

Some new spectrum of Computable Models

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Abstract

We will discuss the problems about characterization of spectrums of computable models for Uncountably Categorical theories. The basic notions can be find in [2]. From J. Baldwin [1] and A.Lachlan we have that all models of any uncountably categorical theory T can be listed into the following chain, denoted by $chain(T)$, of elementary embeddings:

$$A_0 \prec A_1 \prec A_2 \prec \dots A_\omega,$$

where A_0 is the prime model of T , A_ω is the saturated model of T , and each A_{i+1} is proper extension of A_i .

As we are interested in constructive models of T the following definition is central

Definition 1. *Let T be an uncountably categorical theory. The spectrum of constructive models of T , denoted by $SCM(T)$, is the set*

$$\{i \mid \text{the model } A_i \text{ of } T \text{ has a computable copy up to isomorphism}\}.$$

Thus a natural open problems about computable models of uncountably categorical theories is the following:

Problem

Characterize all the subsets X of $\omega \cup \{\omega\}$ for which there exist uncountably categorical theories T such that $SCM(T) = X$.

Problem 1. *Characterize uncountably categorical theories that have computable models.*

The situation for uncountably categorical theories is more complicated. In general, existence of a computable model for an uncountably categorical theory T does not imply that all models of T have computable copies. Indeed,

S.Goncharov [3] showed that there exists a uncountably categorical theory T for which

$SCM(T) = \{0\}$, that is, the only prime model of T has a constructivization.

K.Khudeibergenov [5] extended this result by showing that for every $n \geq 0$ there exists an uncountably categorical T such that $SCM(T) = \{0, 1, \dots, n\}$.

B.Khoussainov, A.Nies and R.Shore [4] proved that there exist uncountably categorical theories T_1 and T_2 such that $SCM(T_1) = \omega$, and $SCM(T_2) = \omega \cup \{\omega\} \setminus \{0\}$.

A.Nies [6] constructed a uncountable categorical theory T such that $Spec(T) = \{1\}$.

A.Nies find some upper bound for these spectrums. He proved that for a uncountable categorical theory T $Spec(T) \in \Sigma_{\omega+3}^0$.

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Problem 2. *Is $\text{Spec}(T)$ arithmetical if the theory T has computable non-saturated model?*

Theorem 1. *For any $n \in \omega$ there exists a uncountably categorical theory T_n for which $\text{SCM}(T_n) = \{n, n + 1, n + 2, \dots, \omega\}$.*

References

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