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Computing Science

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A Critical Review: Exploring Optimization Strategies in Board Game Abalone for Alpha-Beta Search

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Abstract

Over time, many heuristics have been proposed to improve the performance of the alpha-Beta algorithm. Two of these heuristics are combined move ordering and non-linear aspiration windows, which are introduced in another paper. This thesis places that paper under scrutiny and reimplements its heuristics to verify their effectiveness. While the other paper often lacks detail, its contributions are significant. Notably, combined move ordering appears highly effective, although this cannot be stated going off our results because we used an incomplete evaluation function and/or a slow base implementation. The description of non-linear aspiration windows lacked details required to reimplement it and therefore was unable to be verified.

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Chapter 1

Introduction

In the IEEE Conference on Computational Intelligence and Games, CIG 2012, Papadopoulos, Toumpas, Chrysopoulos et al. present a paper (from hereon: source paper) on applying heuristics to an alpha-beta agent for the game Abalone [1].
This thesis has two goals. The first goal is to verify and reproduce the results of the source paper based on what was written about the implementation. The second goal is to investigate the methods used and the decisions made in the source paper.

Unfortunately, the source paper does not contain all the data to back up their claims due to a reported page limit that the authors had to abide by. As a result, many claims go unfounded, corners were cut in the method descriptions and peculiar decisions were made that go unexplained.
This study aims to clarify the unfounded and unexplained parts of the source paper and perform new measurements to see how they compare to the results reported in the source paper. This should make it easier for future research to build on the source paper as we will have filled the gaps that have been left behind by the source paper.

To make measurements, we implement an Abalone alpha-beta game-playing agent based on the source paper. In doing so, it becomes apparent which aspects of the source paper are not backed up properly or explained poorly. By running the game-playing agent with different settings, we can make the necessary measurements to compare the performance of our implementation with the claims from the source paper.

This thesis is structured as follows. In chapter 2 the required knowledge to understand the rest of this material is explained. This is followed by the methodology in chapter 3. Chapter 4 contains a critical review of the source paper. Chapter 5 puts forward potential improvements to the evaluation function metrics. In chapter 6 the obtained results are compared to the results of the source paper. Next, the conclusions of this study are listed in chapter 7. Finally, the discussion is held in chapter 8.

The source code, release build, readme and raw results of the reimplementation of the Abalone AI can be found at `https://github.com/MichielVerloop/AbaloneAI`.

Chapter 2

Preliminaries

This chapter covers the required theory to understand the rest of this thesis. It covers the goal and rules of Abalone, along with its technical characteristics. In addition, the theory of minimax and alpha-beta is provided. Finally, heuristics, extensions to the alpha-beta algorithm and other beneficial techniques that are used are explained.

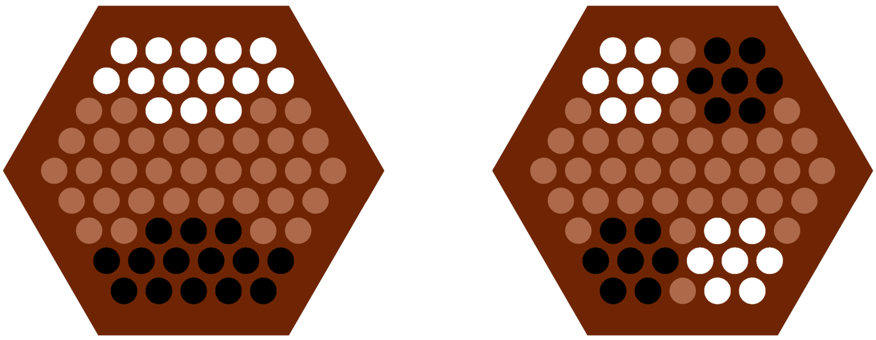
2.1 Abalone

Abalone is a 2-player strategy board game with perfect information and no chance elements. Both players control 14 opposing black or white marbles that are situated on a hexagonal board. The goal is to push six of the opponent's marbles off the edge of the board.

2.1.1 Rules

Starting positions

There are many starting positions that can be used, but only two of them are relevant in this thesis: the default starting position and the Belgian Daisy starting position. The default starting position has each player's marbles near the edge of the board, as shown in figure 2.1a. This starting position



(a) The standard starting layout for Abalone.

(b) The Belgian daisy starting layout.

Figure 2.1: The standard and Belgian daisy starting layout.

makes it easy to play very defensively, even allowing knowledgeable players to force draws. This is an unfortunate oversight, which is accounted for in the default starting position for tournaments: the Belgian Daisy starting position.
In the Belgian Daisy starting position, half the marbles of each player are isolated from their other half, as shown in figure 2.1b. This starting position creates more 'interesting' games: within a few turns, marbles can be threatened and getting all of your marbles close to each other is more difficult to achieve.

Moves

There are a few types of moves. The players take turn moving their marbles, with black always making the first move.
First, the *broadside move*, which allows 2 or 3 marbles in a straight unbroken line to move one space in a direction parallel to the axis of the line. This move cannot push any marbles, so there cannot be any marbles at the destination.
Second, there is the *inline move*, which allows 2 or 3 marbles in a straight unbroken line to move one space along the axis of the line. If there are opponent marbles at the destination, then they can be pushed by the move, provided the pushing marbles outnumber the opponent's marbles and the opponent's marbles are not followed by another of your own marbles. The opponent's marbles can be pushed off the board by inline moves.
Moves can also consist of one marble, which can be moved one space in any direction, thus treated as a combination of an inline and broadside move. Because it can never outnumber any marbles, it can only move to empty spaces.
Within this thesis, broadside moves are referred to as *sidestep moves*, inline moves are referred to as *sumito moves* and moves consisting of a single marble are also considered a sumito move. If a move pushes a marble off the board, it is considered a *capture move*.

Game endings

The game is won by the first player that pushes six of their opponent's marbles off the board.
The game can also end in a draw, although the original rules that were shipped with the physical Abalone game do not mention draws. There is also no authority online that can be referred to for the rules, so instead, we contacted the authors of the game, Michel Lalet and Laurent Levi, for a clarification of the rules surrounding draws.
Together with the eight times Abalone world champion, Vincent Frochot, they sent their response which can be found in the appendix A.1.
Essentially, a draw can occur after a sequence of eight moves. If both players

make and then undo their moves, they know that in response to each other's moves, they will undo their original moves, leading to the same game state after four moves. If the players then repeat the same sequence of moves, the game ends in a draw. If either player wishes to avoid a draw, they must make at least one different move.

2.1.2 Technical characteristics

Abalone reportedly has a branching factor (number of possible moves per game state) of 60 and an average game length of 87 moves [2, page 5], with most games appearing to last anywhere between 50 and 200 moves [3].

For the implementation, there is a subtle but important difference between game states, boards and moves. *Game state* refers to all information that is known about a game of Abalone that is being played and includes the current board. The *board* encompasses only the information that can be inferred from looking at an Abalone board at a certain point in time. Notably, this does not include information on whose turn it is.
A *move* consists of a set of coordinates and a direction, independent of a board or game state. Notably, it does not consist of marbles which is elaborated on in section 3.2. A move can be applied to a board or game state if it is a legal move for that board or game state. Moves must be defined in such a way such that the history heuristic can function, as explained on page 10.

2.1.3 Known Abalone AI implementations

The most well-known and strongest Abalone AI implementation is Aba-Pro from Aichholzer, Aurenhammer and Werner [4].
All other AIs that have been made for Abalone either have never been published or are no longer publicly available. The AI created in the source paper is among this group of 'forgotten' AIs. Despite publishing years after the release of Aba-Pro and referencing the Aba-Pro article on multiple occasions, the source paper does not state that their AI played against Aba-Pro.

2.2 Minimax

For zero-sum games with perfect information, there are several different ways to determine the best move in a given game state. The minimax algorithm is one such approach to determine the best move. It works by exploring all possible moves in a depth-bound depth-first search (DFS) manner. For the final depth layer, all the game states (leaf nodes) are evaluated using heuristics. The results propagate back to the moves at a depth of 1 where the move that leads to the best game state is picked.

The propagation of moves is not completely straightforward as your opponent will be trying to maximize their score and work against you. Hence, the algorithm assumes the worst-case scenario when predicting what the opponent would do [5, ch. 10].

2.2.1 Evaluation

The previous paragraph briefly mentioned the evaluation of game states. We seek to find a function which can approximate how advantageous a certain game state is for the player. In essence, we want a method to quantify how likely a player is to win in a certain game state. Such a function can then be used by the minimax algorithm to determine what the 'best move' is when looking ahead a certain depth.

For Abalone, the source paper differentiates between material and spatial components in the evaluation function. The only material component is the number of marbles captured by each player. The win condition is based solely on this component, but that does not mean that the spatial components can be ignored. Take the *distance from center* spatial component for example. It describes how close the marbles are to the center of the board. The closer marbles are to the center, the safer they are as it would take many moves before such a marble could be captured. Hence, it feeds into the material component: a low distance from the center leads to a low likelihood of losing marbles soon thereafter.
The spatial components are not absolute indicators of how 'good' a game state is, but they tend to approximate whether someone has an advantageous board layout.

2.2.2 Alpha-beta

Alpha-beta is an adaptation to the minimax algorithm which filters out moves that cannot lead to better moves than moves that have previously been explored. It does so by saving the best values acquired at a given sub-tree, then skipping branches that have a worse value.

