

# Matrix Calculations

## Assignment 2, Tuesday, Feb. 16, 2016

**Exercise teachers.** Recall the following split-up of students:

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All (blue) delivery boxes are located in the Mercator building on the ground floor where computing science is located.

**Handing in your answers:** There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box. Before putting your solutions in the box make sure:
  - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 2*'. This e-mail should only contain a single PDF document as attachment. Before sending an e-mail make sure:
  - the file is a PDF document that is well readable
  - your name is part of the filename (for example MyName\_assignment-2.pdf)
  - your name and student number are included in the document (since they may be printed).

**Deadline: Monday, February 22, 12:00 sharp**

**Goals:** After completing these exercises successfully you should be able to determine the set of solutions of a set of (non)homogeneous equations, using the notion of pivot. You should be able to determine the (in)consistency of a set of equations and the (in)dependence of a set of vectors. The total number of points is 20.

1. (4 points) A system of linear equations is given in the following echelon form:

$$\left( \begin{array}{cccc|c} 2 & 3 & 1 & 2 & 1 \\ 0 & 4 & 1 & 1 & 2 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

- (i) How many basic solutions does the corresponding homogeneous system have? Why? Provide the solutions.
  - (ii) Find a (particular) solution of the non-homogeneous system.
  - (iii) Describe all solutions of the non-homogeneous system.
2. (4 points)  
Find the values of the parameter  $a$  and  $b$  such that the following system of linear equations:  
(1) has a unique solution, (2) has more than one solution, (3) is inconsistent:

$$\begin{aligned}
-2x_1 - 2x_2 - ax_3 &= -b - 9 \\
x_1 + (a + 2)x_2 + 2x_3 &= 6 \\
3x_1 + 3x_2 + ax_3 &= b + 11
\end{aligned}$$

**Hint:** Perform Gaussian elimination where you keep parameter  $a$  and  $b$  in the matrix. Distinguish the number of pivots, depending on the values of  $a$  and  $b$ .

3. (4 points) Which conditions have to be satisfied for parameters  $a$ ,  $b$  and  $c$  such that the following system of linear equations is consistent?

$$\begin{aligned}
4x_1 + 2x_2 + (b + 1)x_3 &= 3 \\
-12x_1 - 4x_2 + (-a - 4b + 1)x_3 &= 5a + c - 9 \\
8x_1 + 3x_2 + (a + 3)x_3 &= -5a + 6
\end{aligned}$$

**Hint:** Recall the previous hint.

4. (4 points) Find a polynomial function  $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ , which hits the following points:  $(1, 5)$ ,  $(2, 3)$ ,  $(0, 1)$  and  $(-1, -1)$ .
5. (4 points) Check if the following vectors are linearly dependent/independent. Explain your answers briefly:

$$\text{(i)} \quad \left( \begin{pmatrix} 5 \\ 0 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \end{pmatrix} \right) \quad \text{(ii)} \quad \left( \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -14 \\ 20 \\ 7 \end{pmatrix} \right)$$