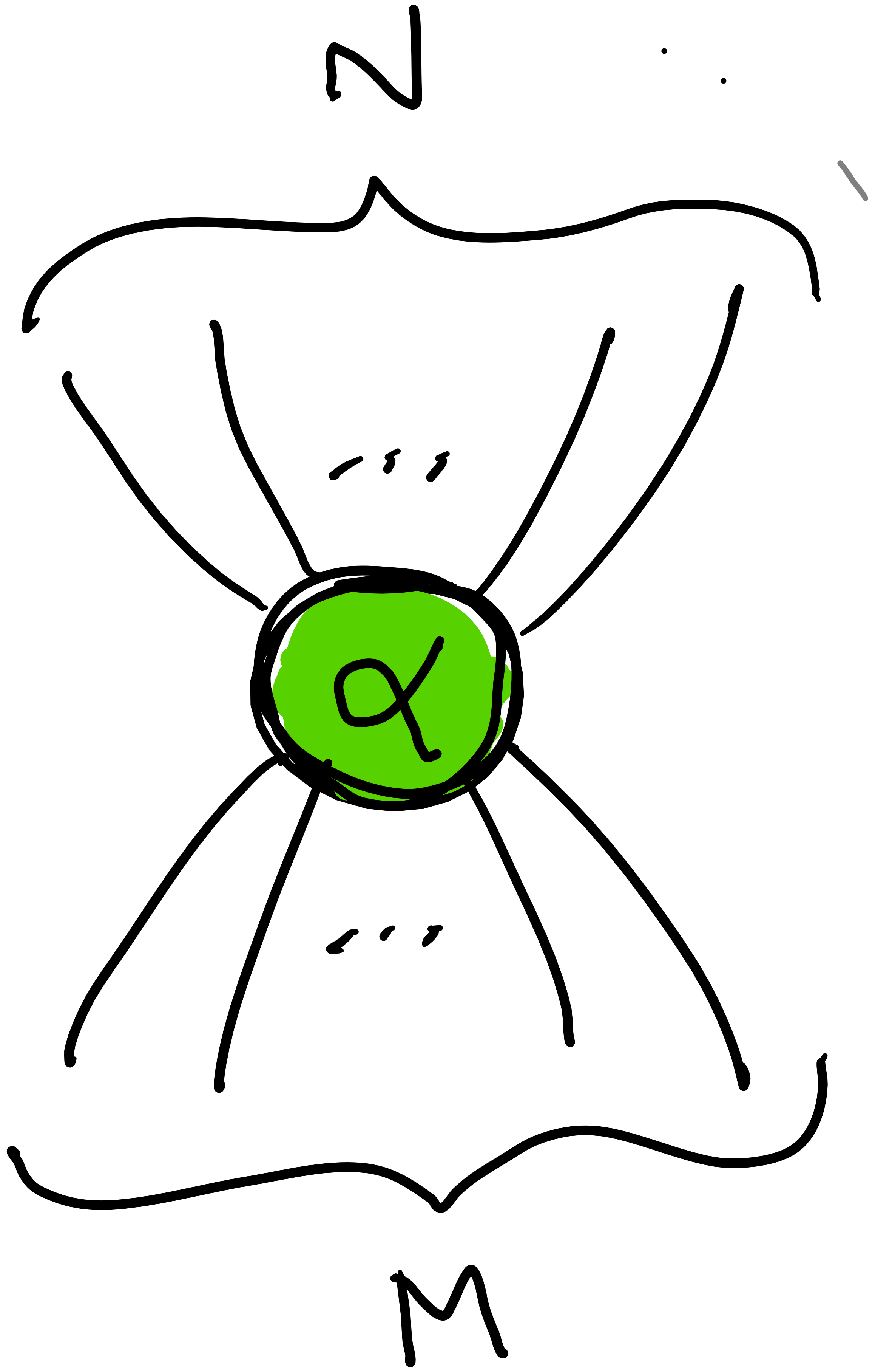
PICTURING QUANTUM PROCESSES II

ALEKS KISSINGER