To reason about the time save that alpha-beta pruning provides over minimax, we have to establish a few assumptions. We must assume a constant branching factor b and a variable depth d.
The time save provided by alpha-beta pruning is dependent on the order in which the nodes are discovered. In the worst-case scenario, the nodes are discovered in order from worst to best meaning that no branches can be pruned. The complexity that this yields, is the same time-complexity as minimax: $\mathcal{O}(b^d)$. In the best-case scenario, most nodes are pruned, yielding a time-complexity of $\mathcal{O}(\sqrt{b^d})$ [6, ch. 5].

2.2.3 Iterative deepening

Iterative deepening is a search technique which searches the best move for a given depth, then increases the depth limit and repeats this process until either the final depth is reached or it exceeds the allocated amount of time. Exploring the tree with iterative deepening will go more slowly than without it as more work has to be performed, but due to the exponential nature of minimax algorithms, it does not reduce the performance by an enormous amount. The reduction in speed is bounded by a constant determined by the branching factor. More specifically, the extra work has an upper bound of $(1 - \frac{1}{b})^{-2}$ where b is the branching factor [7].
Abalone has an average branching factor of 60 as noted in section 2.1.2. When using the alpha-beta search, the effective branching factor lowers to anywhere between 15 and 28. If additional pruning heuristics are applied, the branching factor can further lower to anywhere between 5 and 8 [4]. This translates to a maximum of 56.3% 'wasted' work at a branching factor of 5, and a maximum of 7.5% wasted work at a branching factor of 28.

The reason that iterative deepening is desirable despite slowing the calculations down is twofold. The option to cut off the search after a fixed duration is highly desirable for practical applications. For example, this makes the algorithm suitable for play against humans.
The second reason is that in practice, the results from shallower depths can be used to speed up the exploration of subsequent depths and thereby mitigate how much longer iterative deepening DFS takes compared to DFS on its own.

2.3 Transposition table

Evaluating a game state is a relatively expensive operation and only becomes more expensive as the evaluation function becomes more advanced. A property of minimax algorithms is that the same game state is often visited repeatedly - after only two moves from each player, the same game state can be reached using different moves. Hence, it becomes tempting to store the outcome of the evaluation function on these game states in a technique called memoization. With this technique, storage space and a bit of overhead is traded for a significant reduction in the number of evaluation calls. In addition to storing the rating of a game state, the result of evaluating an entire branch can be stored. Storing branches yields even better time saves as branches can be reused and need not be re-explored, making it as effective as pruning branches.

To store this data efficiently, a hash table with a caching mechanism is ideal as it provides a constant access time and bounded memory usage. The bounded memory is mandatory because the hash tables will easily exceed the available memory if no measures are taken to restrict the memory usage. The stored information can be accessed with a hash key obtained from Zobrist Hashing.

2.4 Zobrist hashing

Zobrist hashing is a common hashing technique to create a hash for a game state that can be applied to many board games. In constructing a Zobrist hash, random values are assigned to specific combinations of material and spatial components. Then, for all such components that exist in a certain game state, the hashes are combined using bitwise XOR operation. The result of this is the Zobrist hash of that game state [8].
Because XOR is self-inverse, the Zobrist hash of a game state resulting from a move can be created by simply applying a bitwise XOR to the Zobrist hash of the previous game state and the 'Zobrist hash of the move'. The Zobrist hash of the move is what you get if you only consider the values of the combinations that change and XOR all of those.
All this gives Zobrist hashing two very desirable properties. First, it is very cheap to generate a hash because it only costs a few XOR operations to do so. Second, it allows the hash of a game state to be made before the game state is visited, which can be useful in combination with hashing the game results, evaluation sorting and history sorting.

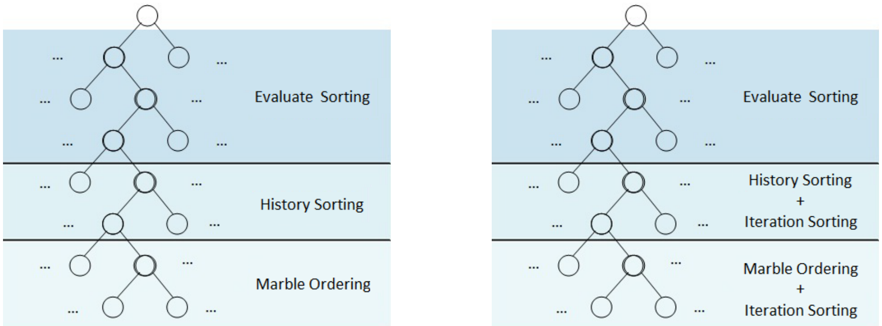
In the context of Abalone, the combination of the color of a marble and its position generate the random value. If later the same combination of marble color and position occurs, the same value is returned as the first time.

2.5 Heuristics

For the scope of this thesis, we consider alpha-beta the 'base algorithm' on top of which different heuristics are tested. These heuristics are presented in this section.

2.5.1 Combined move ordering

Due to the nature of the alpha-beta algorithm, the order in which the moves are supplied to the algorithm determines how quickly it terminates. The practice of ordering the moves that alpha-beta agents take as an input is called move ordering.
There is a balance to be struck between how computationally expensive the move ordering heuristics are and how useful they are at a given depth. For example, at the start of the tree, it is important to have the best ordering possible because branches that are pruned early on prune more work than



(a) Combined move ordering

(b) Advanced combined move ordering

Figure 2.2: (Advanced) combined move ordering imagery taken from the source paper. Both examples have evaluate sorting active on depths 1-3, history sorting at depth 4-5 and marble ordering at depths 6-7. Advanced combined move ordering additionally has iteration sorting active at depths 4-7.

branches at deeper depths. At the end of the tree, only one depth worth of nodes can be pruned, meaning that it is more efficient to use less accurate, yet cheaper sorting heuristics.
The source paper puts forward the technique of combined move ordering which defines sections in the tree use distinct move ordering methods that best suit that depth [1]. After introducing Iteration Sorting, they also put forward the notion of advanced combined move ordering. Examples of the different sections are shown in figure 2.2.

Evaluate sorting

Evaluate sorting is ordering the moves based on the evaluation of the game state of the child nodes. Of all marble ordering techniques, this is by far the most costly way to sort because it requires visiting and evaluating all child nodes.
The accuracy of the evaluate sorting method scales with the accuracy of the evaluation function, provided that game states that score high lead to game states that are even better. Since we have no reason to believe this requirement does not hold, we assume that it holds and treat the evaluation function as the most accurate move ordering method.

History heuristic sorting

History heuristic sorting orders the moves based on the number of prunes they caused. A move prunes branches when it leads to a branch with a good enough alpha or beta score to cause other branches to not be evaluated.

evaluate			45		5		3				80			20	21				
kills	6	11	3	9	10	7	32	4	68	1	45	4	21	8	5	0	3	18	44

Filter

evaluate	45	5	3	80	20	21
kills	3	10	32	45	8	5

Sort

evaluate	80	45	21	20	5	3
kills	45	3	5	8	10	32

History Sorting

evaluate		80		3				5		20			21			45			
kills	68	45	44	32	21	18	11	10	9	8	7	6	5	4	4	3	3	1	0

Reorder front one-third and back one-sixth

evaluate	80	45						5		20			21						3
kills	45	3	68	44	21	18	11	10	9	8	7	6	5	4	4	3	1	0	32

Change relative position

evaluate	80	45						21		20			5						3
kills	45	3	68	44	21	18	11	5	9	8	7	6	10	4	4	3	1	0	32

Figure 2.3: Iteration sorting example taken from the source paper, altered for increased clarity. First, a move ordering heuristic (here: history sorting) is applied based on its metrics (here: kills/prunes). Afterwards, the moves with known scores are filtered and sorted. This is used to reorder the front one-third and back one-sixth of the moves, after which the relative position of the remaining moves with known scores are sorted.

This heuristic is based on the assumption that a move that is good in one board state is also good in similar board states [9].
This sorting method is cheaper than evaluate sorting as child nodes do not have to be visited nor evaluated. Instead, the hash of a child node can be created by combining the hash of the current game state and the hash of the move that leads to the child node, as explained in section 2.4. This hash is then used to look up the number of pruned branches it caused. The moves are then sorted from high to low based on the number of branches they pruned.

Marble ordering

This heuristic is specific to Abalone and was originally introduced in the source paper [1]. This heuristic sorts moves by their types, sumito and broadside moves, and on how many marbles are involved in the move.
The intuition behind the heuristic is that moves that involve more marbles are more likely to impact the game in a significant manner as the turn is utilized to a greater extent.
The heuristic is the cheapest of the move ordering heuristics as the sorting process is very cheap. The moves can be sorted in linear time using a counting sort or otherwise generated by type in linear time.

