n-c.e. sets CEA in c.e. degrees

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A set $A \subseteq \omega$ is called 2-c.e. if there are computable enumerable (c.e.) sets A_1 and A_2 such that $A = A_1 - A_2$. A set D is n-c.e. if $D = D_1 - D_2$ for a c.e. set D_1 and a $(n-1)$-c.e. set D_2. A Turing degree is called n-c.e. *degree* if it contains a n-c.e. set; for $n \geq 2$ it is called *properly* n-c.e. if it is n-c.e. but not $(n-1)$-c.e. degree.

A set A is relatively c.e. in a set B if we can computable enumerate A using B as an oracle, in this case we write $A \in \Sigma_1^B$. I my talk I will consider sets and degrees which are relatively c.e. in c.e. sets and degrees.

It is well-known that any 2-c.e. set is relatively c.e. in a c.e. degree below it. Also, obviously, any set is relatively c.e. in any degree above it. Therefore, it is natural to ask the following question: is there for any 2-c.e. set A a c.e. set B such that A is relatively c.e. in B, and A is Turing incomparable with B? It is easy to see that for 2-c.e. sets the answer is negative: for any c.e. set B just take $\omega - B$. The most interesting case is that when the given set A has a properly 2-c.e. degree. The following theorem gives an answer.

Theorem 1. For any 2-c.e. set A from a properly 2-c.e. degree there exists a c.e. set B such that $A \in \Sigma_1^B$ and $A|_T B$.

Also, there are two well-known theorems.

Theorem (*Arslanov, LaForte, Slaman*). Let A and C be sets such that C is c.e., A is c.e. in C, and A is ω-c.e. Then $deg(A)$ is 2-c.e.

Theorem (*Downey, LaForte, Lempp*). There exists a 3-c.e. set A such that for all c.e. sets W, A has Σ_1^W degree if and only if $W \equiv_T 0'$.

The first theorem says that we can't consider for properly $n-$c.e. $(n \geq 3)$ sets relative c.e. in c.e. sets below it. The second theorem which is proved by a $0'''$-priority argument establishes a stronger result on sets which are relatively c.e. in c.e. sets.

Using $0'$-priority argument we proved that

Theorem 2. There exists a 3-c.e. set A such that for all c.e. sets W, $A \in \Sigma_1^W$ if and only if $W \equiv_T 0'$.

Since the proof of this theorem uses only $0'$-priority argument it can be combined to obtain 3-c.e. sets with this and some additional properties. For instance, it can be made of a low degree, it can be found above any noncomplete c.e. degree, it can be found in any jump c.e. class and so on. Also in this theorem we can require that the degree of A is properly 3-c.e.