ISR 2019

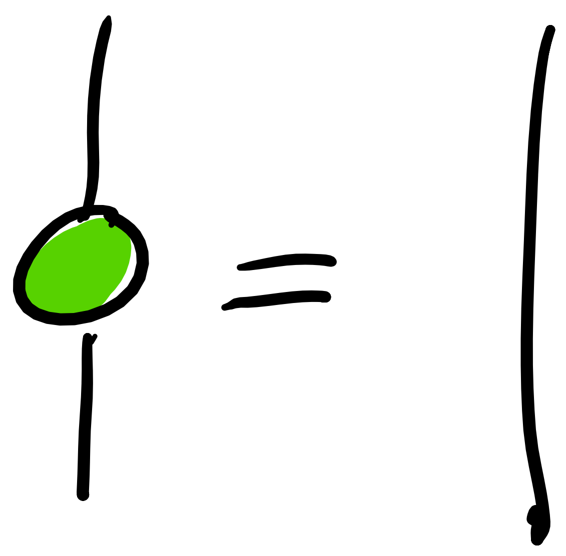
Paris

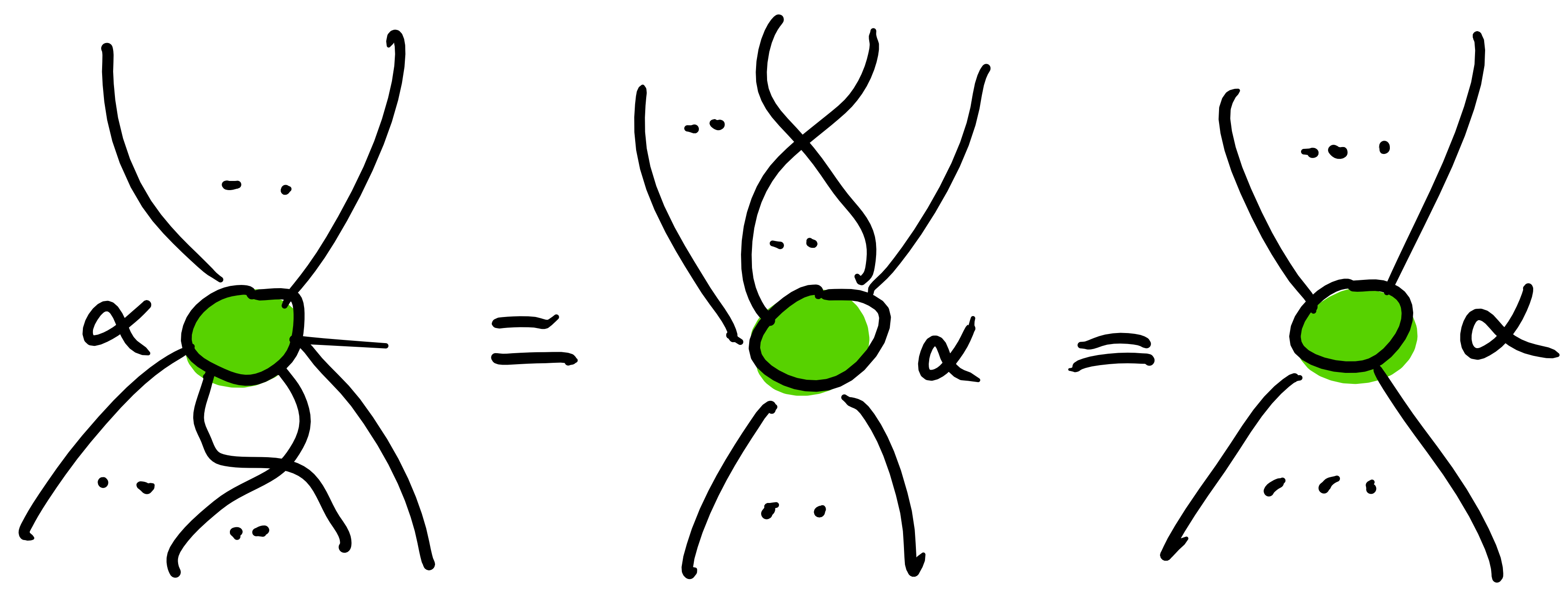
