EMBEDDING LATTICES INTO SUBSEMIGROUP LATTICES

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For an algebraic structure A, let $\operatorname{Sub} A$ denote the substructure lattice of A. The problem of characterizing of the class $\mathbf{S}(\operatorname{Sub}\mathcal{K})$ of lattices embeddable into substructure lattices of structures belonging to some class $\mathcal{K}$; then $\mathcal{K}$ is *lattice-universal*, if $\mathbf{S}(\operatorname{Sub}\mathcal{K})$ coincides with the class of all lattices.

We consider this problem restricted on different classes of semigroups. V. B. Repnitskii proved in [1] that several classes of semigroups are lattice-universal. Problem 28.14.2 in [3] asks about a description of the class $\mathbf{S}(\operatorname{Sub}\mathcal{N}_n)$, $n < \omega$, where $\mathcal{N}_n$ denotes the class of n-nilpotent semigroups. We provide a syntactic descriprion of this class.

Theorem 1. *For any $n < \omega$, the class $\mathbf{S}(\operatorname{Sub}\mathcal{N}_{n+1})$ is a finitely based variety. The class of finite members of $\mathbf{S}(\operatorname{Sub}\mathcal{N}_{n+1})$ is defined by a finite set of identities within the class of finite lattices, whence it is a pseudovariety.*

Problem 28.14.2 in [3] asks about a description of the class $\mathbf{S}(\operatorname{Sub}\mathcal{K})$, where $\mathcal{K}$ is the class of free (commutative) (2-nil)semigroups or the class of free semilattices. V. B. Repnitskii provided in [2] a description of finite lattices belonging to the above classes. We present the following

Theorem 2. *For any infinite cardinal κ, the follwing statements are equivalent:*

(1) *$L \in \mathbf{S}(\operatorname{Sub} F(\kappa))$, where $F(\kappa)$ is a free semigroup of rank κ;*
(2) *$L \in \mathbf{S}(\operatorname{Sub} FC(\kappa))$, where $FC(\kappa)$ is a free commutative semigroup of rank κ;*
(3) *$L \in \mathbf{S}(\operatorname{Sub} FN(\kappa))$, where $FN(\kappa)$ is a free 2-nilsemigroup of rank κ;*
(4) *$L \in \mathbf{S}(\operatorname{Sub} FCN(\kappa))$, where $FCN(\kappa)$ is a free commutative 2-nilsemigroup of rank κ;*
(5) *$L \in \mathbf{S}(\operatorname{Sub} FSL(\kappa))$, where $FSL(\kappa)$ is a free semilattice of rank κ;*
(6) *L embeds into $\prod_{\xi<\kappa} L_\xi$, where L_ξ is a finite lower bounded lattice for any $\xi < \kappa$.*

In particular,

$$\mathbf{S}(\operatorname{Sub}\mathcal{F}) = \mathbf{S}(\operatorname{Sub}\mathcal{FC}) = \mathbf{S}(\operatorname{Sub}\mathcal{FN}) = \mathbf{S}(\operatorname{Sub}\mathcal{FCN}) = \mathbf{S}(\operatorname{Sub}\mathcal{FSL}) = \mathbf{SP}(LB_{\text{fin}}).$$

References

[1] *V. B. Repnitskiĭ*, On the representation of lattices by lattices of subsemigroups, Russian Math. (Izv. VUZov) **40** (1996), 55–64.
[2] *V. B. Repnitskiĭ*, On finite lattices which are embeddable in subsemigroup lattices, Semigroup Forum **46** (1993), 388–397.
[3] *L. N. Shevrin, A. J. Ovsyannikov*, Semigroups and their subsemigroup lattices, Dordrecht, Kluwer Academic publishers, 1996.

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