Iteration sorting

The authors of the source paper also put forward Iteration sorting [1], which combines a cheaper version of the evaluation sort with the other sorting

methods. This sorting method consists of two parts.
The first part consists of copying the list of legal moves and sorting both lists using two separate sorting mechanisms. The first list is sorted based on the hashed results from previous evaluation calls. The moves that haven't yet been hashed are ignored, thus reducing the size of the sorted list.
The second list is sorted using either history heuristic sorting or marble ordering.
The second part is to take the two sorted lists and combine them. Combining these lists is done by taking the best one-third and the worst one-sixth moves from the first list. All moves in these parts of the list are removed from the second list, after which the best one-third and the worst one-sixth moves are added to the front and back of the second list, respectively.
An example of the combination of the history heuristic and iteration sorting can be found in figure 2.3
The authors chose to move the worst one-sixth to the back because "*in case one of [the moves] turns out to be the only good one, being at the absolute back will slow down [the] algorithm significantly*". No reason was provided for why the first one-third specifically is moved to the front.
To work at all, iteration sorting has to make use of hashing. To make it faster, it can be combined with iterative deepening. Doing so pays off because interesting branches from lower depths get a more appropriate ordering, which leads to a lower branching factor, which improves the overall speed.
All in all, iteration sorting allows you to reap the benefits from evaluation sort, namely that it is the most accurate sorting method at our disposal, without having to deal with its downside, which is that it is the most expensive of the sorting methods.

2.5.2 Quiescence search

Quiescence search is an extension to the alpha-beta search that, unlike most heuristics, does not provide a speed-up to the search, but rather improves the quality of the search. It does so by locally increasing the depth for leaf nodes that have an 'unstable' score assigned to them, which causes the score assigned to that node to be refined.
In the context of Abalone, a leaf node is considered to be *quiet* (stable) if no marble captures occurred in the move leading up to it and it is considered to be *not* quiet (unstable) when a marble capture did occur in the move leading up to it.
The intuition behind quiescence search is that it alleviates the problems caused by the horizon problem (also known as the horizon effect). This is an issue that can occur due to the limited depth that the algorithm evaluates. A game state can appear to be good in the leaf node, but might lead to the loss of a marble in the next (few) moves.

2.5.3 Aspiration windows

Aspiration windows is a heuristic that provides the largest speed-up of all the mentioned heuristics, at the cost of move quality. It works by pruning all remaining branches when a move has been found that is considered 'good enough'.
The lower the threshold for 'good enough', the worse the move will be, but the faster that move will be found. Conversely, if the standards for a good enough move are very high, the move will be better but take longer to be found. If the threshold is too high, no move will be found and a re-search at a lower threshold has to be made.

Note that this differs from what the source paper considers to be aspiration windows. The reason for this discrepancy is explained in section 4.4.

Chapter 3

Methodology

In this chapter, we discuss our approach in critically analyzing and trying to reproduce the results from the source paper.

3.1 Approach

To achieve the first goal of this thesis, verifying and reproducing the results from the source paper, an implementation of the Abalone AI described in the source paper had to be made. This implementation should allow for different heuristics to be fairly compared against each other. An in-depth view of how this is achieved is given in section 3.5.
The second goal of this thesis, to critically analyze the paper is aided by the fact that the implementation was made without any help from the authors of the source paper. This ensures that the contents of the paper are evaluated without any external biases, making it easier to critically analyze the contents of the paper.

3.2 Implementation considerations

There are some strict requirements for the Abalone implementation.
First of all, it must be possible to abort the computation of a move for time-based Iterative Deepening to function.
In general, the representation of a move should be unambiguous. If there are two move representations that describe the same move, the branching factor will be unnecessarily increased, which leads to exponential performance losses.
For the History Heuristic, it is also important that a move is represented independently from the board state, otherwise the History Heuristic cannot function.
The move representation must also allow for Zobrist hashing. Zobrist hashing requires that when the hash of a board state and the hash of a move are combined using the XOR operation, the result should equal the hash of the move applied to the board state.
For performance considerations, it is mandatory to make it possible to undo a move. If no undo functionality is present, a copy of the entire game state would have to be made for almost every move that is made during the search which would be incredibly expensive.

Next to the requirements, there are some decisions that should be made. The first decision is which language and programming paradigm to use. In this thesis, Java was chosen as the programming language because of the author's experience and affinity with that programming language.
Next, a trade-off has to be made between performance and code maintainability. The more primitive and efficient the board and legal move generation are, the faster the AI will run as most of the computation time is spent generating all legal moves for board states. The downside is that implementing other features may prove more difficult. In our implementation, the decision was made to go for a less efficient but more easily modifiable program structure. This negatively impacts the absolute timing performance results, but we hypothesize that the relative timing performance results will be similar to that reported in the source paper.

3.3 Identifying criticisms

To identify weak points in the source paper, the question 'why' was asked for every claim. If neither the context nor common (domain-specific) knowledge could answer the question, it was added to the list of criticisms.
Criticisms were also identified during the implementation process as lacking implementation details become especially apparent during that step.

3.4 Addressing the criticisms

After identifying the criticisms, the next step is to find out to which extent they can be addressed. For some of the criticisms it makes more sense than others to extensively address them. In this thesis, we only comprehensively addressed the evaluation function as the source paper outlined that for future research.

3.5 Comparing heuristics

There are several different ways in which to compare heuristics. One can compare quality, speed, and the combination of these: performance.

To compare the quality of a heuristic, two identical agents repeatedly play against each other, except one uses the heuristic that is to be tested, while the other does not. There will not be a source of nondeterminism for this measurement, so only 4 unique games can be created: two for each layout, with both agents playing as the first player once. Because of this small sample size, the results should be taken with a grain of salt.
To test speed, two agents, again with one differing heuristic will generate

valid moves on a set of predetermined game states. Since both agents operate on identical game states, the comparison is not affected by the quality of moves.

To test performance, the different agents repeatedly play against each other on a time limit. As a result, during one execution the agent may return the move computed at a depth of 3, whereas during another execution it returns the move that was computed at a depth of 4. This leads to different moves being used, and thereby branching of the games. This branching is desirable as it allows more games to be evaluated than just the 4 in the case of quality measurements.

Naturally, the win/loss ratio achieved by the quality and performance measurements is used to determine the effectiveness of the heuristics used in these measurements.

Chapter 4

Critical review

This chapter delves into the decisions and claims made in the source paper that are not self-evident or lack proper argumentation.

4.1 Training the evaluation function

The evaluation function as introduced by the source paper has 6 metrics that determine the score of game states. These metrics should be assigned weights to dictate their importance in the final score, else the evaluation function would be a poor estimation of how valuable a game state is. The source paper makes no mention of these weights or how they identify the optimal values for these weights.

4.2 Formation break calculation

The source paper states the following on the formation break metric:

> *"Placing your marble among the opponent's marbles has many advantages. Not only your marble cannot be attacked, but also the formation of your opponent's marbles gets split in half. Thus, they are less capable of attacking and become much more vulnerable to being captured. This parameter is calculated as the number of marbles that have opponent marble in opposite sides."*
>
> (Source paper, page 2)

In figure 4.1 there are three examples where the black marble is in formation break, in line with the given definition. The above definition considers all of these formation breaks equal. It does not consider multi-axis formation breaks better or worse. In section 5.1, we investigate whether it is worth differentiating between single- and multi-axis formation breaks and we discover numerous other factors to consider.

4.3 Marble capturing danger

The source paper introduces *single marble capturing danger* as follows:

> *"When one of the player's marbles touches the edge of the board, while having neighboring opponent marbles, there is a possibility*

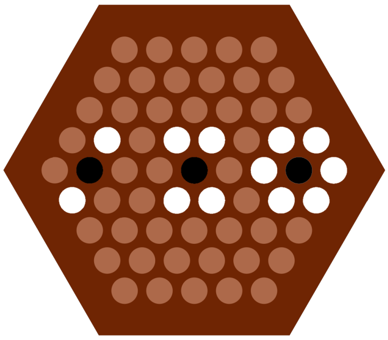


Figure 4.1: Three examples where the black marble in formation break. From left to right, the formation break occurs on one, two and three axes.

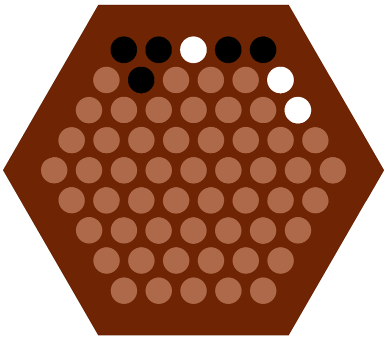
> *of being captured. If there is only one, the reduction is minor. If there are more than one opponent marbles the danger is imminent and the reduction in the evaluation is high."*
>
> (Source paper, page 2)

The source paper introduces *double marble capturing danger* as follows:

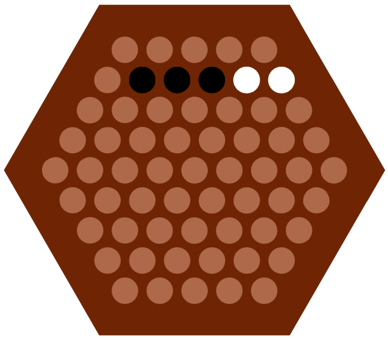
> *"The same goes when two of the player's marbles touch the edge of the board and have more than one opponent marbles on the opposite side. This time the evaluation reduction is even higher than before, because there are more than one marbles in danger."*
>
> (Source paper, page 2)

From these definitions we can derive that the goal of the single and double marble capturing danger metrics is to judge whether there is a high possibility of one or multiple marbles being captured.

The problem with this metric is that it does not seem to reliably achieve this goal. The metric creates a lot of false positives and false negatives alike. One can devise many situations where the metric backfires and gives the opposite score of what one would want. Two examples of this are given in figure 4.2. Furthermore, the definition from the source paper does not appear to differentiate between 2, 3 or 4 adjacent opponent marbles. In figure 4.2a, the situation would be a lot worse for white if the white marble in the top row were to be threatened by two additional marbles. The extra marbles would mean that the white marble is trapped, allowing it to be captured within two moves.
There is also no consideration for whether the marble that is in danger can be pushed off in one move.



