Matrix Calculations

Assignment 3, Tuesday, September 19, 2017

Exercise teachers. Recall the following split-up of students:

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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 3*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-3.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, September 25, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to give the general solution to a system of non-homogeneous equations and determine whether certain sets form vector spaces (or subspaces).
The total number of points is 20.

1. **(5 points)** Express the vector $\boldsymbol{v} = (4, 3, 2) \in \mathbb{R}^3$ as a linear combination of the following vectors:

$$\boldsymbol{v}_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \boldsymbol{v}_2 = \begin{pmatrix} 3 \\ -6 \\ -1 \end{pmatrix} \qquad \boldsymbol{v}_3 = \begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}$$

2. **(10 points)** A system of linear equations is given by the following augmented matrix in echelon form:

$$\left(\begin{array}{ccccc|c} 2 & 3 & 1 & 2 & 1 & 1 \\ 0 & 0 & 4 & 1 & 1 & 2 \\ -2 & -3 & 3 & -1 & 2 & 7 \end{array}\right)$$

   (i) How many basic solutions does the corresponding homogeneous system have? Why? Provide basic solutions.

   (ii) Find a particular solution of the non-homogeneous system.

(iii) Give the general solution of the non-homogeneous system. (That is: give the set of all solutions as a parametrisation.)

3. **(5 points)** Which of the following subsets of $\mathbb{R}^n$ are subspaces?

   (i) $S \subseteq \mathbb{R}^3$ defined by $S = \{(x, 2x, 3x) \mid x \in \mathbb{R}\}$.

   (ii) $S \subseteq \mathbb{R}^2$ defined by $S = \{(x, x + y) \mid x, y \in \mathbb{R}\}$

   (iii) $S \subseteq \mathbb{R}^2$ defined by $S = \{(x, x + 1) \mid x \in \mathbb{R}\}$

   (iv) For any $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^n$, $S \subseteq \mathbb{R}^n$ defined by: $\{a \cdot \boldsymbol{v} + b \cdot \boldsymbol{w} \mid a, b \in \mathbb{R}\}$.

   (v) $\mathbb{N} \subseteq \mathbb{R}$

   If a set is a subspace, prove it. If it is not, give an argument why not.

4. **(0 points)** *Extra exercise, for those interested*

   Prove that the following, alternative definition of a vector space is equivalent to the one given in the slides:

   Definition 1. A *vector space* $(V, +, \cdot, \mathbf{0})$ is a set V, a special element $\mathbf{0} \in V$ and operations $+, \cdot$ satisfying the following properties:

   (a) $\boldsymbol{v} + \boldsymbol{w} = \boldsymbol{w} + \boldsymbol{v}$

   (b) $(\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w} = \boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w})$

   (c) $\boldsymbol{v} + \mathbf{0} = \boldsymbol{v}$

   (d) $(a + b) \cdot \boldsymbol{v} = a \cdot \boldsymbol{v} + b \cdot \boldsymbol{v}$

   (e) $a \cdot (\boldsymbol{v} + \boldsymbol{w}) = a \cdot \boldsymbol{v} + a \cdot \boldsymbol{w}$

   (f) $a \cdot (b \cdot \boldsymbol{v}) = ab \cdot \boldsymbol{v}$

   (g) $1 \cdot \boldsymbol{v} = \boldsymbol{v}$

   (h) for all $\boldsymbol{v} \in V$ there exists $-\boldsymbol{v} \in V$ such that $-\boldsymbol{v} + \boldsymbol{v} = \mathbf{0}$

   That is, assuming (a)–(g), condition (h) is equivalent to the equation $0 \cdot \boldsymbol{v} = \mathbf{0}$.