Reducing the Cost of Probabilistic Knowledge Compilation

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Outline

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The Problem

Exact inference is nice,
but...

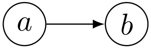
knowledge compilation is
computationally intensive.

Outline

Running Example

a	b	$P(X)$
1	1	0.4
1	2	0.4
2	1	0.05
2	2	0.05
3	1	0
3	2	0.1

Joint distribution



$$P(X) = P(b|a)P(a)$$

Bayesian network

$P(a=1)$	$P(a=2)$	$P(a=3)$
0.8	0.1	0.1

a	$P(b=1\|a)$	$P(b=2\|a)$
1	0.5	0.5
2	0.5	0.5
3	0	1

CPTs

Encoding

Let a BN be defined over variables x. We encode it as Boolean function f by adding for $x \in X$:

$$\underbrace{(x_1 \vee \cdots \vee x_n)}_{\text{at-least-once}} \wedge \underbrace{\bigwedge_{i=1}^{n} \bigwedge_{j=i+1}^{n} (\overline{x_i} \vee \overline{x_j})}_{\text{at-most-once}} .$$

Unique symbolic weights ω_j identify distinct probabilities local to x's CPT. We introduce into f clauses that imply each weight ω_j.

Encoding

$P(a=1)$	$P(a=2)$	$P(a=3)$
0.8	0.1	0.1

a	$P(b=1 \mid a)$	$P(b=2 \mid a)$
1	0.5	0.5
2	0.5	0.5
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$P(a=1)$	$P(a=2)$	$P(a=3)$
ω_1	ω_2	ω_2

a	$P(b=1 \mid a)$	$P(b=2 \mid a)$
1	ω_3	ω_3
2	ω_3	ω_3
3	ω_4	ω_5

Encoding

$P(a=1)$	$P(a=2)$	$P(a=3)$
0.8	0.1	0.1

$P(a=1)$	$P(a=2)$	$P(a=3)$
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a	$P(b=1\|a)$	$P(b=2\|a)$
1	ω_3	ω_3
2	ω_3	ω_3
3	ω_4	ω_5

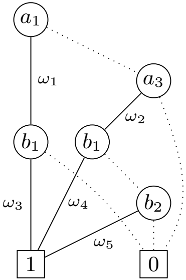
Variables:

$$(a_1 \vee a_2 \vee a_3) \wedge (\overline{a_1} \vee \overline{a_2}) \wedge (\overline{a_1} \vee \overline{a_3}) \wedge (\overline{a_2} \vee \overline{a_3}) \wedge (b_1 \vee b_2) \wedge (\overline{b_1} \vee \overline{b_2})$$

Probabilities:

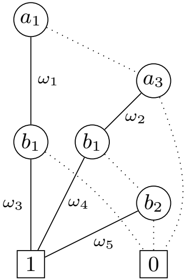
$$(\overline{a_1} \vee \omega_1) \wedge (\overline{a_2} \vee \omega_2) \wedge (\overline{a_3} \vee \omega_2) \wedge (\overline{a_1} \vee \overline{b_1} \vee \omega_3) \wedge (\overline{a_1} \vee \overline{b_2} \vee \omega_3) \wedge (\overline{a_2} \vee \overline{b_1} \vee \omega_3) \wedge (\overline{a_2} \vee \overline{b_2} \vee \omega_3) \wedge (\overline{a_3} \vee \overline{b_1} \vee \omega_4) \wedge (\overline{a_3} \vee \overline{b_2} \vee \omega_5).$$

Inference by Weighted Model Counting

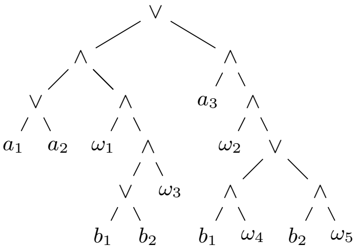


WPBDD

Inference by Weighted Model Counting

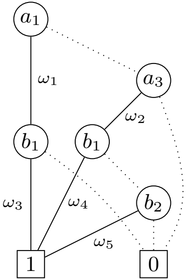


WPBDD

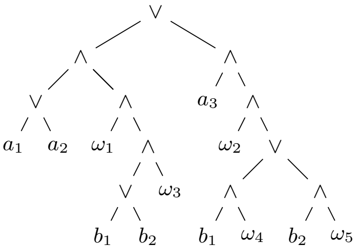


Logical circuit

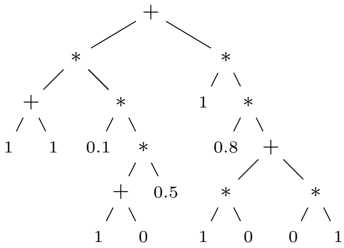
Inference by Weighted Model Counting



WPBDD



Logical circuit



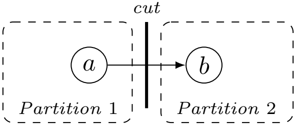
Instantiated arithmetic circuit for $P(b = 1)$