(a) White would count three single marble capturing dangers and one double marble capturing danger, despite being in a position where they can push off an opponent marble and are not in danger if it is the white player's turn.



(b) White would count zero marble capturing dangers, while clearly being at risk of losing a marble if it is the black player's turn.

Figure 4.2: Two example situations where the marble capturing danger metrics fail to evaluate the situation well.

The description of how the marble capturing danger metrics are implemented is also lacking. There is no information on how the two interact nor on which numbers should be assigned to 'minor' and 'high' score reductions.
Finally, the single and double marble capturing danger metrics seem very similar, which makes it an odd choice to create two metrics that are this similar rather than to create one 'danger' metric.
To address these concerns, we put forward an alternative to these metrics in section 5.2.

4.4 Non-linear aspiration windows

As was mentioned in the preliminaries, section 2.5.3, there is a difference between the described aspiration windows and the aspiration windows that were implemented by the source paper.
The approach taken by the source paper does not appear to align with what is generally considered to be aspiration windows. While there is no clear, concise and complete definition that we could find on aspiration windows, the following seems to be a common pattern among various sources [9]–[12]:

The aspiration windows set a demand for how much better moves have to be than the root node at a given depth. Iterative Deepening can be used to better estimate the optimal value for this demand: the estimation of the

best score at a given depth can be based on the best score at the previous depth.
To reduce the branching factor, moves have to be significantly better than the current best move as determined by alpha and beta. This reduces the quality but saves a lot of time. To still provide quality guarantees, a move is only used if it can guarantee a score equal to or better than the demand that was set for the current maximum depth. If no such move is found, the search is repeated with lower requirements until such a move is found or the requirements cannot further be relaxed. This is called re-searching. If in the end no move was found that can match the demand, the best move in the last re-search is used.

All this vastly differs from what is described in the source paper. No mention is made of either iterative deepening or re-searches in the context of aspiration windows. The source paper only seems to implement or mention the "moves have to be significantly better than the current best move as determined by alpha and beta" aspect. They implement this by exploiting "*the use of the Alpha-Beta pruning, amplifying the window "cuts" with the alpha-beta "cuts". Thus, a and b can be derived as follows:*"

$$
\begin{gathered}
estimation = evaluate(game) \\
a = max(a, estimation - windowsize) \\
b = min(b, estimation + windowsize)
\end{gathered}
$$

The issue is that they never mention re-searches nor do they mention iterative deepening in the context of aspiration windows, despite that being the standard way to implement aspiration windows, to the best of the author's knowledge. Deviating from the standard is not necessarily bad. However, given that non-linear aspiration windows are introduced as a novel concept, the description and implementation of this should be clear and unambiguous. Differences between the standard and new approach should be outlined, and the differences in performance should be compared. Else, those that come across the paper cannot make an informed decision between the standard or altered approach to aspiration windows.

Another peculiar thing is that the speed-up claims differ each time performance comes up. In the section on non-linear aspiration windows, they state that

> *"The performance increases up to tenfold compared to a simple search in the same depth, sacrificing only a reasonable amount of the quality of the result."* (Source paper, page 4)

However, in the experimental results section, they state

> *"Our player becomes almost twenty times faster, while losing some of its quality due to the "guessing" procedure mentioned*

in III. [...] With this kind of "node economy", you can search at least two levels deeper in the same time, and thus increase the quality again to a greater proportion."

(Source paper, page 7)

This raises the question, which of these statements are correct?

Now let's consider Aichholzer, Aurenhammer and Werner [4] and their implementation of 'linear aspiration windows', which they refer to as 'heuristic alpha-beta'. They use a non-standard approach to aspiration windows, on which the source paper seems to have based their approach. In table 1 of that paper, the claim is made that it reduces the branching factor from 13 to 5 in the opening positions and from 13 to 8 in the ending positions. In figure 5 of that paper, we can see that this corresponds to evaluating depth 9 with heuristic alpha-beta faster than depth 6 without.
The speed increase is far superior to that of the source paper's approach to aspiration windows, despite both implementations being trained to have an agent with aspiration windows at a depth of $d + 1$ 'marginally' beat an agent without aspiration windows at a depth of d.
Given that the source paper allegedly built on [4], it is the author's opinion that the source paper should have compared the results.

Finally, the source paper does not provide adequate information on how to train the aspiration windows. Below is the only statement on the training of aspiration windows:

"The most commonly used procedure for the determination of the windows' width is their gradual narrowing up to the point that the Aspiration algorithm can marginally beat a one-level deeper Min-Max Search. This way, the low degradation of the quality of the player is guaranteed." (Source paper, page 3)

Note that nothing was cited to back this up or provide further instructions and that the word 'marginally' says very little in this context. It could mean that the aspiration agent can beat the one-level deeper alpha-beta agent only rarely, a little over 50% of the time, or almost always, where in most matches it can barely manage to win.

4.5 Two-step hashing

In their section on transposition tables in the source paper, the authors state the following about a flag to indicate whether a variable has been used before:

"Every entry of the table can have a third variable, which will act as a flag, indicating that the value has been used at least once

> *before. This way, when a second-level hashing collision occurs, the stored value is replaced with a new one, accordingly. The purpose of this technique is to keep in store the more frequent moves and replace the ones that have never been used."*
>
> (Source paper, page 4)

Their replacement scheme is uncommon and not as practical as some of the other choices [13]. If they clear their transposition table after each turn, they are unable to benefit from previously explored turns. If instead they keep their transposition table between turns, then getting to a game state twice sets the flag and will prevent all future game states with the same 22-bit key from being stored, even if the original game state later becomes infeasible or even impossible to reach.

Several of the replacement schemes outlined by Breuker, Uiterwijk and Herik in [13] already solve the issue of keeping in store more frequent moves, although they each come with their own disadvantages compared to the replacement scheme of the source paper. For example, the replacement scheme *deep* solves the above issue at the cost of storing a number instead of a boolean. It is odd that the choice of replacement scheme goes unexplained in the source paper.

4.6 Dealing with missing implementation details

In this chapter, we showed for several cases that there are lacking implementation details. In this section, we describe how the lack of information is dealt with in the implementation.

The weights of the evaluation are untrained. The values assigned to it are solely an estimate on the author's part of what might be reasonable weights. It would have been preferable to properly train these weights, but that has been considered out of scope.

For marble capturing danger, we cannot derive an unambiguous implementation, so we implement two versions. One which we believe is the closest to what the authors describe, simply referred to as single- and double marble capturing danger. One which we believe best achieves the goal of the described heuristic, introduced in section 5.2 and referred to as immediate marble capturing danger.

Non-linear aspiration windows are more difficult to get right. Because the source paper does not go into how the non-linear aspiration windows or the evaluation function were trained, and because the description of aspiration windows lacks too many details, the decision was made to not implement aspiration windows. Any implementation based on the description from the source paper is unlikely to be similar to the implementation of the source paper.

For two-step hashing, we decided to go with the ‘new’ replacement scheme for the sake of simplicity.

Chapter 5

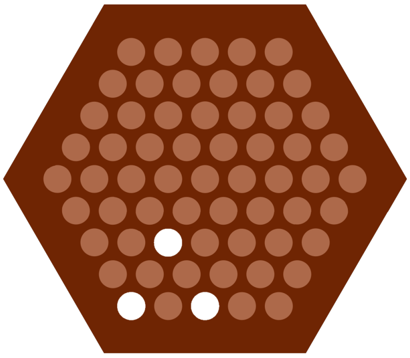
Evaluation Function

This chapter puts forward potential improvements to the metrics of the evaluation function as per the criticisms presented in chapter 4.
The contributions of this chapter are purely theoretical. Whether the metrics provide the hypothesized effects and whether they actually improve performance is left as future work.

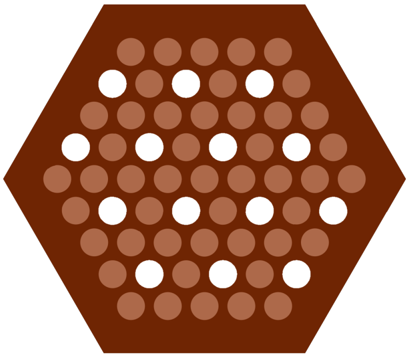
5.1 Formation break calculation

As explained in section 4.2, the number of axes in which a formation break occurs is not considered in the implementation of the source paper.
To judge whether it would be better to consider the number of axes when computing formation break, we contacted Vincent Frochot, who won the Abalone World Championship 8 times as of 2020. Frochot was kind enough to reply and grant permission to attach his reply in the appendix, section A.2.
Frochot states that (paraphrasing) a formation break on multiple axes can be beneficial because it forces the opponent into a line of play *if* they want to take control of the position that the marble in formation break occupies. This grants the player that owns the marble in formation break a time advantage, expressed in moves, which allows them to improve their position.

