

# Information Sources Logic

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We consider a framework for information collecting and processing which consists of a set of sources providing information about formulas, and a processor. For *each* formula  $\varphi$  (not only an atomic one, like in [3]!), a source can say that  $\varphi$  is true (1), or false (0), or that it knows nothing about  $\varphi$ . The processor collects information from the sources, and combines it using some strategy to form a new valuation  $v_P$ . This valuation uses four logical truth values, corresponding to the four possible sets of the classical values provided by the sources:  $\mathbf{t} = \{1\}$  (truth),  $\mathbf{f} = \{0\}$  (falsity),  $\top = \{0, 1\}$  (inconsistency),  $\perp = \emptyset$  (uncertainty). This framework can give rise to many scenarios. We explore the most basic cases, e.g. the one in which the only assumption concerning  $v_P$  is that the processor observes the most general rules compliant with the properties of classical connectives, like assigning 0 to  $\varphi \vee \psi$  iff it assigns 0 to both  $\varphi$  and  $\psi$ , and 1 if (but not only if) it assigns 1 to either of them. We show that in this case the resulting logic corresponds to a four-valued logical *non-deterministic* matrix [2, 1]. Other cases lead to other non-deterministic matrices. We provide complete, cut-free sequent calculi for the various logics we get, using a general method we have developed.

## References

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