One reducibility on Admissibles

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In [1] Yu. Ershov introduced a notion of Σ-definability for admissible sets as some generalization of constructivizability. He described all the structures which is Σ-definable in $\mathbb{HF}_{\mathrm{L}}$ where L is dense linear ordering. Recall that a hereditarily finite set $\mathbb{HF}_{\mathcal{M}}$ over structure $\mathcal{M}$ is the least by inclusion admissible set containing $\mathrm{dom}(\mathcal{M})$ as a subset. However, Σ-definability does not preserve computable invariants. Here we introduce some reducibility which preserves such computable invariants as effective subsets of natural numbers and computable collections of such subsets. We say that an admissible set $\mathbb{A}$ is Σ-*reducible* to an admissible set $\mathbb{B}$ if there a surjection map $\nu : \mathrm{dom}(\mathbb{B}) \to \mathrm{dom}(\mathbb{A})$ such that $\nu^{-1}(C)$ is $\mathbb{B}$-c.e., for any $\mathbb{A}$-c.e. predicate C. As usual, C is called $\mathbb{A}$-c.e. if it is definable by some Σ formula in $\mathbb{A}$.

We build a transformation which corresponds any admissible set to a hereditarily finite set, Σ-equivalent to one. Moreover, this transformation preserves a certain list of descriptive set theoretical properties. As corollary, the structure $\mathcal{D}$ of all Σ-equivalent classes of admissible sets is an upper semilattice. It is showed that the semilattice $\mathcal{L}_e$ of enumeration degrees is embedded into $\mathcal{D}$. This embedding is natural namely it corresponds any enumeration degree to some admissible set having this e-degree. Furthermore, Σ-reducibility is determined by using Σ-definability. In [2, 3, 4, 5] we study computability properties of the least elements of some natural segments of $\mathcal{D}$.

We also introduced the jump operation $\mathcal{J}$ on Admissible Sets. It is proved that any enumeration degree $\mathbf{a}$, $\overline{\varnothing'} \leqslant_e \mathbf{a}$, is in the image of $\mathcal{J}$. Moreover, a locally constructible structure can be chosen as some pre-image of $\mathbf{a}$.

References

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