Outline

The Framework

1. Partition
2. Compilation
3. Assembly
4. Inference

Partition and Compile

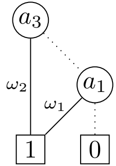


Partitioning

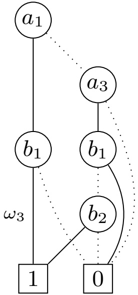
$P(a=1)$	$P(a=2)$	$P(a=3)$
ω_1	ω_2	ω_2

a	$P(b{=}1 \mid a)$	$P(b{=}2 \mid a)$
1	ω_3	ω_3
2	ω_3	ω_3
3	ω_4	ω_5

Capture symbolic structure in CPTs



Partition 1. Compilation with order: $a_3 \leq a_2 \leq a_1$



Partition 2. Compilation with order: $a_1 \leq a_2 \leq a_3 \leq b_1 \leq b_2$

Partition and Compile: What to compile?

Partition 1:

$$(a_1 \vee a_2 \vee a_3) \wedge (\overline{a_1} \vee \overline{a_2}) \wedge (\overline{a_1} \vee \overline{a_3}) \wedge (\overline{a_2} \vee \overline{a_3}) \wedge$$

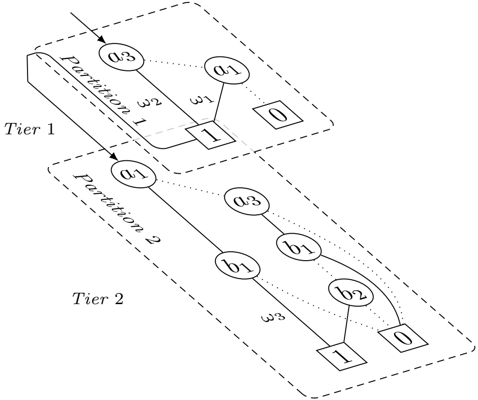
$$(\overline{a_1} \vee \omega_1) \wedge (\overline{a_2} \vee \omega_2) \wedge (\overline{a_3} \vee \omega_2)$$

Partition 2:

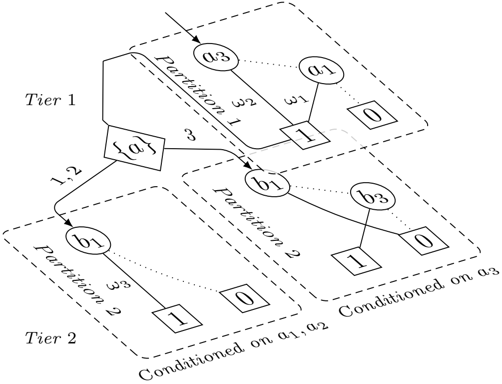
$$(a_1 \vee a_2 \vee a_3) \wedge (\overline{a_1} \vee \overline{a_2}) \wedge (\overline{a_1} \vee \overline{a_3}) \wedge (\overline{a_2} \vee \overline{a_3}) \wedge$$
$$(b_1 \vee b_2) \wedge (\overline{b_1} \vee \overline{b_2}) \wedge$$

$$(\overline{a_1} \vee \overline{b_1} \vee \omega_3) \wedge (\overline{a_1} \vee \overline{b_2} \vee \omega_3) \wedge (\overline{a_2} \vee \overline{b_1} \vee \omega_3) \wedge$$
$$(\overline{a_2} \vee \overline{b_2} \vee \omega_3) \wedge (\overline{a_3} \vee \overline{b_1} \vee \omega_4) \wedge (\overline{a_3} \vee \overline{b_2} \vee \omega_5).$$

Assemby and Inference



Dynamic conditioning



Static conditioning

Outline

An upperbound

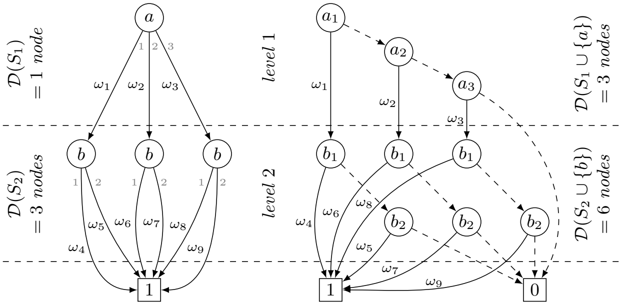
Let a Bayesian network be defined over n variables X.

