Example of computing a principal type:

$$\lambda x^{\alpha}.\lambda y^{\beta}.\underbrace{y^{\beta}(\lambda z^{\gamma}.\overbrace{y^{\beta}x^{\alpha}}^{\delta})}_{\varepsilon}$$

1. Assign type vars to all variables: $x : \alpha, y : \beta, z : \gamma$.
2. Assign type vars to all applicative subterms: $y\,x : \delta$, $y(\lambda z.y\,x) : \varepsilon$.
3. Generate equations between types, necessary for the term to be typable: $\beta = \alpha{\to}\delta$ $\qquad$ $\beta = (\gamma{\to}\delta){\to}\varepsilon$
4. Find a most general unifier (a substitution) for the type vars that solves the equations: $\alpha := \gamma{\to}\delta,\ \ \beta := (\gamma{\to}\delta){\to}\varepsilon,\ \ \delta := \varepsilon$
5. The principal type of $\lambda x.\lambda y.y(\lambda z.yx)$ is now

$$(\gamma{\to}\varepsilon){\to}((\gamma{\to}\varepsilon){\to}\varepsilon){\to}\varepsilon$$

Exercise: Compute principal types for $\mathbf{S} := \lambda x.\lambda y.\lambda z.x\,z(y\,z)$ and for $M := \lambda x.\lambda y.x(y(\lambda z.x\,z\,z))(y(\lambda z.x\,z\,z))$.