In his reply, Frochot also mentions another metric which he calls *triangulation*. This metric is closely related to the formation break metric.
Triangulation occurs when three marbles form the corners of a triangle, with one possibly empty field between each pair of marbles, as illustrated in figure 5.1a. Such a position protects the marbles from 2 directions each because it is not possible to fit 2 or more of the opponent's marbles between the marbles. With an extra marble, two triangulations can be formed to protect the existing marbles from another direction, while the extra marble would enjoy the protection of the existing marbles in these same directions. When this idea is taken to the extreme, the result is a perfect triangulation, as illustrated in figure 5.1b. In this situation, no white marbles can ever be pushed.
Note that if a black marble were placed in between the marbles of a white triangulation, the evaluation function will consider the triangulation of white a good thing, while also considering the formation break of black to be good. The explanation for this is simple: both a formation break and triangu-



(a) An example of three marbles forming a single triangulation. Note that the marbles on the edge of the board can only be captured from one direction.



(b) One of the three perfect triangulations that are possible in Abalone. Regardless of where one can place black marbles, white marbles are unable to be pushed.

Figure 5.1: Examples of triangulation.

lation make it expensive (in terms of moves) to take over the location of the marbles that form a formation break or triangulation. If there is no desire from the opponent to take over the position, then the marbles metrics achieve nothing and may even be counterproductive by restricting the movement of your own marbles.

We conclude that it is better to replace the question "Should a formation break on multiple axes be considered better than a formation break on one axis?" with "When is an opponent likely to want to take control of a position?". This leads to a notion of *desired positions* which could refine the formation break and triangulation metrics.

5.2 Marble capturing danger

The issue with the marble capturing danger metric used by the source paper is that it is prone to false positives and false negatives, as explained in section 4.3.
We have addressed this by putting forward our own version of the metric that we call *immediate marble capturing danger*.
Immediate marble capturing danger is triggered if and only if a marble can be pushed off within one move by the opponent.
By definition, there will be no false positives or false negatives for this metric. This does not mean, however, that it is objectively better than single and double marble capturing danger. These metrics are less accurate but apply to far more situations. As a result, they may be indicative of a future

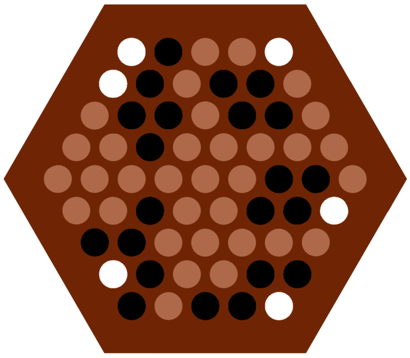


Figure 5.2: Five example situations in which white marbles can be captured, regardless of the actions white performs.

'immediate marble capturing danger' and steer the AI into a more aggressive sequence of moves that allows it to capture the opponent's marble.
Marble capturing danger can further be improved upon with another metric. If a marble is immediately threatened, it can be checked whether that marble can escape. If it cannot escape, you can assign most of the points that you normally get for capturing the marble to the rating of the game state. This then allows you to improve your position elsewhere on the board and only decide to capture the marble if there is no better move. A few example situations in which a marble cannot escape are given in figure 5.2.

5.3 Coherence

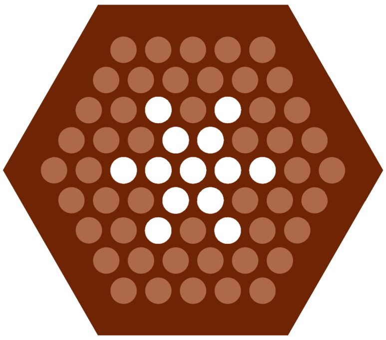
Our implementation of the coherence metric and the definition of the metric differ as the definition was misinterpreted early on. The difference was not caught until after generating the results, which is too expensive to redo. Consider the definition of the coherence metric from the source paper:

> *"[Coherence] is calculated as the number of neighbors each marble has, giving also an extra bonus in three marbles in-line formations."* (Source paper, page 2)

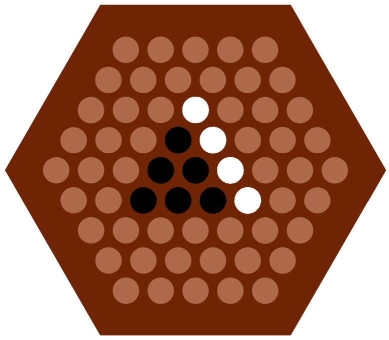
Compare this to the way the metric was implemented:

> The coherence score of a marble is the number of allied marbles surrounding it in a star pattern (figure 5.3a), where the outer marbles only count if there is an allied marble between it and the original marble.

The key difference between the two is that our implementation overly rewards long lines. The marble in the center of the star pattern is given the



(a) The middle marble improves its coherence score for all of the other marbles on the board, for a total score of 12.



(b) A typical formation that can be observed when our AI (white) plays against a stronger opponent. Because of the coherence metric, it forms lines that cannot influence the board in a meaningful way.

Figure 5.3: Problems with our implementation of the coherence metric.

maximum score, while in reality it is often not a good use of marbles to create such formations. After analyzing the games played by our AI, we found that stronger opponents often amass a ‘powerful triangle’, next to which our AI would form lines of marbles as illustrated in figure 5.3b. These lines are vulnerable to being broken up by the powerful triangle without being able to break up the powerful triangle itself.
To avoid overly rewarding long lines, we propose the following definition:

> For each axis, for each marble, assign a score of 0 if it forms no line, a score of 2 if it forms a line of length 2, and a score of 3 if it forms a line of length 3 or more. The coherence score of a marble is the sum of the scores of all its axes.

No bonus has to be awarded for reaching a length of 3 as this is inherently taken care of. Because each marble in the line contributes to the score, there is a large reward for creating a line of 3 marbles. When a fourth marble is added, however, it no longer improves the score of all the previous marbles, making it less valuable. If the marble is instead placed adjacent to two of the other marbles, the score increases.

5.4 Distance to middle

The distance to middle metric is as simple as it can get: sum up the Manhattan distance to the middle for each marble. The intuition being that the middle gives versatility and safety. From the middle, marbles can quickly deploy to attack the opponent on any side without being in immediate risk of being captured.

An alternative to distance to middle has been put forward by Aichholzer, Aurenhammer and Werner [4] which they used as the sole metric for the evaluation function of their Abalone game-playing agent, Aba-Pro. Instead of computing the distance to the middle, this metric computes the distance to the weighted average of the middle and the centers of mass of the black and white marbles, respectively. The intuition is that "*a strong player will keep the balls in a compact shape and at a board-centered position.*"

Vincent Frochot suggested *pathfinding distance* to middle as another potential alternative. Such a metric would reward and punish based on the freedom of movement of a marble in addition to staying safe. However, the metric could be too expensive to be worth using.

Combining the two alternatives results in a *weighted pathfinding distance to middle* metric which encourages a player's marbles to stick together, be close to the middle and close to the opponent's marbles, while additionally taking into account freedom of movement.

Chapter 6

Results

We compare our performance results with the results of the source paper.

6.1 Internal consistency

Before we can produce results of any value, we must show that our method to generate the data is not flawed. As explained in the methodology, internal consistency for timing is guarded by a consistent background load for the computer[1] and multiple executions of the same game. All runs that measured performance, including the runs for other sections, were given an arbitrarily chosen time limit of 15 seconds.
We observed that even after rerunning the same game 30 times, the minimum time spent computing a turn is around 4% slower with a background load active. The minimum time spent computing the same turn is used instead of the average as this better reflects the best possible performance on the used hardware. Any results slower than the minimum time spent computing are caused by small background processes that are infeasible to eliminate. Even though we are mainly interested in the minimum time spent, it is worth noting how much the runs improve when there is no background load, as depicted in figure 6.1.

6.2 First-player bias

When putting two identical agents at odds against each other using iterative deepening depth-first search to get nondeterministic executions, we observed that the first player won 6 and 12 out 30 games on the Belgian daisy and standard layout, respectively. Many of these games are barely different from each other, to the point where (nearly) all moves are identical and the games even end on the same turn. Such duplicate games are not interesting in determining whether there is a bias, so we filter the runs to only leave in a single of such duplicate runs. In this scenario, the first player wins 5 out of 15 games on the Belgian daisy layout and 11 out of 19 games on the standard layout.

[1] All results were generated on a Windows 10 system with an Intel Core i5-4590 cpu with a clock speed of 3.3GHz running Java through AdoptOpenJDK JRE with Hotspot 11.0.9.11 (x64) which was given 4 GB of RAM.

