Pauli-X: 

Pauli-Y: 

Complementary Observables

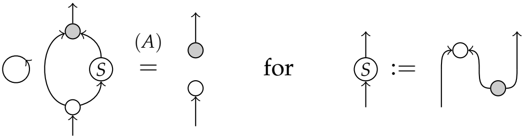
- X and Z are called *complementary* if maximal knowledge of one implies minimal knowledge of the other. In other words, if we measure Z in the X basis (or vice versa), all outcomes occur with equal probability.

$$\forall i, j \,.\, |\langle x_i | z_j \rangle|^2 = 1/D$$

- E.g. position and momentum, or (more relevant in quantum info) orthogonal spin-directions of a particle.

Complementary Observables, Diagrammatically

- The unbiasedness condition is equivalent to a simple graphical identity on the induced observable structures  and  of X and Z:



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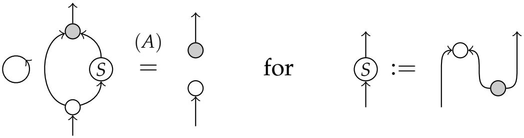
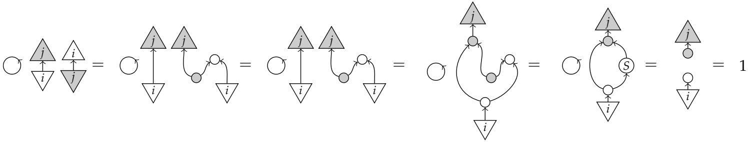


- Proof $(A) \Rightarrow$ unbiased:



...so $\mathrm{tr}(\hat{1})\langle x_j|z_i\rangle\langle z_i|x_j\rangle = D \cdot |\langle x_j|z_i\rangle|^2 = 1$.

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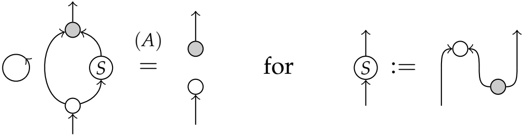

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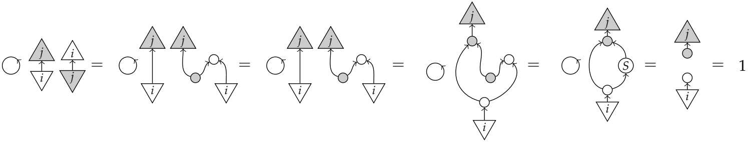

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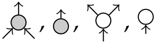
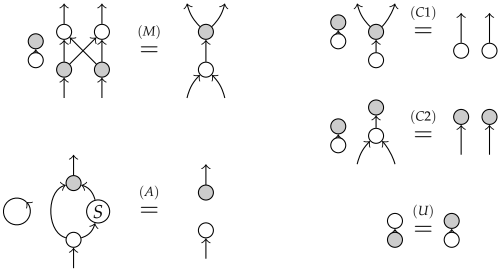
- $\Leftarrow$ is also true, assuming "enough classical points".

Strong Complementarity

- Two observables are called *strongly complementary* if () forms a *scaled Hopf algebra*.



- Under the assumption of "enough classical points", (B), $(C1)$, and $(C2)$ imply (A).

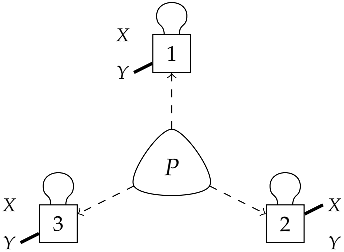
Classification of Strongly Complementary Observables

- While classification of complementary observables in all dimensions is still an open problem, the classification of *strongly* complementary observables is particularly simple:

Theorem

Pairs of strongly complementary observables in a Hilbert space of dimension D are in 1-to-1 correspondence with the Abelian groups of order D.

Mermin Setup



- Perform four separate experiments, with the following measurement settings:

$$\begin{cases} 1. & X & X & X \\ 2. & X & Y & Y \\ 3. & Y & X & Y \\ 4. & Y & Y & X \end{cases}$$

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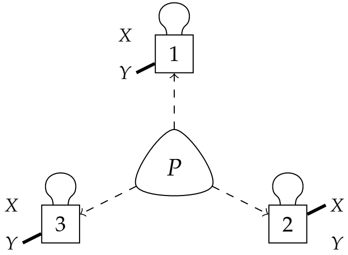

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- Assume (for contradiction): This setup admits a local hidden variable model.