1. We formulate it as a set of *constraints* $C_x \in C$, where C_x represents the dependencies of variable $x \in X$:
$$C_x = \{x\} \cup \textsc{Parents}(x).$$
2. Compile using ordering $O = \{o_1, \ldots, o_n\}$ imposed on X by π, where $o_i = \pi(x)$, $o_j \leq o_k$ for $j < k$.
3. The upperbound is based on *spanning variables* $S_l \in S$ associated with each evaluation depth, or level l:
$$S_l = \{o_1, \ldots, o_{l-1}\} \ \cap \bigcup_{\{C_x \in C \ : \ \{o_1, \ldots, o_{l-1}\} \subset C_x\}} C_x$$
4. A generalized upperbound is computed by:
$$\sum_{l=1}^{n} \mathcal{D}(S_l), \text{ where } \mathcal{D}(Y) = \prod_{x \in Y} |x|.$$

An Upperbound

$P(a=1)$	$P(a=2)$	$P(a=3)$
ω_1	ω_2	ω_3

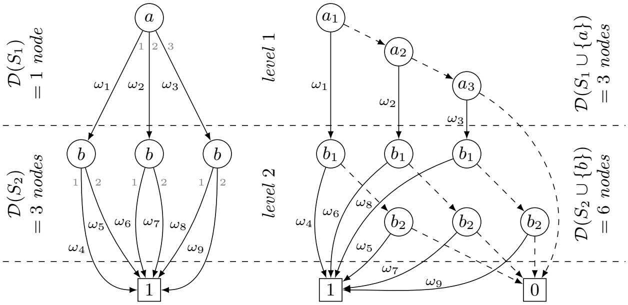
a	$P(b=1\|a)$	$P(b=2\|a)$
1	ω_4	ω_5
2	ω_6	ω_7
3	ω_8	ω_9



An Upperbound

$P(a{=}1)$	$P(a{=}2)$	$P(a{=}3)$
ω_1	ω_2	ω_3

a	$P(b{=}1 \mid a)$	$P(b{=}2 \mid a)$
1	ω_4	ω_5
2	ω_6	ω_7
3	ω_8	ω_9

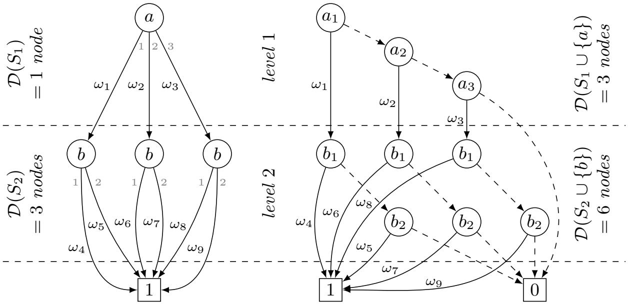


$$C_a = \{a\}, C_b = \{a, b\}.$$

An Upperbound

$P(a{=}1)$	$P(a{=}2)$	$P(a{=}3)$
ω_1	ω_2	ω_3

a	$P(b{=}1\|a)$	$P(b{=}2\|a)$
1	ω_4	ω_5
2	ω_6	ω_7
3	ω_8	ω_9



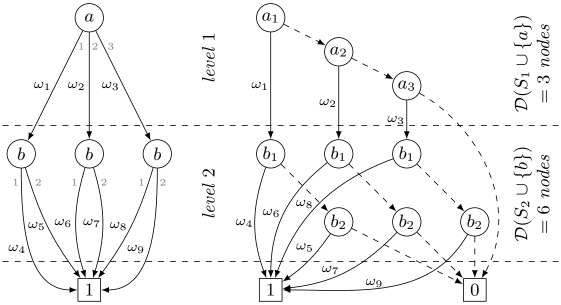
$$C = \; C_a = \{a\}, C_b = \{a, b\}.$$

l	O	S_l	$\mathcal{D}(S_l)$	$\sum_{i=1}^{l+1} \mathcal{D}(S_i)$	$\mathcal{D}(S_l \cup O_l)$	$\sum_{i=1}^{l+1} \mathcal{D}(S_i \cup O_l)$
1	a	$\{\}$				
2	b	$\{a\}$				

An Upperbound

$P(a{=}1)$	$P(a{=}2)$	$P(a{=}3)$
ω_1	ω_2	ω_3

a	$P(b{=}1 \mid a)$	$P(b{=}2 \mid a)$
1	ω_4	ω_5
2	ω_6	ω_7
3	ω_8	ω_9



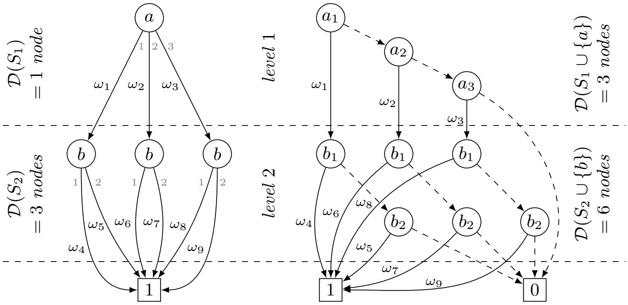
$$C = \; C_a = \{a\}, C_b = \{a, b\}.$$

l	O	S_l	$\mathcal{D}(S_l)$	$\sum_{i=1}^{l+1} \mathcal{D}(S_i)$	$\mathcal{D}(S_l \cup O_l)$	$\sum_{i=1}^{l+1} \mathcal{D}(S_i \cup O_l)$
1	a	$\{\}$	1	1	3	3
2	b	$\{a\}$				