Spiders

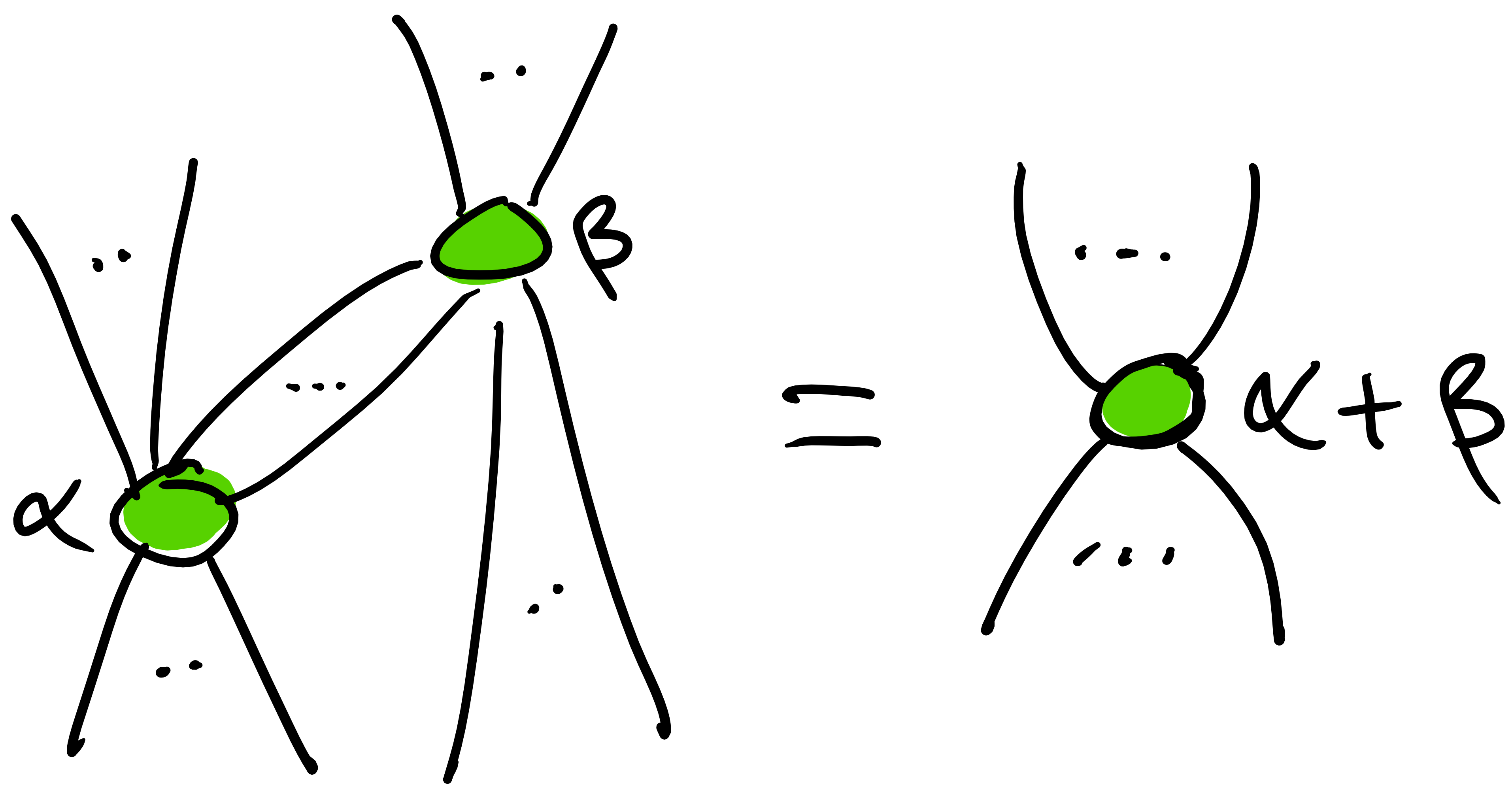


$$:= \begin{pmatrix} 1 & & \\ & \emptyset & \\ & & e^{i\alpha} \end{pmatrix}$$

SPIDERS

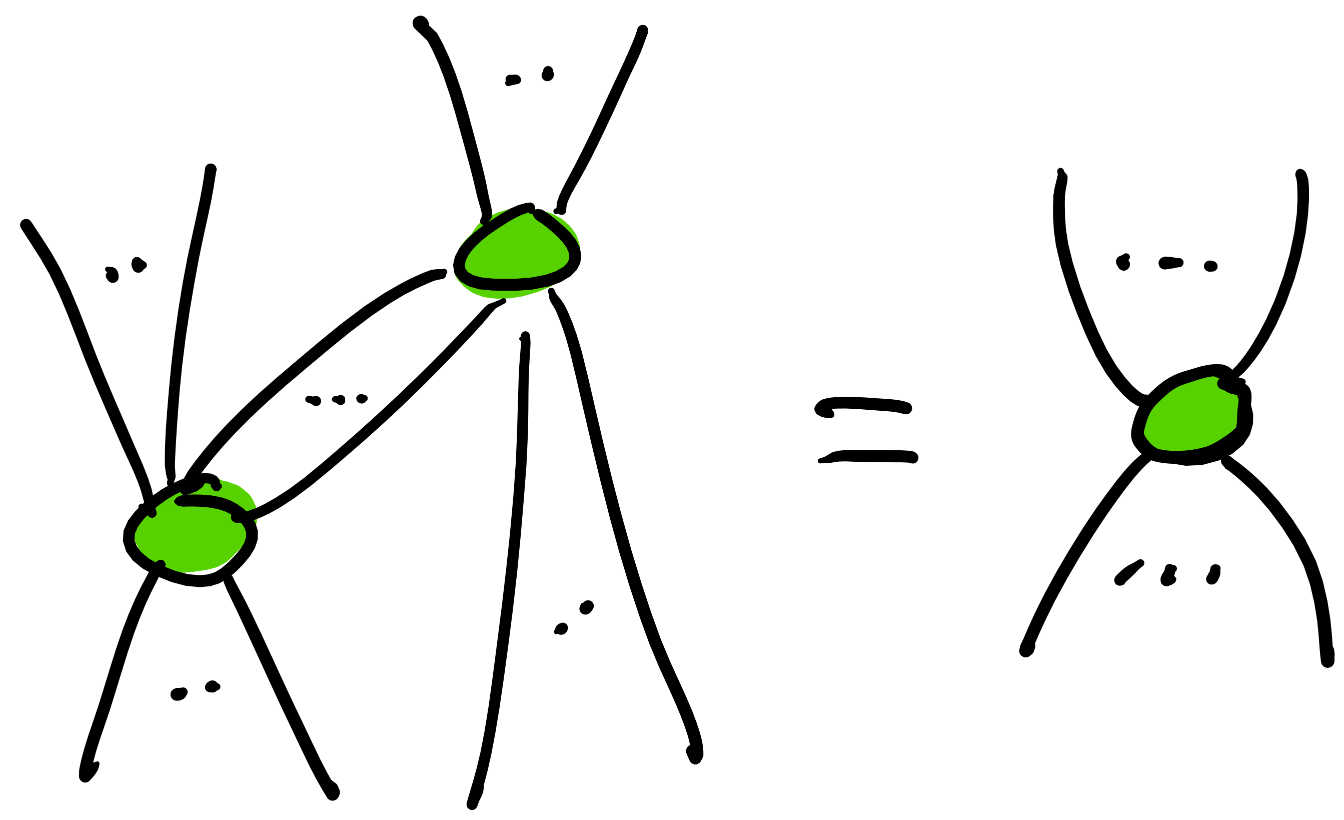