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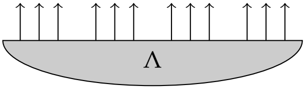
- A probability distribution over such hidden states looks like a Born vector $|\Lambda)$ with 12 wires:



Local Hidden States

- We now turn to imposing the restriction of locality on a global hidden state. A local hidden state encodes outcomes at the level of local measurement settings.

$$|\lambda') = |\underbrace{\overbrace{+}^{X}\ \overbrace{-}^{Y}}_{\textit{system 1}}\ \underbrace{\overbrace{-}^{X}\ \overbrace{+}^{Y}}_{\textit{system 2}}\ \underbrace{\overbrace{-}^{X}\ \overbrace{+}^{Y}}_{\textit{system 3}})$$

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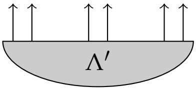
- A local hidden state is then a Born vector with 6 wires:



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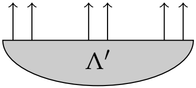

- Note how this is a much smaller space than distributions over global hidden states ($A^{\otimes 6}$ vs. $A^{\otimes 12}$). If we can find a suitable embedding $E : A^{\otimes 6} \to A^{\otimes 12}$, then we can define locality as being in the image of E.

Embedding Local States

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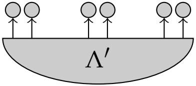

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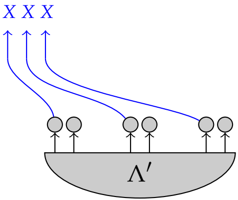

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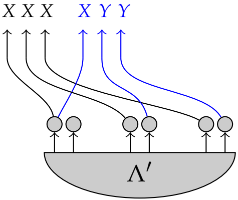

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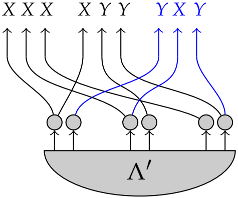

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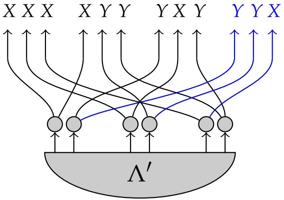
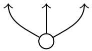


GHZ States

- A GHZ state is a sum over all of the perfectly correlated triples of eigenstates of an observable: $\sum |z_i\rangle \otimes |z_i\rangle \otimes |z_i\rangle$. Abstractly, it can be constructed using a spider:



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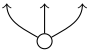
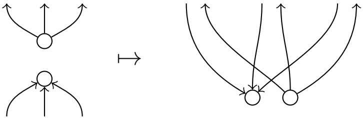
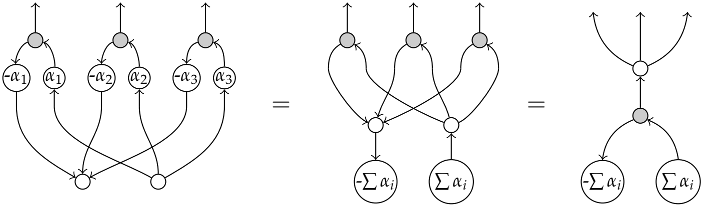


- Pure states are represented by doubling: $|\psi\rangle \mapsto |\psi\rangle\langle\psi|$. For GHZ:



Measuring GHZ States

- Let  define a basis for a GHZ state, and  a strongly complementary basis. If we measure within a (white) phase of , we can compute correlations with a few diagram rewrites.



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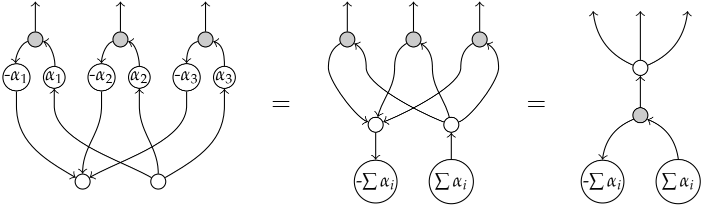
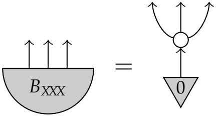
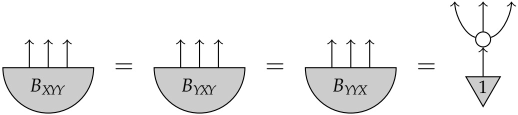


- Notice how the choice of measurements has a purely global effect. In particular, permuting our choice of measurement angles does not effect the outcome.

Measuring GHZ States: Examples

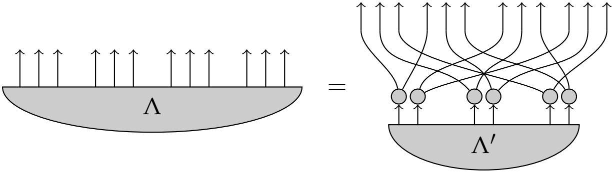
- Using this trick, we can simplify the distributions of measurement outcomes on GHZ states.



