SOME RESULTS ON MEDVEDEV DEGREES OF SEPARABILITY.

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Definitions.

1.(Medvedev) Any class $\mathcal{A}\subseteq\omega^\omega$ of total functions is called *mass problem.*

Let $\mathcal{A}$ and $\mathcal{B}$ be mass problems. Then $\mathcal{A}\leqslant_M\mathcal{B}$ if and only if there is a computable functional Φ, such that $(\forall g \in \mathcal{B})(\exists f\in\mathcal{A})(graph(f) = \Phi^{graph(g)})$.

2. Let $S_{A,B} = \{\chi_R|(A \subseteq R)\&(B \cap R = \emptyset)\}$ be mass problem of separation of a disjoint pair of sets (A, B). Then $(A, B) \leqslant_S (C, D) \iff S_{A,B} \leqslant_M S_{C,D}$. The class $\mathbf{a} = \{(C, D)|(C, D) \equiv_S (A, B)\}$ is called *the S-degree of the pair* (A, B). For S-degrees $\mathbf{a}$ and $\mathbf{b}$ define $\mathbf{a} \leqslant_S \mathbf{b}$ if and only if there are pairs $(A, B) \in \mathbf{a}$ and $(C, D) \in \mathbf{b}$, such that $(A, B) \leqslant_S (C, D)$. Let D_S denotes the structure of all S-degrees of disjoint pairs (A, B) and let D_T denotes the structure of all Turing degrees.

3. A pair (A, B) is called a *computably enumerable (c.e.) pair*, if A and B are c.e. sets. Accordingly, S-degree $\mathbf{a}$ is *c.e.*, if there is a c.e. pair $(A, B) \in \mathbf{a}$. Let D^0_S denotes the structure of all c.e. S-degrees and let D^0_T denotes the structure of all c.e. Turing degrees.

4. For any set A, the pair $(A, \overline{A})$ is called *regular.*

Denote $C_0 = \{x| \quad \phi_x(x) = 0\}$, $C_1 = \{x| \quad \phi_x(x) = 1\}$.

Facts.

1. D_S and D^0_S are upper semilattices under the relation $\leqslant_S$.

2. There is isomorphic embedding $i : D_T \longrightarrow D_S$, $i(deg_T A) = deg_S(A, \overline{A})$, because $A \leqslant_T B \iff (A, \overline{A}) \leqslant_S (B, \overline{B})$.

3. For any c.e. pair (A, B) we have $(A, B) \leqslant_S (C_0, C_1)$.

Theorem 1. Any non-zero c.e. S-degree does not contain a regular pair.

Theorem 2 (lower density). For any non-zero S-degree $\mathbf{b}$ there is an S-degree $\mathbf{a}$, such that $\mathbf{0} <_S \mathbf{a} <_S \mathbf{b}$.

Corollary 2.1. There are no minimal S-degrees.

Corollary 2.2. D_T is not isomorphic to D_S.

Theorem 3 (splitting). For any c.e. pair $(X, Y) \notin \mathbf{0}$ there are c.e. sets A_0 and A_1, such that $A_0 \cup A_1 = X$, $A_0 \cap A_1 = \emptyset$ and $(X, Y) \nleqslant_S (A_i, Y)$, where i=0,1.

Corollary 3.1. For any non-zero c.e. S-degree $\mathbf{b}$ there are non-comparable c.e. S-degrees $\mathbf{a}_0$ and $\mathbf{a}_1$, that $\mathbf{b} = \mathbf{a}_0 \cup \mathbf{a}_1$.

Corollary 3.2. There are no minimal c.e. S-degrees.

Corollary 3.3. D^0_S is infinite.

Theorem 4 (density). Let $\mathbf{a}$, $\mathbf{b}$ are any non-zero c.e. S-degrees and $\mathbf{a} <_S \mathbf{b}$. Then exists c.e. S-degree $\mathbf{c}$, such that $\mathbf{a} <_S \mathbf{c} <_S \mathbf{b}$.