Matrix Calculations

Assignment 7, Tuesday, October 17, 2017

Exercise teachers. Recall the following split-up of students:

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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 7*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-7.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, October 23, 16:00 sharp!

Goals: After completing these exercises, you should be able ot compute eigenvalues, eigenvectors, and the diagonalisation of a matrix.

1. **(3 points)**

   We consider the weather forecast predictions for the next day: a rainy day R, a cloudy day C or a sunny day S. Assume predictions follow this distribution rule:

Forecast R	70% stay at R	20% go to C	10% go to S
Forecast C	20% go to R	60% stay at C	20% go to S
Forecast S	20% go to R	40% go to C	40% stay at S

   (a) Provide the transition matrix $\boldsymbol{A}$.

   (b) If there is a 50% probability of rain today and 10% probability of sun, what is the probability that it will be cloudy the day after tomorrow?

2. **(10 points)** Consider the following "student transition matrix", denoting the fraction of RU students that will stay at / leave the RU and the fraction of non-RU students that will come to / not come to the RU:

$$\boldsymbol{S} = \begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix}$$

(a) Find eigenvalues and eigenvectors of $\boldsymbol{S}$.

(b) Let the eigenvectors form a basis $\mathcal{B}$. Write $\boldsymbol{S}$ as a diagonal matrix $\boldsymbol{D}$ with respect to basis $\mathcal{B}$.

(c) Compute the basis transformation matrices $\boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}}$ and $\boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$ and show that:

$$\boldsymbol{S} = \boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}} \cdot \boldsymbol{D} \cdot \boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$$

(d) What is the second iteration of the student transition matrix? (Use the diagonal matrix to compute this.)

(e) What will happen if the number of iterations goes to infinity (find $\lim_{n\to\infty} \boldsymbol{S}^n$)?

3. **(7 points)** Consider the following matrix:

$$\boldsymbol{F} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix}$$

(a) What is the characteristic polynomial of $\boldsymbol{F}$?

(b) Find the eigenvalues of the matrix.

(c) Compute the eigenvectors corresponding to each of these eigenvalues.