Quantum Processes and Computation

Assignment 2, Monday, February 19, 2018

Exercise teachers:
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Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Friday, February 23, 17:00

Goals: After completing these exercises you know how to reason with cups, caps, and process-state duality in string diagrams. The total number of points is 100, distributed over 7 exercises. Material covered in book: everything up to section 4.2

Exercise 1 (3.38 & 3.40) (10 points): Suppose that there is a unique zero process for all possible types.

1. Show that for each type the zero process is unique.
2. We call two processes f and g with the same inputs and outputs *equal up to a number* (written $f \approx g$) if there exist non-zero numbers λ, μ such that $\lambda f = \mu g$. Suppose a process theory has *no zero divisors*. That is, it satisfies the following property: $\lambda f = 0$ if and only if $\lambda = 0$ or $f = 0$. Show that $f \approx 0$ if and only if $f = 0$.

Exercise 2 (4.6) (10 points): Characterise the functions in **functions** that are $\circ$-separable and as a consequence show that identities (i.e. plain wires) in **functions** are $\circ$-non-separable in general.

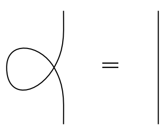
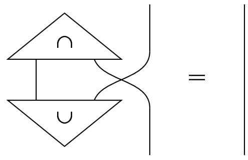
Exercise 3 (4.10 & 4.16) (20 points):

- Show that process-state duality does not hold for **functions**.
- Prove that in **relations**, the following relations on a set A:

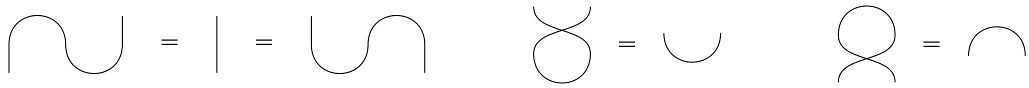
 $:: * \mapsto \{(a,a) \mid a \in A\}$  $:: \forall a \in A : (a,a) \mapsto *$

  satisfy the *yanking equations* (eq. 4.11 in the book), and thus that **relations** has process-state duality.

Exercise 4 (4.12) (10 points): Prove that

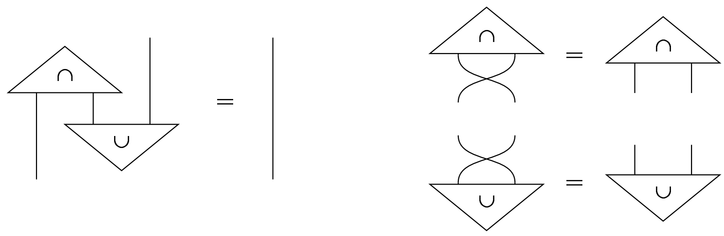
 or written differently: 

follows from the following 3 equations:

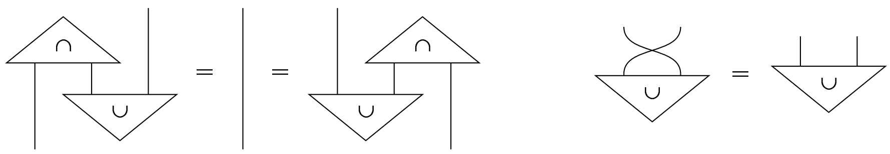


Exercise 5 (4.14) (20 points): Prove that the following two sets of equations are equivalent:

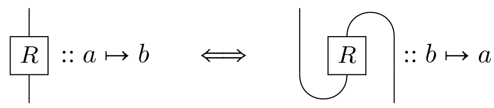
(i) a state and an effect satisfying:



(ii) a state and an effect satisfying:

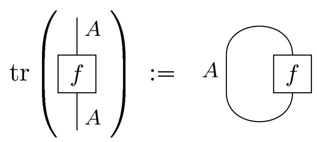


Exercise 6 (4.27) (10 points): Prove that in **relations** the following holds



i.e when a is related to b on the LHS, then b is related to a on the RHS. Note: In the next lecture we will see that the RHS is an instance of the *transpose* of a process.

Exercise 7 (4.37) (20 points): For a transformation $f : A \to A$ in a process theory we define its *trace* as follows:



Show that the trace is independent of the particular choice of cup and cap, i.e. that any state and effect on two systems satisfying the yanking equations defines the same trace operation tr(−).

Hint: Write the trace with respect to the first (i.e. the usual) cup and cap pair, then expand a piece of straight wire using the yanking equations of the new state/effect pair.