Derivation rules for minimal prpositional logic, $\sigma$ and $\tau$ are propositions.

$$\frac{\sigma\to\tau \quad \sigma}{\tau}\ \to\text{-E} \qquad \frac{\begin{array}{c}[\sigma]^j\\ \vdots\\ \tau\end{array}}{\sigma\to\tau}\ {}^{[j]}\!\to\text{-I}$$

An example derivation with nested prooftrees.

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{[\alpha\to\beta\to\gamma]^3 \quad [\alpha]^1}{\beta\to\gamma} \qquad \cfrac{[\alpha\to\beta]^2 \quad [\alpha]^1}{\beta}}{\gamma}}{\alpha\to\gamma}\ 1}{(\alpha\to\beta)\to\alpha\to\gamma}\ 2}{(\alpha\to\beta\to\gamma)\to(\alpha\to\beta)\to\alpha\to\gamma}\ 3$$

A definition of a set of derivation rules.

| | | |
|---|---|---|
| (axiom) | $\Delta \vdash \phi$ | if $\phi \in \Delta$ |
| ($\to$-introduction) | $\dfrac{\Delta \cup \phi \vdash \psi}{\Delta \vdash \phi\to\psi}$ | |
| ($\to$-elimination) | $\dfrac{\Delta \vdash \phi\to\psi \quad \Delta \vdash \phi}{\Delta \vdash \psi}$ | |
| ($\forall$-introduction) | $\dfrac{\Delta \vdash \phi}{\Delta \vdash \forall x{:}\sigma.\phi}$ | if $x{:}\sigma \notin \mathrm{FV}(\Delta)$ |
| ($\forall$-elimination) | $\dfrac{\Delta \vdash \forall x{:}\sigma.\phi}{\Delta \vdash \phi[t/x]}$ | if $t : \sigma$ |
| (conversion) | $\dfrac{\Delta \vdash \phi}{\Delta \vdash \psi}$ | if $\phi =_\beta \psi$ |