

GLIVENKO THEOREMS FOR SUBSTRUCTURAL LOGICS

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Glivenko's theorem states that a formula ϕ is provable in classical propositional logic iff its double negation is provable in intuitionistic logic, in symbols $\vdash_{\mathbf{Cl}} \phi$ iff $\vdash_{\mathbf{Int}} \neg\neg\phi$. Nevertheless, $\vdash_{\mathbf{Int}}$ is not the only relation that has this property relative to $\vdash_{\mathbf{Cl}}$, nor is $\vdash_{\mathbf{Cl}}$ the only relation that admits a double negation interpretation in another logic. For example, it is shown in [1] that, for every formula ϕ , $\vdash_{\mathbf{Cl}} \phi$ iff $\vdash_{\mathbf{SBL}} \neg\neg\phi$, and that, for every formula ϕ , $\vdash_{\mathbf{Lu}} \phi$ iff $\vdash_{\mathbf{BL}} \neg\neg\phi$, where \mathbf{SBL} is the extension of Hajek basic logic \mathbf{BL} by the axiom $\psi \wedge \neg\psi = \perp$ and \mathbf{Lu} is Łukasiewicz infinite valued logic.

All the logics discussed so far are substructural logics. In this talk, we extend Glivenko's theorem and discuss pairs of substructural logics for which a double negation translation (in one of four different forms) holds. Moreover, we show that for every involutive substructural logic there exists a minimum substructural logic that contains the first via a double negation interpretation. Our presentation is algebraic (we use the fact that substructural logics over the full Lambek calculus are algebraizable) and is formulated in the context of residuated lattices. The contents of the talk are part of joint work, [2] and [3], with H. Ono.

REFERENCES

- [1] R. Cignoli and A. Torrens, *Hájek basic fuzzy logic and Łukasiewicz infinite-valued logic*, Arch. Math. Logic **42** (2003), 361–370.
- [2] N. Galatos and H. Ono, *Algebraization, parameterized local deduction theorem and interpolation for substructural logics over FL*, to appear in Studia Logica 83 (2006), pp 32.
- [3] N. Galatos and H. Ono, *Glivenko theorems for substructural logics over FL*, submitted.

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