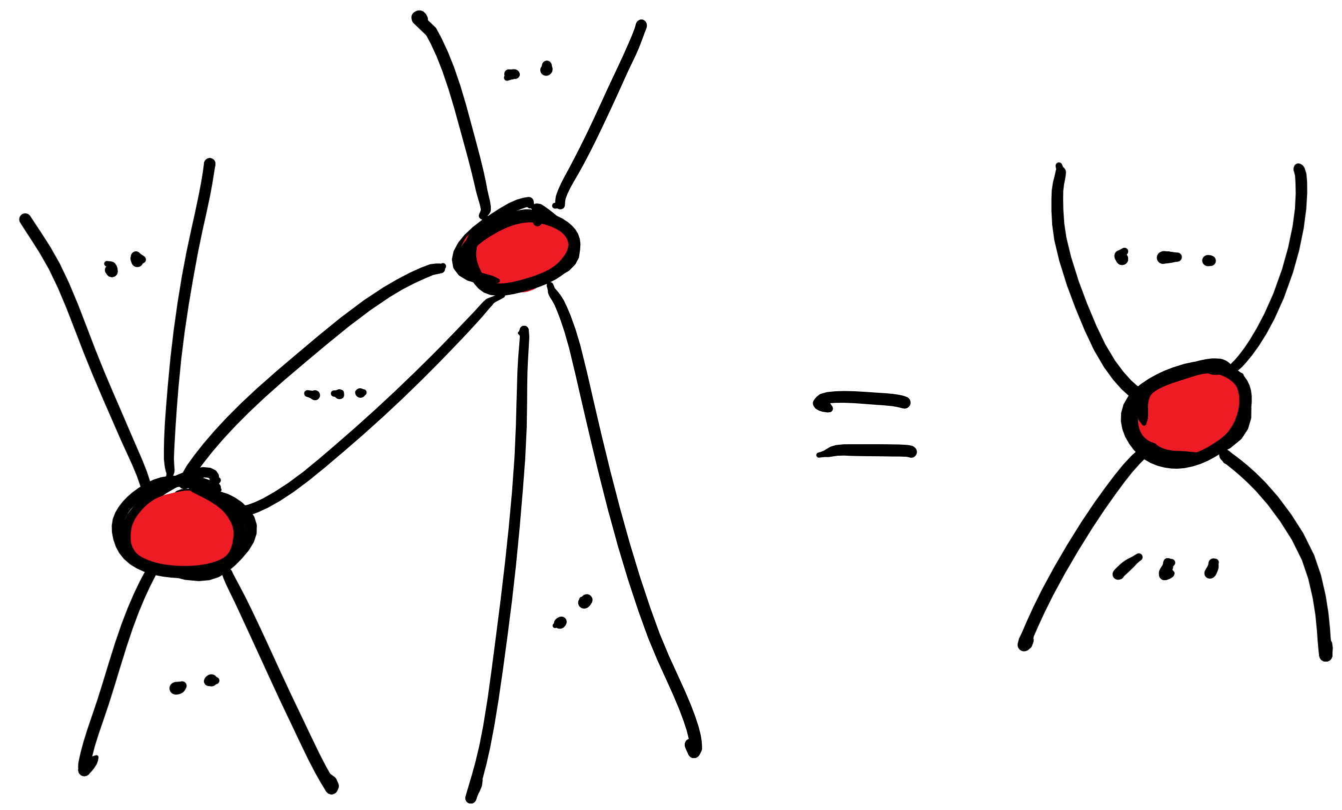


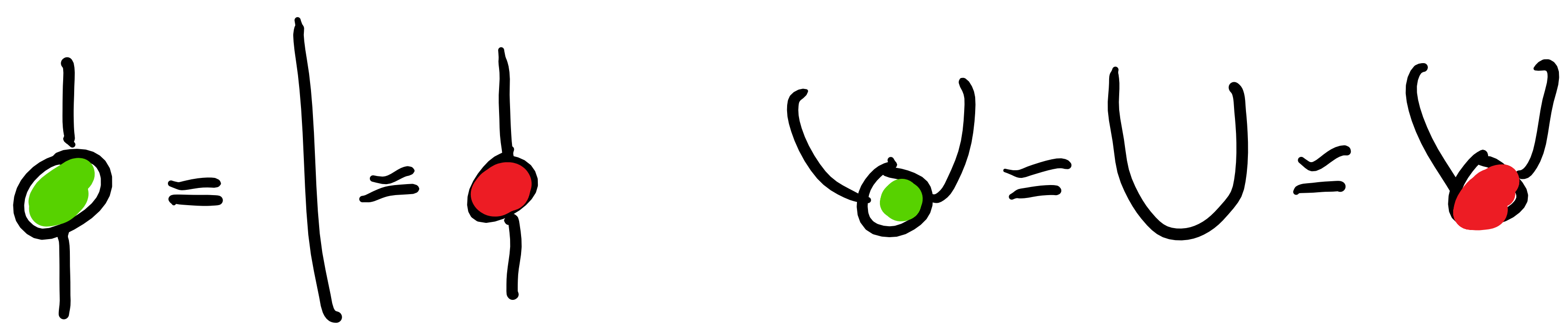


