

Model Theory and Lie Algebras: Infinite dimensional representations over $sl_2(K)$

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This report aims at illustrating the main results of two joint works, the former with Mike Prest [3] and the latter with Ivo Herzog [2], both devoted to a model theoretic investigation of certain simple infinite dimensional representations over $sl_2(K)$ –the Lie algebra of trace zero 2×2 matrices over an algebraically closed field K of characteristic zero– and inspired by Herzog’s paper [1] showing several remarkable results about the ring of definable scalars of finite-dimensional representations of $sl_2(K)$.

In detail, we deal with

- Verma modules,
- pseudo-finite dimensional representations of $sl_2(K)$

(meaning infinite dimensional representations satisfying the same first order sentences as finite dimensional representations of $sl_2(K)$).

First we examine the ring of definable scalars of a Verma module and, among other things, we show that it is Von Neumann regular; also, we obtain some information on the Ziegler spectrum of a Verma module.

Then we consider a pseudo-finite dimensional representation V over $sl_2(K)$. It is known that its endomorphism ring is a field extension K_V of K , so we want to characterize all elements of this field K_V . Here some intriguing connections with exponentiation arise. We also discuss some open problems in this framework.

References

- [1] I. Herzog, *The pseudo-finite dimensional representations of $sl(2, k)$* , *Selecta Mathematica*, 7 (2001), 241-290

- [2] S. L’Innocente, I. Herzog, *Pseudo-finite dimensional representation over $sl_2(K)$ and exponentiation*, in progress
- [3] S. L’Innocente, M. Prest, *Rings of definable scalars of Verma modules*, submitted