AX THEOREM IN ARBITRARY CHARACTERISTIC

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Recall a theorem of Ax, which is a differential version of the Schanuel Conjecture.

Theorem. *Let (K, ∂) be a differential field of characteristic 0 and $C = \partial^{-1}(0)$ its field of constants. Assume $x, y \in K^n$ satisfy*

$$\partial(x_1) = \frac{\partial(y_1)}{y_1}, \ldots, \partial(x_n) = \frac{\partial(y_n)}{y_n}$$

and $\partial(x_1), \ldots, \partial(x_n)$ are linearly independent over $\mathbb{Q}$. Then

$$\operatorname{trdeg}_C(x, y) \geqslant n + 1.$$

The above statement clearly does not hold when K has positive characteristic, since K is algebraic over C then. If we replace a derivation ∂ with a Hasse derivation D, then K is not algebraic over C anymore, but there are still problems caused by the lack of the exponential function.

Nevertheless, we formulate a characteristic-free version of the Ax Theorem in terms of Hasse derivations and logarithmic derivatives of commutative algebraic groups and discuss its connections with the Conjecture on the Intersection of Tori.