# Decidability Variants of Linear Temporal Logic LTL w.r.t. Logical Consecutions

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We study linear temporal logic LTL in standard language with operations **U** (Untill) and **N** (Next) and its extension by operations **S** (Since) and $\mathbf{N^{-}1}$ (Previous). $LTL^{\mathbf{U}}_{\mathbf{N}}(\mathcal{N})$ is the logic with **U** and **N** generated by the frame $\mathcal{N}$ of all natural numbers as Kripke/Hinttikka structure, $LTL^{\mathbf{U}}_{\mathbf{N}}(\mathcal{Z})$ is the linear logic based on the frame $\mathcal{Z}$ of all integer numbers, and $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ is the similar logic which uses **S** and $\mathbf{N^{-1}}$. Logical consecution is a structural inference rule $\mathbf{A_1}, \ldots, \mathbf{A_n}/\mathbf{B}$ (where $\mathbf{A_i}$ and **B** are some formulas in the language of given logic).

**Theorem 1.** *Linear temporal logic* $LTL^{\mathbf{U}}_{\mathbf{N}}(\mathcal{N})$ *is decidable w.r.t. admissible logical consecutions.*

**Theorem 2.** *Linear temporal logics* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{N})$ *and* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ *do not have the finite model property.*

**Theorem 3.** *There is an algorithm recognizing inference rules admissible in* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$*. So,* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ *is decidable w.r.t. admissible inference rules, as a consequence,* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ *itself is decidable.*

These theorems extends results from [1] and [2] to LTL with standard operations **U** (Untill) and **N** (Next) and its extension by operations **S** (Since) and $\mathbf{N^{-}1}$ (Previous).

## References

[1] Rybakov V.V. Logical Consecutions in Discrete Linear Temporal Logic. *Journal of Symbolic Logic, V.70, No 4 (2005), pp. 1137-1149.*

[2] Rybakov V.V. Logical Consecutions in Intransitive Temporal Linear Logic of Finite Intervals. *Journal of Logic Computation, Vol. 15 No. 5 (2005) pp. 633 -657.*