

Matrix Calculations

Assignment 6, Tuesday, March 15, 2016

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email
Abdullahi Ali	HG00.310	abdullahi154@gmail.com
Michiel de Bondt	HG00.086	debondt@math.ru.nl
Bart Gruppen	HG01.028	b.gruppen@student.ru.nl
Sander Uijlen	HG00.086	s.uijlen@cs.ru.nl

All (blue) delivery boxes are located in the Mercator building on the ground floor where computing science is located.

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box. Before putting your solutions in the box make sure:
 - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 6*'. This e-mail should only contain a single PDF document as attachment. Before sending an e-mail make sure:
 - the file is a PDF document that is well readable
 - your name is part of the filename (for example MyName_assignment-6.pdf)
 - your name and student number are included in the document (since they may be printed).

Deadline: Monday, March 21, 12:00 sharp!

Goals: After completing these exercises successfully you should be able to compute eigenvalues and eigenvectors of matrices and you should be able to compute the equilibrium division of a Markov chain. The total number of points is 20.

1. **(3 points)**

We consider the weather forecast predictions for the next day: a rainy day R , a cloudy day C or a sunny day S . Assume predictions follow this distribution rule:

Forecast R	70% stay at R	20% go to C	10% go to S
Forecast C	20% go to R	60% stay at C	20% go to S
Forecast S	20% go to R	40% go to C	40% stay at S

- (a) Provide the transition matrix A .
- (b) If there is a 50% probability of rain today and 10% probability of sun, what is the probability that it will be cloudy the day after tomorrow?

2. **(10 points)** Consider the following “student transition matrix”, denoting the fraction of RU students that will stay at / leave the RU and the fraction of non-RU students that will come to / not come to the RU:

$$\mathbf{S} = \begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix}$$

- (a) Find eigenvalues and eigenvectors of \mathbf{S} .
 - (b) Find the diagonal matrix, which is the representation of \mathbf{S} wrt. the eigenvector basis that you have found by (a)
 - (c) What is the second iteration of the student transition matrix? (Use the diagonal matrix to compute this.)
 - (d) What will happen if the number of iterations goes to infinity (find $\lim_{n \rightarrow \infty} \mathbf{S}^n$)?
 - (e) Find the equilibrium starting from (3000, 25000)
3. **(7 points)** Consider the following matrix:

$$\mathbf{F} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix}$$

- (a) What is the characteristic polynomial of \mathbf{F} ?
- (b) Find the eigenvalues of the matrix.
- (c) Compute the eigenvectors corresponding to each of these eigenvalues.