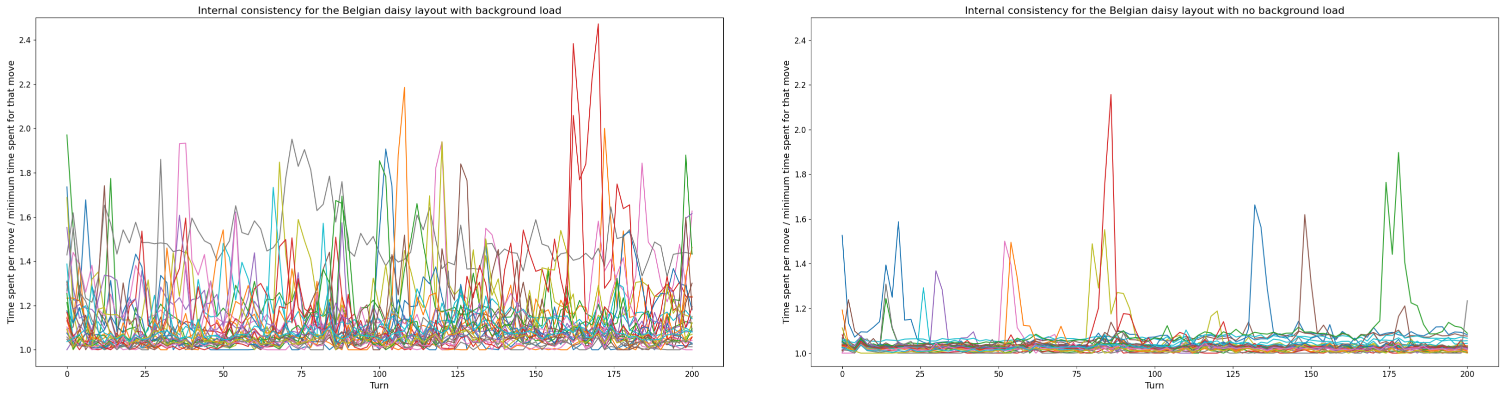


Figure 6.1: Internal consistency runs for Belgian daisy. In both graphs, the computation times for 30 runs are plotted against the minimum computation time for each turn. The difference between the graphs clearly show that a background load has major influence on the computation times.

An odd pattern is that while games appear to be decided earlier, they are also more diverse in the standard layout. We reason that the games are decided earlier in the standard layout because both players want to take the same positions, yet only one of them succeeds and gets the advantage. In Belgian daisy, the AI finds multiple desired positions as there are two clusters of marbles.
We suspect that the games are more diverse in the standard layout because there are more turns where nothing game-deciding happens. The small variations create disjoint games before the game it branches from is decided. Additionally, for Belgian daisy we see (in appendix B.1) that the agents always trade a marble at the start, which means that there are less marbles to play with, leading to more similar games.

All in all, we conclude that there is a bias in the second player's favor. To mitigate this, we will be running all quality and performance measurements twice as often so both players get to be the first and second player equally often. For brevity's sake, this will not be mentioned for each of the results.

6.3 Consider enemy position

A feature that was not considered in the source paper but which we implemented is to toggle the computations of metrics for the opponent's marbles. We consider two agents: 'consider self', which only considers its own marbles when rating the game states, and 'consider enemy' which considers both its own marbles and that of the enemy when rating the game states. As expected, the quality is much worse for consider self than consider enemy: on

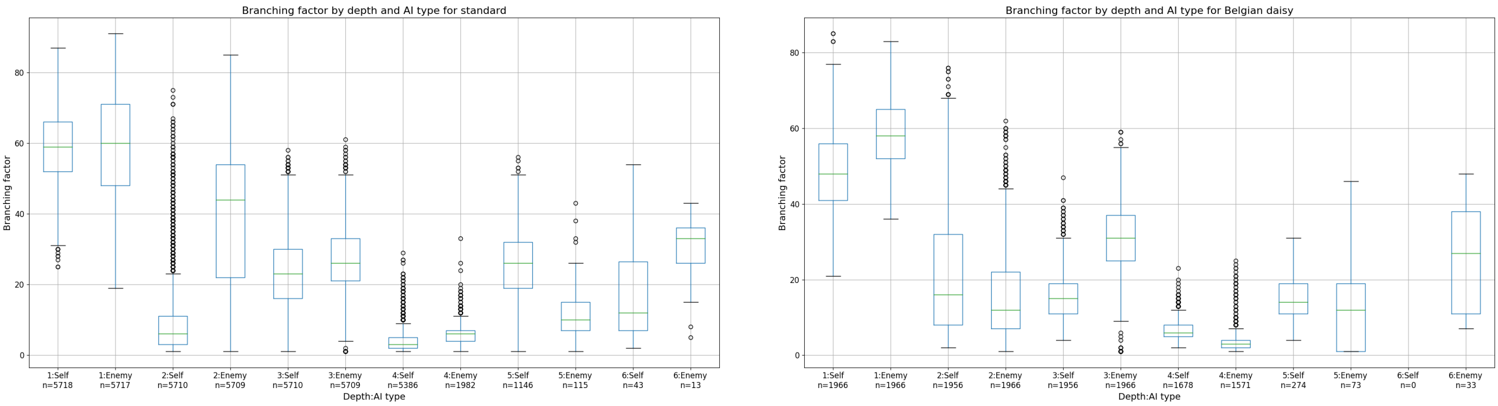


Figure 6.2: The branching factor at depths 1-6 on both starting layouts, for the AI that only considers its own marbles and for the AI that considers both marbles. n refers to sample size; the number of turns where this depth was reached.

both the Belgian daisy and the standard layout, consider enemy wins 4/4 times.
The performance, on the other hand, is far superior for consider self on the Belgian daisy layout as it wins 20 out of 20 games. For the standard layout, however, both agents are similar in strength: 9 out of 20 games are won by consider self.
We reason that consider self is weaker on the standard layout due to both AIs trying to get the same position (the middle), while only one of them can attain this position. The AI that controls the middle generally wins.
The success of consider self on the Belgian daisy layout is explained by the decrease in computation time. This decrease is a consequence of only having to compute the metrics for half the marbles, and of a branching factor that is, on average, lower than that of consider enemy, as can be seen in figure 6.2. The lower average branching factor is presumably caused by the fact that if there are less marbles to consider, the scores are more likely to overlap, and therefore get pruned. In a way, this makes it similar to aspiration windows: quality is sacrificed for speed to increase the overall performance.

6.4 Speed comparisons

To verify the claims made by the source paper we compare the speed of different heuristics as described in the methodology. In figure 6.3, the computation times of the heuristics that each ran at a depth of 4 are plotted against each other on a logarithmic scale for the standard layout. Because the graph for the Belgian daisy loadout has a similar pattern to the standard layout, it is included in the appendix instead of this section.

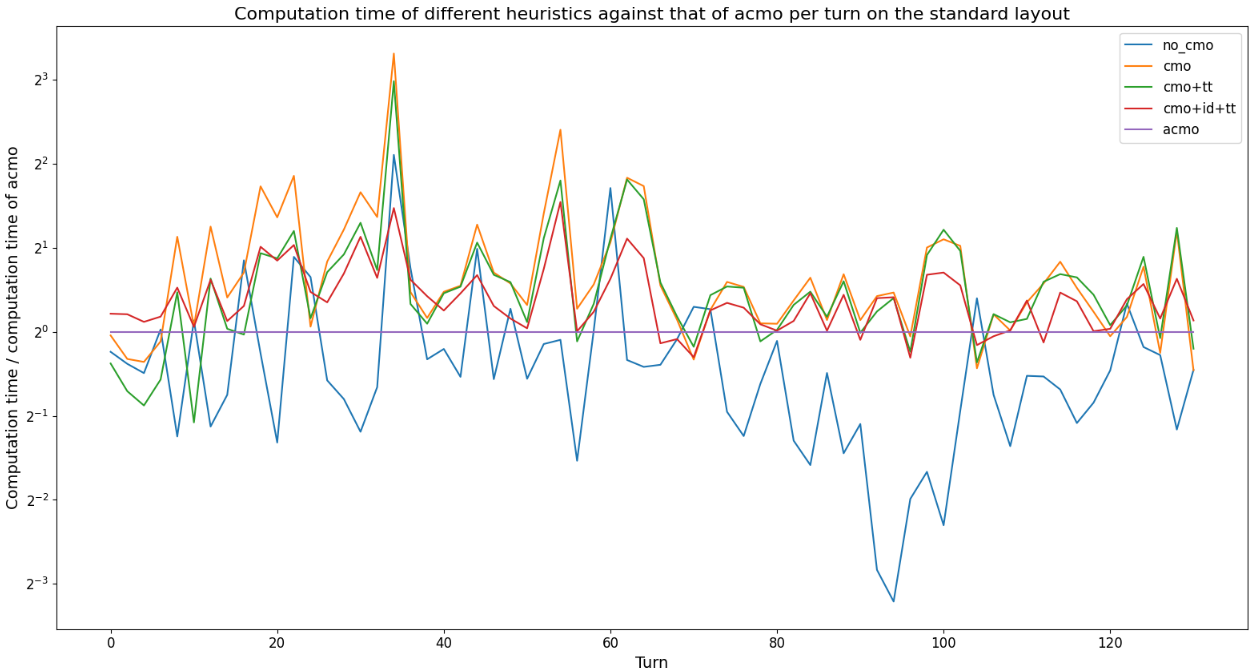


Figure 6.3: Different sets of heuristics applied to the same game on the standard layout. For every heuristic, the game was rerun 30 times with a depth limit of 4. Out of these runs, the minimum time it took to compute a move at a specific turn is plotted against the minimum time of the acmo heuristic of that turn.

From the figure we observe that acmo is quite reliably faster than all the other heuristics, with the exception of no_cmo. No_cmo is chaotic and often faster than the other metrics. Meanwhile, cmo+id+tt is faster than cmo+tt, which itself is slightly faster than cmo.
While one might expect that no_cmo gets a massive advantage with picking the minimum time spent instead of the average, in reality, this is not the case as the order in which moves are generated for the same game state is deterministic.

With the results from this section, we can finally compare our results against that of the source paper in the next sections.

Heuristics	Combined time spent (s)	
	Standard layout	Belgian daisy layout
no_cmo	735	398
cmo	1770	785
cmo+tt	1492	716
cmo+id+tt	1267	667
acmo	907	522

Table 6.1: The sum of the minimum time spent on each turn in 30 runs of the same game for different sets of heuristics on the standard and Belgian daisy layout.

