Tentamen Matrix Rekenen 2014

Wednesday, April 9, 2014, 12:30 – 15:30

- Please write clearly, and put on **each page**: your name, your student number, your field (IC=informatica, IK=informatiekunde, KI=kunstmatige intelligentie, other), and your exercise class teacher (Tim Steenvoorden, Sander Uijlen, Bram Westerbaan).
- The exam is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English.
- This exam consists of **4 questions**, printed on one page. Each (sub)question indicates how many points it is worth. You can score a maximum of **100 points**.
- It is advised to explain your approach and to check your answers yourself.

1. **(32 points)** Consider subspace $U \subseteq \mathbb{R}^4$ spanned by the vectors $u_1 = (1,1,1,1)$, $u_2 = (1,1,2,4)$ and $u_3 = (1,2,-4,-3)$
   (a) **(6 points)** Show that u_1, u_2, u_3 form a basis of U.
   (b) **(6 points)** Compute the length of u_2, $\|u_2\|$.
   (c) **(6 points)** Compute $\cos(\gamma)$, where γ is the angle between u_1 and u_2.
   (d) **(8 points)** Apply the Gram-Schmidt procedure to find an orthogonal basis (v_1, v_2, v_3) of U.
   (e) **(6 points)** Compute the projection of the vector $w = (2,4,-4,6)$ onto the space U.

2. **(20 points)** Consider the matrix A and the vector v:
$$A := \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix} \quad v := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$
   (a) **(6 points)** Compute $v^T \cdot A^T$.
   (b) **(7 points)** Compute the determinant of A.
   (c) **(7 points)** Does A have an inverse? If no: why not? If yes: give the inverse.

3. **(18 points)** We consider the following map $F : \mathbb{R}^4 \to \mathbb{R}^3$
$$F(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 - x_3 + 6x_4, 3x_1 + 8x_2 + 9x_3 + 10x_4, 2x_1 - x_2 + 2x_3 - 2x_4).$$
   (a) **(6 points)** Write down the matrix for F and transform it to echelon form.
   (b) **(6 points)** Determine the kernel of F, $\ker(F)$.
   (c) **(6 points)** Determine the dimension of the kernel of F and the dimension of the image of F, $\dim(\ker(F))$ and $\dim(\mathrm{im}(F))$.

4. **(30 points)** Consider the matrix
$$M := \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}$$
   (a) **(6 points)** What is the characteristic polynomial of M?
   (b) **(6 points)** What are the eigenvalues of M?
   (c) **(6 points)** Compute for each eigenvalue an eigenvector of M.
   (d) **(6 points)** Determine the basis-transformation matrix $T_{E \Rightarrow B}$ from the *eigenvector basis* E to the standard basis $B = ((1,0),(0,1))$, and the basis-transformation matrix $T_{B \Rightarrow E}$ from the standard basis B to the eigenvector basis E.
   (e) **(6 points)** Compute M^{10}, using the eigenvalue matrix and the basis transformations.