Matrix Calculations

Assignment 2, Tuesday, Feb. 7, 2017

Exercise teachers. Recall the following split-up of students:

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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 2*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-2.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, February 13, 12:00 sharp!

Goals: After completing these exercises successfully you should be able to determine of a homogeneous or non-homogeneous systems of linear equations: whether it is (in)consistent, whether it has a unique solution, and what its general solutions are. The total number of points is 20.

1. **(5 points)** A homogeneous system of linear equations is given:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\ 2x_1 + 4x_2 + 8x_3 + 10x_4 &= 0 \\ 3x_1 + 6x_2 + 11x_3 + 17x_4 &= 0 \end{aligned}$$

   (i) Perform Gauss elimination on the associated coefficient matrix to obtain an echelon form.

   (ii) Compute basic solution(s).

   (iii) Give the general solution in the format $(x_1, x_2, x_3, x_4) = c_1 \cdot \boldsymbol{v}_1 + \ldots + c_p \cdot \boldsymbol{v}_p$, where $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_p$ are the basic solution(s) that you've found under (ii).

2. **(5 points)** A system of linear equations is given by the following augmented matrix in echelon form:

$$\left(\begin{array}{ccccc|c} 2 & 3 & 1 & 2 & 1 & 1 \\ 0 & 0 & 4 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{array}\right)$$

(i) How many basic solutions does the corresponding homogeneous system have? Why? Provide basic solutions.

(ii) Find a particular solution of the non-homogeneous system.

(iii) Give the general solution of the non-homogeneous system. (That is: give the set of all solutions as a parametrisation.)

3. **(5 points)**

   Find the values of the parameter a and b such that the following system of linear equations:

   (i) has a unique solution,

   (ii) is inconsistent,

   (iii) has more than one solution. In this case: describe the general solution.

$$\begin{aligned} x_1 + x_2 + ax_3 &= 2 \\ 2x_1 + x_2 + (2a+1)x_3 &= 5 \\ 3x_1 + (a-1)x_2 + 2x_3 &= b+2 \end{aligned}$$

   Hint: Perform Gaussian elimination where you keep parameter a and b in the matrix.

4. **(5 points)** Find a polynomial function $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$, which hits the following points: $(1, 5)$, $(2, 2)$, $(0, 1)$ and $(-1, -1)$.

5. **(0 points)** *Extra exercise, for those interested*

   Prove that the set of solutions of a homogeneous system of linear equations is closed under scalar multiplication.

   That is, show that if $(s_1, \ldots, s_n)$ is a solution of a homogeneous system of linear equations, then $c \cdot (s_1, \ldots, s_n)$ is also a solution (for any c).