6.4.1 Transposition tables

Are transposition tables providing an approximate speed increase of three times?

> *"All in all, Transposition Tables have a great impact on the performance of the algorithm. With the use of a table with 4Mega-Entries the search becomes approximately three times faster. This is due to the fact that the overhead of the calculation of the keys is negligible, whereas the gain from avoiding calling the evaluation function is significant."* (Source paper, page 5)

In our implementation, at a depth of 4, adding transposition tables to combined move ordering yields a speed increase of 18.6% for the standard layout and 9.6% for the Belgian daisy layout.

6.4.2 Iterative deepening and transposition combined

Is ID + TT 25% slower than just using transposition tables?

> *"In our experiment with Abalone, the use of Iterative Deepening combined with Transposition Tables has consistently reduced our code's execution speed by about 25%."*
>
> (Source paper, page 5)

Adding iterative deepening at a depth of 4 increases the speed on the standard layout by 17.7% and 7.3% on the Belgian daisy layout instead of slowing it down by 25%. This speed increase, while initially unexpected, may be explained by two factors.
First, iterative deepening does not slow down the program as much as it does in the source paper. Due to the evaluation function being untrained and the absence of the non-linear aspiration windows in our implementation, we have a higher branching factor than the source paper. Recall from section 2.2.3 that a higher branching factor leads to less time spent computing previous depths, relatively speaking.
Second, the history sorting heuristic is taking advantage of the repeated executions, thereby overcoming the cost of iterative deepening and even giving a speed increase on top of that.

6.4.3 (Advanced) combined move ordering

Does (advanced) combined move ordering provide a speed boost of approximately 250%?

> *"After encapsulating the Combined Move Ordering in the player's implementation, the result is approximately 250% faster at a seven level search."* (Source paper, page 7)

As we were unable to run games on a depth of 7, we do not expect to see a speed boost as large as 250%. At a depth of 4, we observed that advanced combined move ordering is on average 48.7% faster than combined move ordering on the standard layout and 95.1% faster on the Belgian daisy layout, making it the fastest of the heuristics. However, the search with no heuristics is on average 50.3% faster than advanced combined move ordering on the standard layout and 23.7% faster on the Belgian daisy layout.

We suspect that speed increase of combined move ordering is outweighed by the additional computation cost because the evaluation function is untrained. This means that the assumption of "a good move applied to a good state leads to another good state" is often invalidated, which decreases the speed increase to the point that move ordering is no longer beneficial.
Evaluate sorting still has potential as we can see that when the costs are lower, it provides a speedup. This is evidenced in the acmo vs cmo+id+tt comparison where iteration sorting can reuse the results of evaluate sorting at a negligible cost.

6.5 Aba-Pro versus the source paper

To compare the evaluation function of Aba-Pro [4] and that of the source paper, the evaluation function of Aba-Pro was also implemented and many runs were conducted where Aba-Pro played our reimplementation of the source paper. Aba-Pro performs its calculations 2.9x faster than the source paper because less computations have to be made on each game state. This is in line with expectations.

When measuring quality, both AIs win half the games. As for performance, the source paper reimplementation comes out as the clear winner as it won 13 out of 20 games on the standard layout and 15 out of 20 games on the Belgian daisy layout.

Aba-Pro has 2 hidden weights: one which determines the weight of the center of the board in the computation of ‘reference point R’, one which determines how much captures are valued. The evaluation function of the source paper is more likely to be sensitive to being (un)trained than Aba-Pro because it has more weights, so the suspicion is that when both versions are properly trained, the source paper would outperform our reimplementation Aba-Pro.

Chapter 7

Conclusion

7.1 Verifying and reproducing the results from the source paper

Our results do not align with that of the paper. The source paper reports a reduction in speed for Iterative Deepening, whereas we observe an increase in speed. For transposition tables and advanced combined move ordering, the speedup achieved by the source paper is an order of magnitude higher than the speedup our implementation achieves.

7.2 Critical analysis of the source paper

The source paper does not hold up well. It explains the intuition behind non-linear aspiration windows, but fails to describe it to sufficient detail to reimplement it, which led to the decision to scrap our reimplementation of the heuristic. The paper reports speed increases and decreases on multiple occasions, yet there are no graphs or tables to back up any of these claims. The graphs and tables that are provided only show the number of visited nodes, not the computation time for visiting all these nodes. There is no description of the impact that the quality of the evaluation function has on the results or under which conditions they were generated. The paper was not written in a manner that allows it to be independently verified or even built upon.

7.3 Evaluation function

We introduced alternatives to several metrics defined in the source paper in chapter 5. Based on theoretical examples, we suspect that they more accurately describe the quality of a game state. Verifying whether these metrics are indeed more effective, taking into account computation time, is left as future work.

Chapter 8

Discussion

The source paper did not provide the weights for the evaluation function or information on how to train it, which led to the decision to use arbitrary weights for the evaluation function. As a result, the evaluate sorting heuristic was not nearly as effective as in the implementation of the source paper. This is presumed to be the reason why move ordering heuristics lead to a decrease in computation speed according to our results. Additionally, the absence of the weights prevented qualitative comparisons between the metrics that make up the evaluation metrics and the proposed metrics.

In the implementation, teams were implemented in addition to players. The reason being to support 3- and 4-player games, which was required for the original project from which this thesis grew. In hindsight, it would have been better to remove teams as a whole and keep the focus on 2-player games to reduce the complexity of the code and increase the overall performance.

For our speed comparisons, we did not check whether the AI that the game is originally played by influences the times of the different AIs replaying the game. For example, a game generated by an acmo agent might lead acmo to be faster compared to other AIs than it would be if the game were generated by a cmo agent.

Chapter 9

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Chapter 10

Acknowledgements

Vincent Frochot, for helping me get up to speed with the theory of Abalone and introducing me to many concepts, notably the concept of triangulation.

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My dad, for motivating me to finally finish my thesis.

Appendix A

Emails

This appendix contains relevant parts of the email exchanges with Michel Lalet, Laurent Levi and Vincent Frochot.

A.1 Draw email

Because the rulebook does not state how repeating patterns should be resolved, I asked the co-author of the game, Laurent Lévi how the game should be resolved if both players repeatedly make and undo a move. Laurent Lévi discussed this with the other co-author of the game, Michel Lalet and with Vincent Frochot, 8 times Abalone world champion. They sent back the following reply:
14 May 2020

A.1.1 English version (translated)

If a player repeats the same movement, it can be considered that he is making a tie proposal to his opponent, which the latter can accept by in turn making a movement identical to the one he had already made in the previous round. The draw is made in four rallies where each player will make and break the same move identically.
Example:

Round 1:
Player 1 makes movement A.
Player 2 makes the move B.

Round 2:
Player 1 does the reverse of move A to get back to where he was.
Player 2 does the reverse of move B to get back to where he was.

*** at this time, Player 1 is assumed to have understood that Player 2 wants to play move B if Player 1 plays move A ***

Round 3:
Player 1 repeats the same movement A.
Player 2 repeats the same movement B.

Round 4:
Player 1 does the reverse of movement A to get back to where he was.
*<=> ** Player 1 proposes the tie game ***
Player 2 does the reverse of movement B to return to where he was.
*<=> ** Player 2 accepts the tie game ***

This situation of declaring a "tie" can occur between players, even if one of them leads the score, for example 4 balls out to 1.

You just have to start a new game to decide between yourself!

A.1.2 French version (original)

“Si un joueur réitère un même mouvement, on peut considérer qu’il fait à son adversaire une proposition d’égalité, que ce dernier peut accepter en faisant à son tour un mouvement identique à celui qu’il avait déjà effectué au tour précédent.
La proposition d’égalité s’effectue en quatre échanges où chaque joueur va faire et défaire, à l’identique, un même mouvement.
Exemple :

Tour 1 :
Joueur 1 fait le mouvement A.
Joueur 2 fait le mouvement B.

Tour 2 :
Joueur 1 fait l’inverse du mouvement A pour revenir où il était.
Joueur 2 fait l’inverse du mouvement B pour revenir où il était.

*** à ce moment là, Joueur 1 est censé avoir compris que Joueur 2 veut jouer le mouvement B si Joueur 1 joue le mouvement A ***

Tour 3 :
Joueur 1 refait le même mouvement A.
Joueur 2 refait le même mouvement B.

Tour 4 :
Joueur 1 refait l’inverse du mouvement A pour revenir où il était.
*<=> ** Joueur 1 propose la partie égalité ***
Joueur 2 refait l’inverse du mouvement B pour revenir où il était.
*<=> ** Joueur 2 accepte la partie égalité ***

Cette situation aboutissant à déclarer une « égalité » peut se produire entre les joueurs, même si l’un d’eux mènerait au score, par exemple 4 boules *sorties à 1.*

Vous n'avez plus qu'à débuter une nouvelle partie pour vous départager !"

A.2 Formation Break emails

A.2.1 Mail 1

To answer the question from section 5.1, whether it is a boon to have a formation break on 2 or 3 axes rather than 1, we contacted Vincent Frochot, 8 times Abalone world champion.
Vincent Frochot was kind enough to respond and give permission to add his answer to this thesis. The following is the body of his reply:

12 August 2020
"A "formation break" is useful if you can use it while attacking to create the attack on multiple axes, or if you need time (expressed as moves) (e.g. to reposition a group, or to win before the opponent can): the opponent will have to push this marble to be able to attack on the axes she was.
Thus, a marble totally surrounded by opponent marles is helping you more in the sense [that] the opponent will first need to create a way before he can expect to push outside of his group this marble.

