A constructive type-theoretical Interpretation of the philosophical Methods of Abstraction and Instantiation

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April 8, 2006

Abstract

In the philosophical tradition, methodological procedures are often composed rather than simple: classical examples of such structures are the processes of *analysis* and *synthesis* and those of *abstraction* and *instantiation*. The latter couple is extremely relevant to logical knowledge also in modern formalization, representing the theoretical basis of basic notions such as *singular* and *abstract*, and the related ones of *individual* and *concept*. What does the philosophical analysis provides in the case of these methods when interpreted within type-theoretical systems?

Typed systems present different ways of proceding in abstraction, e.g. polytypic abstraction, data types abstraction and others; on the other hand, the procedure of instantiation is essentially represented either in terms of introduction of constructors or by means of representation of primitive data-types for computation and information processing, or else as the instance of a program within object-oriented programming. In both cases, nevertheless, there is no exact correspondance to the philosophical methods considered above. A specific version of a typed system, namely Constructive Type Theory (CTT), presents instead a rather precise formalization of the mentioned philosophical procedures: a first analogy has been already recognised in the switch from monomorphic to polymorphic versions of the theory, where forgetting and restoring information in respect to types produces either the generalization or specification of data. But the methods of *abstraction* and *instantiation* are represented by means of a different analysis of the theory. It starts by looking at CTT as built up on two levels:

1. the extensional part, roughly corresponding to extensional set theory;
2. the intensional part, being equivalently represented by a programming language.

This structure provides on each side a relevant notion, to be considered as the product of the procedures of data-abstraction or data-instatiation:

- type as *meaning-object* on the extensional level is now a rather precise representation for the product of abstraction, and it works as the introduciton of a *predicable meaning*;
- the *objectual predication* is on the intensional level the corresponding form of individual instantiation, and it works as a method to represent a *constructed meaning*.

This analysis wants to provide an epistemological clarification of these connections, based on the central distinction between the concepts of knowledge and information.

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