Matrix Calculations

Assignment 6, Tuesday, October 10, 2017

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email
John van de Wetering	HG00.114	`wetering@cs.ru.nl`
Justin Reniers	HG01.058	`j.reniers@student.ru.nl`
Justin Hende	HG02.028	`justin.hende@gmail.com`
Bart Gruppen	HG03.632	`b.gruppen@student.ru.nl`

The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 6*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-6.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, October 16, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to take the determinants of matrices, express vectors or matrices in different bases, and translate between bases. The total number of points is 20.

1. **(6 points)** Compute the determinants of the following matrices:

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} 1 & 2 & x \\ 0 & 0 & y \\ 2 & 1 & 1 \end{pmatrix}$$

   (where the determinant of $\boldsymbol{C}$ should be written in terms of x and y)

2. **(6 points)**

   Consider the following 2 bases for $\mathbb{R}^3$:

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \qquad \mathcal{C} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

   Translate the following vectors into the $\mathcal{B}$ basis and the $\mathcal{C}$ basis:

$$\boldsymbol{v} := \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}_{\mathcal{S}} \qquad \boldsymbol{w} := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}_{\mathcal{S}}$$

That is, write each vector in each of these two forms:

$$\begin{pmatrix} * \\ * \\ * \end{pmatrix}_{\mathcal{B}} \qquad \begin{pmatrix} * \\ * \\ * \end{pmatrix}_{\mathcal{C}}$$

3. **(8 points)** Consider the following two bases in $\mathbb{R}^2$:

$$\mathcal{B} = \left\{ \binom{2}{1}, \binom{5}{3} \right\} \qquad \mathcal{C} = \left\{ \binom{1}{-1}, \binom{1}{-2} \right\}$$

   (a) Find the basis transformation matrix $\boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{C}}$ from $\mathcal{B}$ to $\mathcal{C}$.

   (b) Find the basis transformation matrix $\boldsymbol{T}_{\mathcal{C}\Rightarrow\mathcal{B}}$ from $\mathcal{C}$ to $\mathcal{B}$.

   (c) Compute $\boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{C}} \cdot \boldsymbol{T}_{\mathcal{C}\Rightarrow\mathcal{B}}$ and $\boldsymbol{T}_{\mathcal{C}\Rightarrow\mathcal{B}} \cdot \boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{C}}$.

   (d) Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be given with respect to basis $\mathcal{C}$ by the following matrix $\boldsymbol{M}$:

$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}_{\mathcal{C}}$$

   Give the matrix $\boldsymbol{N}$ that describes g with respect to basis $\mathcal{B}$.

   (e) (BONUS) Show that matrix multiplication preserves change-of-basis. That is, if we have two matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ written in basis $\mathcal{B}$, show that transforming each into basis $\mathcal{C}$ THEN multiplying them is the same as multiplying them THEN transforming the result into basis $\mathcal{C}$.