But there is indeed something else to consider, if you extend this notion using more marbles.
Abalone has some non-obvious elegant properties.
Since the field is a hexagon, if we try coloring each square with the minimum total number of different colors, while no neighbouring square shares the same color, we obtain a pattern of triangles.
That's why I chose to name this concept "Triangulation", when I speak about this.
Triangulation will let you create some sort of forced logical path your opponent will have to take before he can take back the control of one position.
This concept is used very often in games between best Abalone players.

I shared with you some use cases from my drive account, in 3 separate folders:

Example 1:
"perfect" triangulation for White (3 perfect triangulations are possible on an Abalone board, in total) while Black is using his 9 remaining marbles to create a partial triangulation.
Notice Black won't be able to do anything until White accepts to give him one of his points, and White would have to go in the edge to be able to start

doing something.

Example 2:
Triangulation from Black gives him enough time to win the match by pushing out the marble in i9 (top right corner).

Example 3:
Sequence of moves from 2018 Abalone MSO championship - final game
We can see triangulation used both as defense and attack."

A.2.2 Mail 2

After the first mail, I sent a reply noting how formation break and triangulation are conflicting metrics. I also asked him if he came up with the concept of triangulation. The following was his response:

13 August 2020
""What I find interesting about formation break and triangulation is that you get conflicting results from the metrics:"
"If white forms a triangulation with 3 marbles, that'd be considered good for white. If black is between one of these marbles, however, it would count towards formation break which is considered good for black."
In the 9th chapter of "Le livre d'Abalone" you can find this idea of the situational value of marble infecting the opponent group. Using it correctly in an algorithm to evaluate a board may not be very easy : The goal is to push out 6 marbles, not to block the opponent.

Triangulation (or formation break) is useful if you need time, or if you plan to use the marbles as a way to create multiple attacks in one move later one. So if you want to consider an advantage of attack later on, you need to make sure the opponent won't be able to prevent the push required to unlock the multiple attacks.
Also, if both White and Black think they need time, I guess probably one of them was less right than the other : It depends, triangulation is not necessarily good. :)

I came up with this idea of "triangulation" indeed, since I am studying the game and writing theory so I have to name things. Also, the color exercise is very cool when I have to explain it to children, they are really surprised to see the "match" :)
The name is not officially validated by any way, but I observed it already started becoming a convention when we talk with other players I know.
Maybe with time another word will be considered more relevant."

Appendix B

Graphs

This appendix contains additional data that that does not fit between the text.

B.1 First-player bias

The graphs in this section are referenced in section 6.2. They denote the score progression from the first player's perspective in many different games.

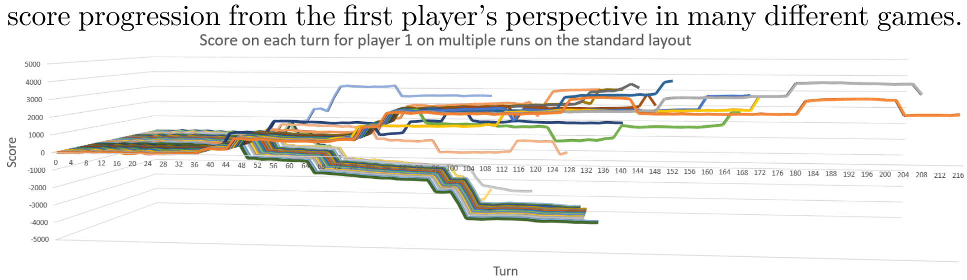

Figure B.1: Score progression for the standard layout.

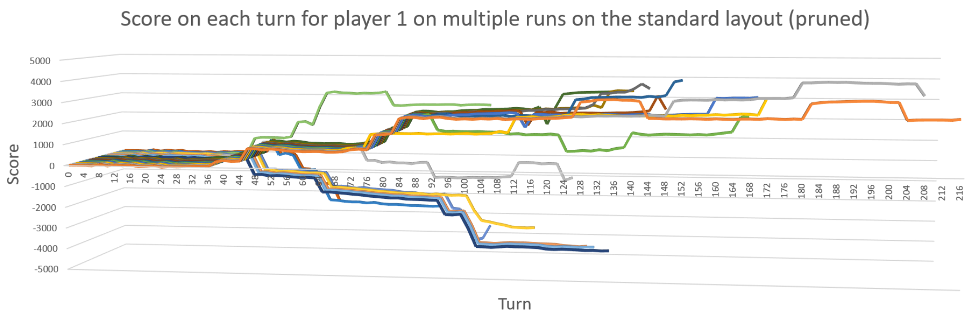

Figure B.2: Score progression for the standard layout, pruned. Many games in the unpruned version share the same score progression. For the pruned version, only one such game is included by filtering out the other games that end on the same turn.

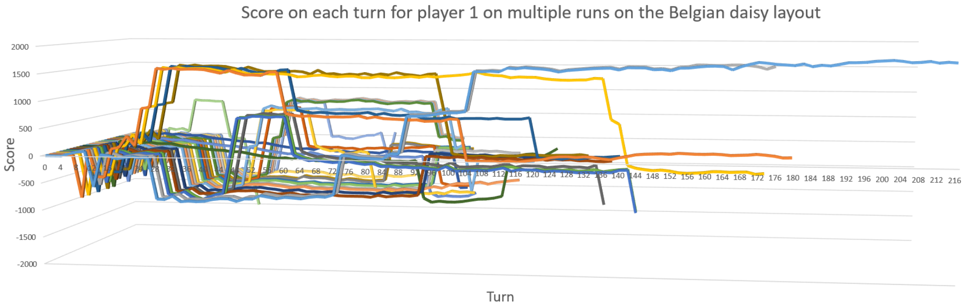

Figure B.3: Score progression for the Belgian daisy layout.

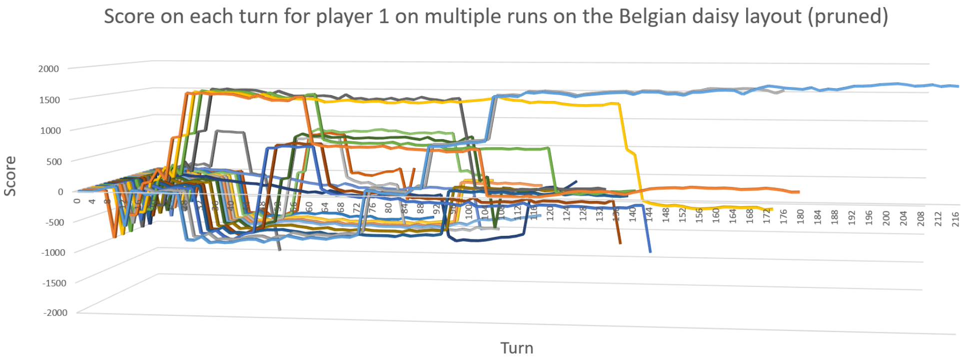

Figure B.4: Score progression for the Belgian daisy layout, pruned. The games are pruned in the same manner as in the standard layout.

B.2 Speed

This section includes a graph comparing the speed of the same heuristics as the graph in section 6.4 for the Belgian daisy layout. The graph is included here instead of in the speed section because it shows the same patterns as the graph for the standard layout.

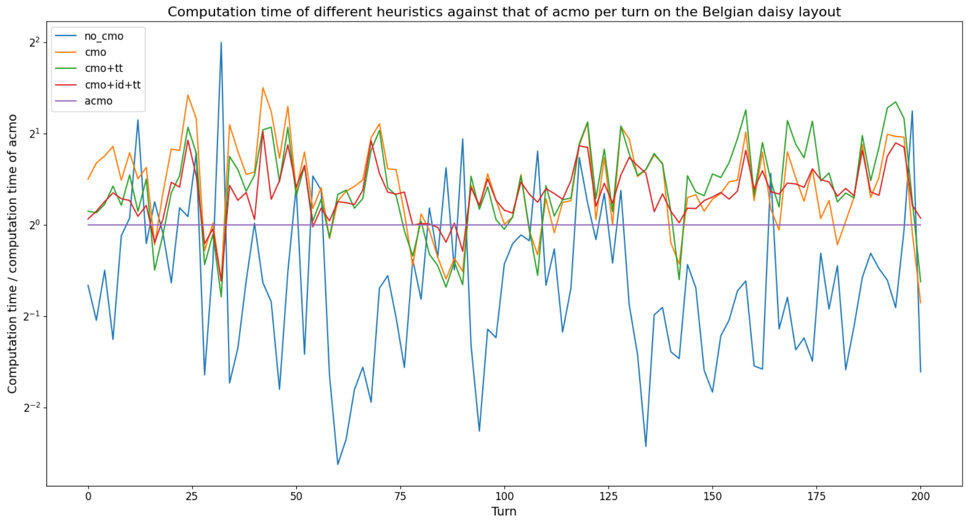

Figure B.5: Different sets of heuristics applied to the same game on the Belgian daisy layout. For every heuristic, the game was rerun 30 times with a depth limit of 4. Out of these runs, the minimum time it took to compute a move at a specific turn is plotted against the minimum time of the acmo heuristic of that turn.