Matrix Calculations

Assignment 5, Tuesday, March 7, 2017

Exercise teachers. Recall the following split-up of students:

teacher	**lecture room**	**email**
John van de Wetering	HG00.065	`wetering@cs.ru.nl`
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 5*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-5.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, March 13, 12:00 sharp!

Goals: After completing these exercises successfully you should be able to compute inverses and determinants use bases transformation matrices to change bases. The total number of points is 20.

1. **(9 points)** Consider the matrix $\boldsymbol{A}$:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

   (a) Compute the inverse of $\boldsymbol{A}$.

   (b) Compute the determinant of $\boldsymbol{A}$.

   (c) Use the inverse of $\boldsymbol{A}$ to solve the following system of linear equations:

$$\begin{aligned} x + 2y + 3z &= -5 \\ 2x + z &= 0 \\ 2x + y + z &= 10 \end{aligned}$$

2. **(4 points)** The sum of two matrices is just the sum of their elements, for example:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a+w & b+x \\ c+y & d+z \end{pmatrix}$$

Give an example of a pair of 2×2 matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ which are both invertible, but where $\boldsymbol{C} + \boldsymbol{D}$ is not invertible.

3. **(7 points)** Consider the following two bases in $\mathbb{R}^2$: $\mathcal{B} = \{\boldsymbol{u}_1, \boldsymbol{u}_2\} = \{(2,1),\ (5,3)\}$ and $\mathcal{C} = \{\boldsymbol{v}_1, \boldsymbol{v}_2\} = \{(1,-1),\ (1,-2)\}$.

   (a) Find the basis transformation matrix $\boldsymbol{P} = \boldsymbol{T}_{\mathcal{B} \Rightarrow \mathcal{C}}$ from $\mathcal{B}$ to $\mathcal{C}$.

   (b) Find the basis transformation matrix $\boldsymbol{Q} = \boldsymbol{T}_{\mathcal{C} \Rightarrow \mathcal{B}}$ from $\mathcal{C}$ to $\mathcal{B}$.

   (c) Compute $\boldsymbol{P} \cdot \boldsymbol{Q}$ and $\boldsymbol{Q} \cdot \boldsymbol{P}$.

   (d) Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be given with respect to basis $\mathcal{C}$ by the following matrix $\boldsymbol{M}$:

   $$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

   Give the matrix $\boldsymbol{N}$ that describes g with respect to basis $\mathcal{B}$.

   (e) (BONUS) Show that matrix multiplication commutes with change-of-basis. That is, if we have two matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ written in basis $\mathcal{B}$, show that transforming each into basis $\mathcal{C}$ THEN multiplying them is the same as multiplying them THEN transforming the result into basis $\mathcal{C}$.