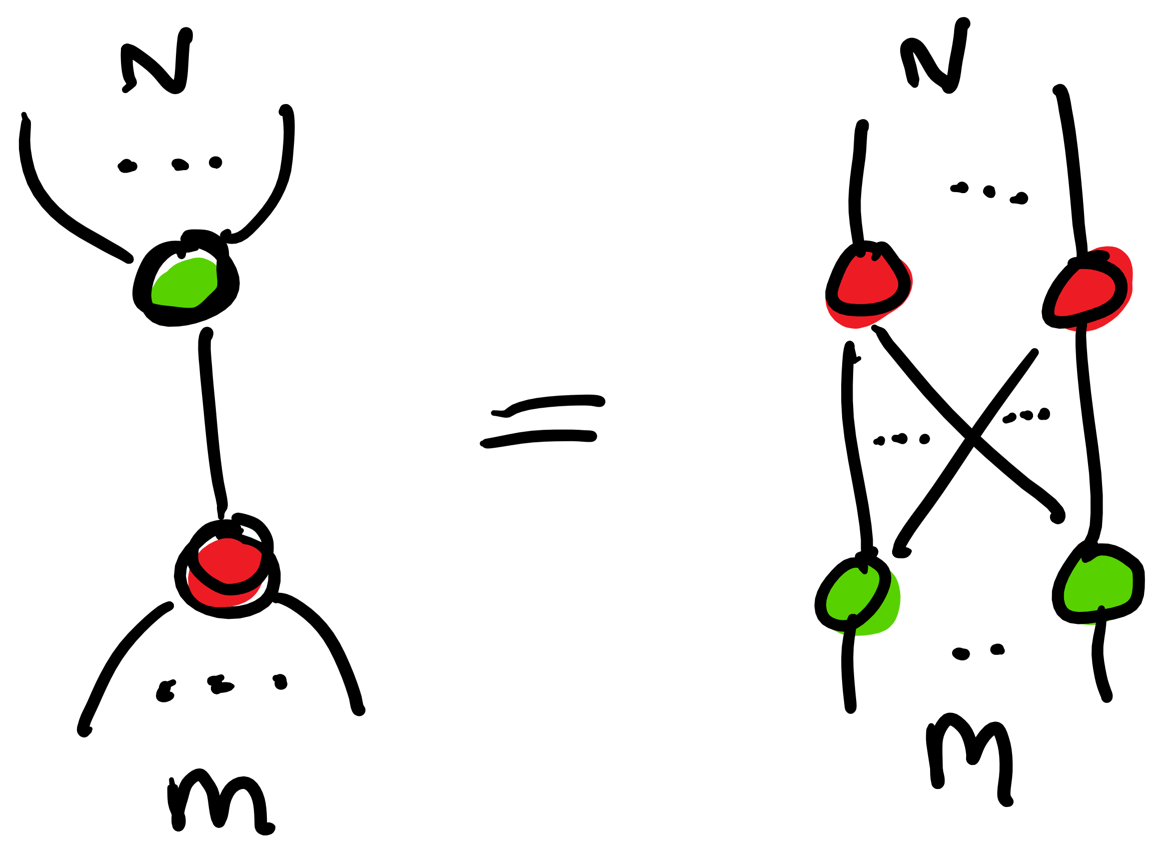
ZX-calculus

PHASE-FREE









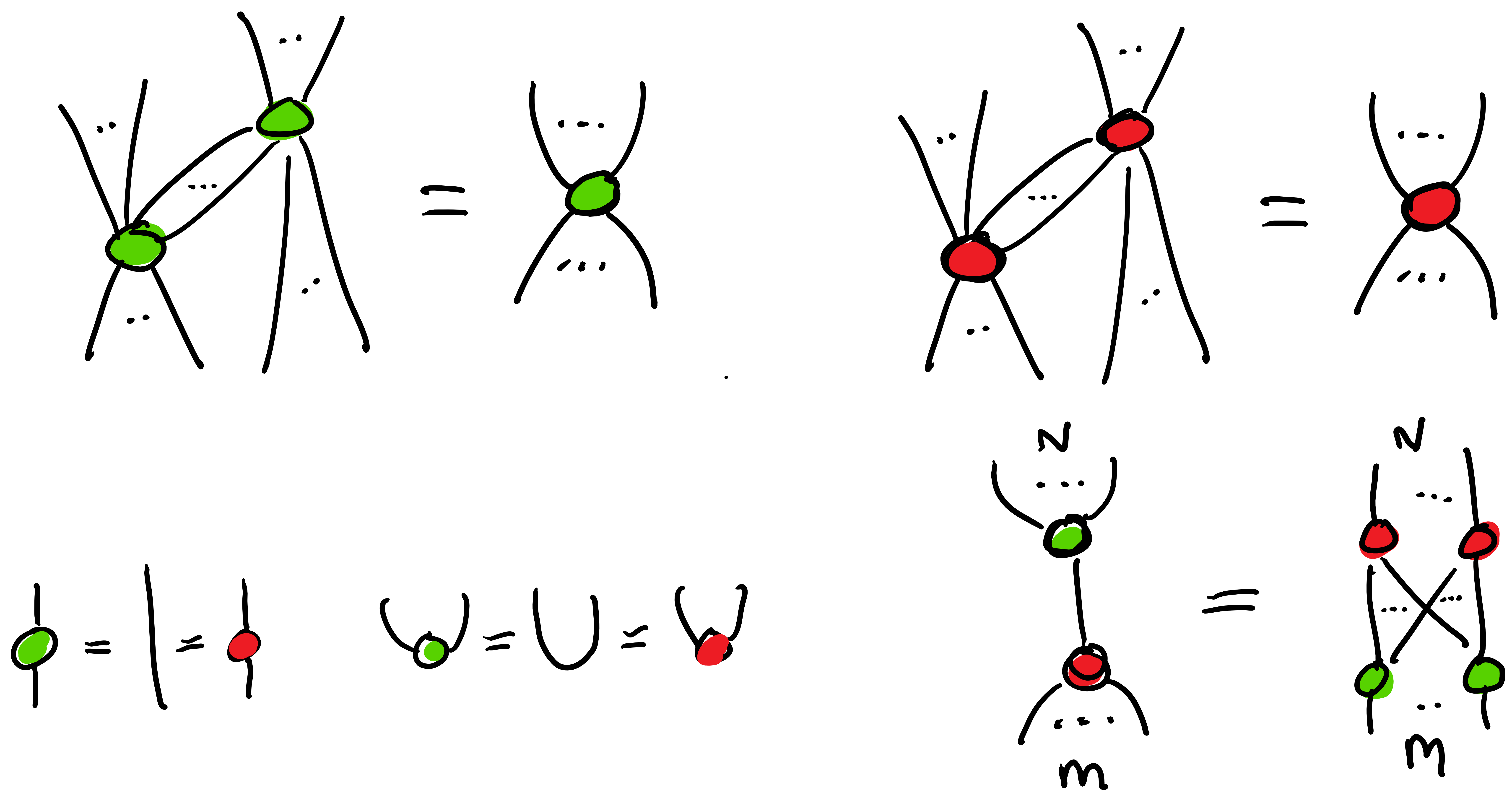
THEOREM The phase-free ZX-calculus ZX_0 is complete for phase-free ZX-diagrams.

