Matrix Calculations

Assignment 3, Tuesday, Feb. 23, 2016

Exercise teachers. Recall the following split-up of students:

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All (blue) delivery boxes are located in the Mercator building on the ground floor where computing science is located.

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box. Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 3*'. This e-mail should only contain a single PDF document as attachment. Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-3.pdf)
   - your name and student number are included in the document (since they may be printed).

Deadline: Monday, February 29, 12:00 sharp!

Goals: After completing these exercises successfully you should be able to prove that a set of vectors forms a basis, the (non-)linearity of maps and you should be able to do basic matrix operations like matrix-vector multiplications. The total number of points is 20.

Note In your answers you should **explain** what you are doing: just a series of matrices or a computation is (often) not sufficient; you should **use words and sentences** to give an argument or draw a conclusion.

1. **(4 points)** Show, that the following vectors are basis in $\mathbb{R}^2$. Explain in detail how you proceed.
$$v_1 = (1, 2) \qquad v_2 = (1, 3)$$
2. **(2 points)** Prove explicitly that the following map is linear
$G\colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by $G(x, y) = (ax + by, cx + y, b^2 x)$, (for parameters $a, b, c \in \mathbb{R}$).
3. **(4 points)** Show that the following maps are *not* linear
   (i) $F_1\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by $F_1(x, y, z) = (x + y, x\,y)$;
   (ii) $F_2\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F_2(x, y) = (x + 2(y + 3), x + 2y)$.
4. **(6 points)** We consider the following homogeneous sets of equations G and H
$$G : \begin{array}{rcl} x + 4y + z &=& 0 \\ 2x + 5y + z &=& 0 \\ 3x + 6y + z &=& 0 \end{array} \qquad H : \begin{array}{rcl} x - y + 2z &=& 0 \\ 3x - 3y + 6z &=& 0 \\ -2x + 2y - 4z &=& 0 \end{array}$$

(i) Find a basis for the solution space of G. What is its dimension?

(ii) Find a basis for the solution space of H. What is its dimension?

5. **(4 points)** Consider the following matrices and vectors:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 3 & 4 \\ 1 & 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 3 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad D = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad E = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Compute (i) AD (ii) BC, (iii) $B^T E$, (iv) $E^T D$.