An Upperbound

$P(a{=}1)$	$P(a{=}2)$	$P(a{=}3)$
ω_1	ω_2	ω_3

a	$P(b{=}1 \mid a)$	$P(b{=}2 \mid a)$
1	ω_4	ω_5
2	ω_6	ω_7
3	ω_8	ω_9



$$C = \quad C_a = \{a\}, C_b = \{a, b\}.$$

l	O	S_l	$\mathcal{D}(S_l)$	$\sum_{i=1}^{l+1} \mathcal{D}(S_i)$	$\mathcal{D}(S_l \cup O_l)$	$\sum_{i=1}^{l+1} \mathcal{D}(S_i \cup O_l)$
1	a	$\{\}$	1	1	3	3
2	b	$\{a\}$	3	4	6	9

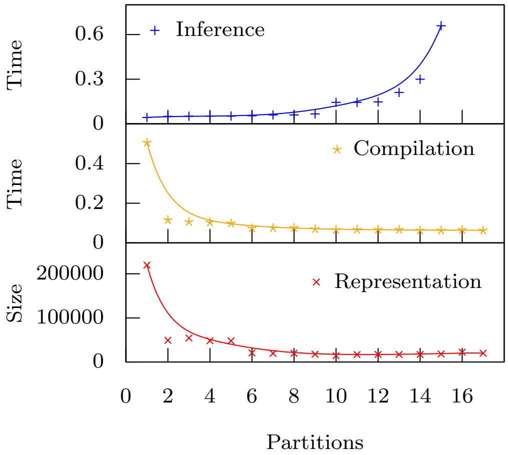
Outline

Compilation Time

Bayesian Network	X	$\mathcal{A}(X)$	P	WPBDD S	WPBDD T	WPBDD S	WPBDD T	SDD S	SDD T	OBDD S	OBDD T
insurance	27	89	2	33183	0.014	348956	0.077	1731415	2.046	1263540	0.289
weeduk	15	90	2	30735	0.406	30733	0.418	-	-	109734	0.176
alarm	37	105	2	2730	0.003	3788	0.003	35004	0.054	10008	0.003
water	32	116	2	49212	0.095	219797	0.506	-	-	-	-
powerplant	40	120	2	2451	0.002	4158	0.003	26662	0.038	11043	0.002
carpo	54	122	2	1937	0.003	2377	0.003	13405	0.028	7179	0.003
win95pts	76	152	2	49405	0.018	810957	0.361	1109210	2.057	4876152	4.997
hepar2	70	162	2	33234	0.025	56574	0.033	188453	2.423	142806	0.161
fungiuk	15	165	2	79682	1.515	234322	7.763	-	-	733551	0.812
hailfinder	56	223	2	225325	0.052	4025502	1.395	10508499	7.51	31493220	12.435
3nt	58	228	2	9844	0.015	858645	0.578	42774722	58.905	15592962	19.493
4sp	58	246	2	83156	0.035	918353	0.352	-	-	20558352	34.598
barley	48	421	2	13721258	11.197	-	-	-	-	-	-
mainuk	48	421	2	9045244	9.002	-	-	-	-	-	-
andes	220	440	2	426513	0.117	-	-	-	-	-	-
pathfinder	135	520	2	143032	0.493	577163	0.717	2287777	23.337	5732988	18.656
mildew	35	616	2	1634250	111.434	5666709	113.264	-	-	-	-
munin1	186	992	4	13196919	6.693	-	-	-	-	-	-
pigs	441	1323	9	3292450	1.534	-	-	-	-	-	-

Compilation cost,
where X and $\mathcal{A}(X)$ are the number of variables in the BN and encoding, S is the number of logical operators that the symbolic representation induces, time T is in seconds, P is number of partitions, and - implies compilation failure due to memory requirements.

Partition vs. Inference Time



Effects of partitioning on the *water* network.

Closing Remarks

- The compilation cost is drastically reduced.
- The representations obtained are much smaller.
- Independent partition orderings increase structure exploitation.
- WMC for huge networks?

Questions?

Email me at
gdal **at** cs.ru.nl
for any further questions.