

Monadic second order logic on equivalence relations

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We study the MSO logic over various special kinds of structures and find some results concerning its decidability and expressive power. For instance, we consider the structures with only unary predicates and equivalence relations in local agreement.

Let $\mathfrak{A} = (|\mathfrak{A}|, P_1, \dots, P_k, R_1, \dots, R_n)$ be a structure where P_1, \dots, P_k are unary predicates, and R_1, \dots, R_n are equivalence relations. Let $[x]_{R_i}$ denote the equivalence class of x according to the relation R_i . We say that R_1, \dots, R_n are in local agreement if for every $x \in |\mathfrak{A}|$ the set $\{[x]_{R_1}, \dots, [x]_{R_n}\}$ is linearly ordered according to the set theoretic inclusion.

It is a well-known fact, that over the class of structures with one equivalence relation the MSO logic is decidable. We show that it is decidable over the class of all structures with unary predicates and equivalence relations in local agreement. Moreover, we show that over these structures, every MSO formula has a translation in the first order language, which has exactly the same models. The translated FO formula is very complex, compared to the original MSO formula.