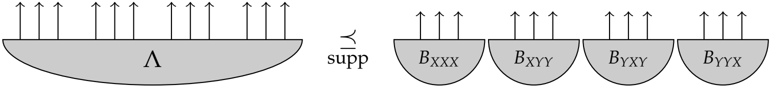


Mermin's Assumptions

- We shall recast the assumptions made by Mermin in our language and derive a contradiction.
- **Assumption 1:** $|\Lambda)$ is a distribution over local hidden states:



- **Assumption 2:** $|\Lambda)$ is (possibilistically) consistent with the QM-predictions $|B_{XXX}) \otimes |B_{XYY}) \otimes |B_{YXY}) \otimes |B_{YYX})$:



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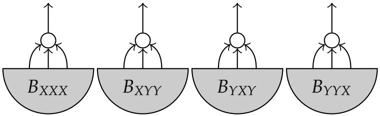
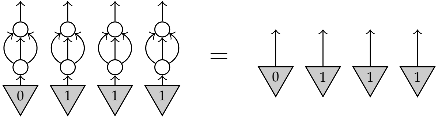
- In two dimensions, $|G| = 2$, so it must be $\mathbb{Z}_2$. This is just normal parity.
- We can compute the parity of lights in each of the four experiments by applying white multiplications:



Parity is an Invariant

- The parity map on the previous slide is a comonoid homomorphism because ( ,  ,  , ) is a bialgebra. We can see that parity is constant as a consequence of specialness of  .



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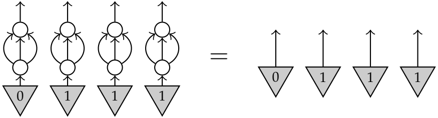
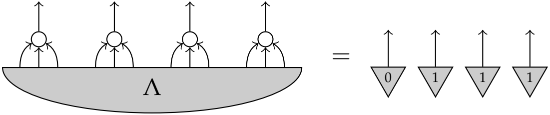


- Since the parity map is constant on the predicted outcomes, we conclude by assumption 2 that:



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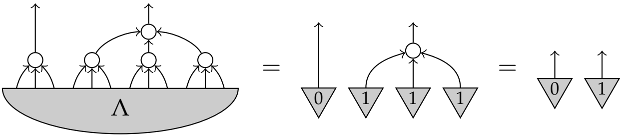
- Mermin derives the contradiction by computing the overall parity of the three experiments involving a Y measurement.



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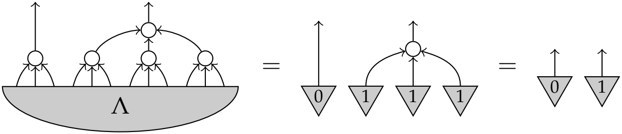
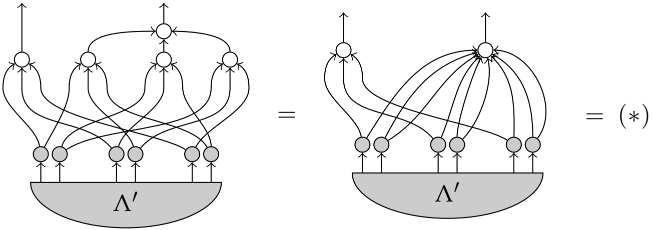


- One can argue in words that the locality assumption forces this parity to be equal to the parity of the first experiment. We can do it in diagrams.

Mermin Locality Violation

- First apply the locality assumption and the spider rule:



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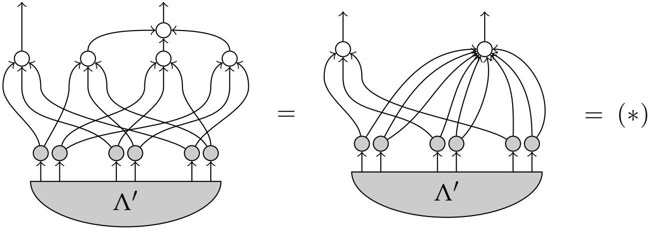
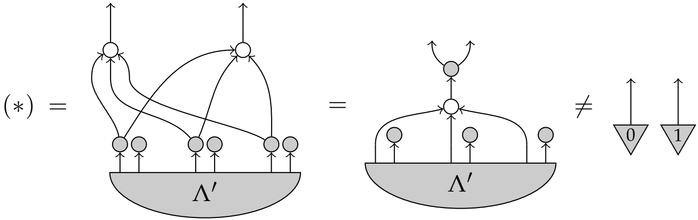


- Note that all of the elements of $\mathbb{Z}_2$ are self-inverse, so $S = 1$. As a consequence of the antipode law for Hopf algebras, parallel edges vanish.



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  2. **Spek** - Spekken's epistemic toy theory

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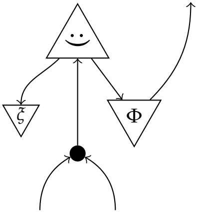
    2. An Abelian group G such that for each round of the experiment, we choose observables such that the group sum of the N outcomes is constant.
- Mermin scenarios extend straightforwardly to higher dimensions and parties, in those cases, we replace $\mathbb{Z}_2$ with a *generalised parity group* G. We replace the final step where pairs of parallel wires vanish with a step where sets of $k = \exp(G) = \max\{|g| : g \in G\}$ parallel wires vanish.
- Since we only use the †-compact structure of the category, along with the classical and phase groups, Mermin scenarios make sense in other *generalised categories of processes*.
    1. **Rel** - sets and relations, "possibilistic" QT
    2. **Spek** - Spekken's epistemic toy theory
    3. abstract †-CCC's with extra structure (e.g. purification)

Thanks!



- Questions?