$$[\![D]\!] = [\![D']\!] \Rightarrow ZX_0 \vdash D = D'$$

matrix of D

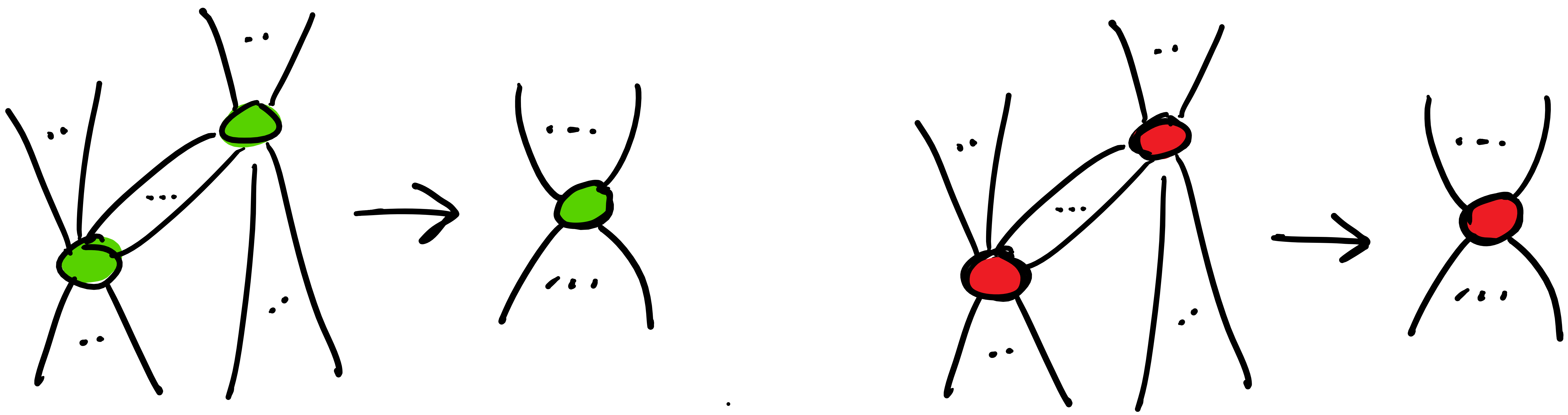
PHASE-FREE

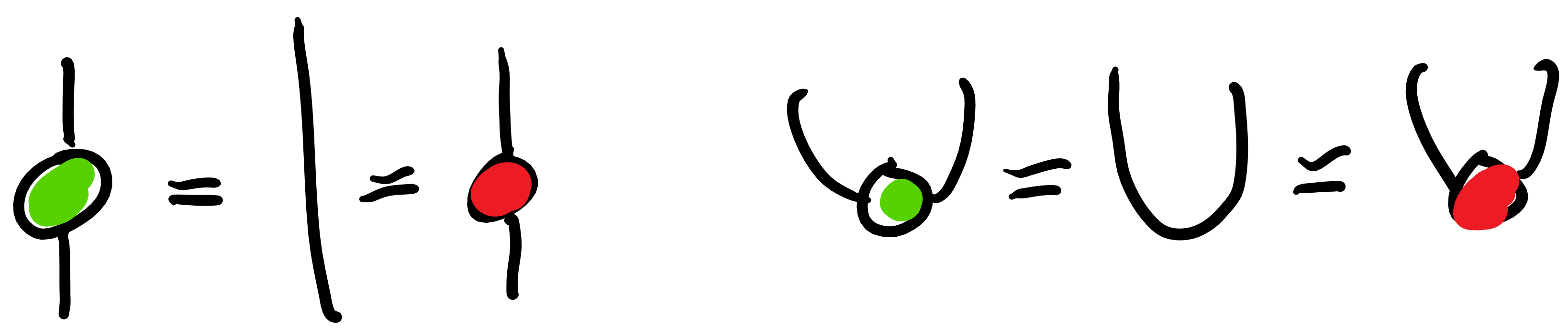
ZX-calculus

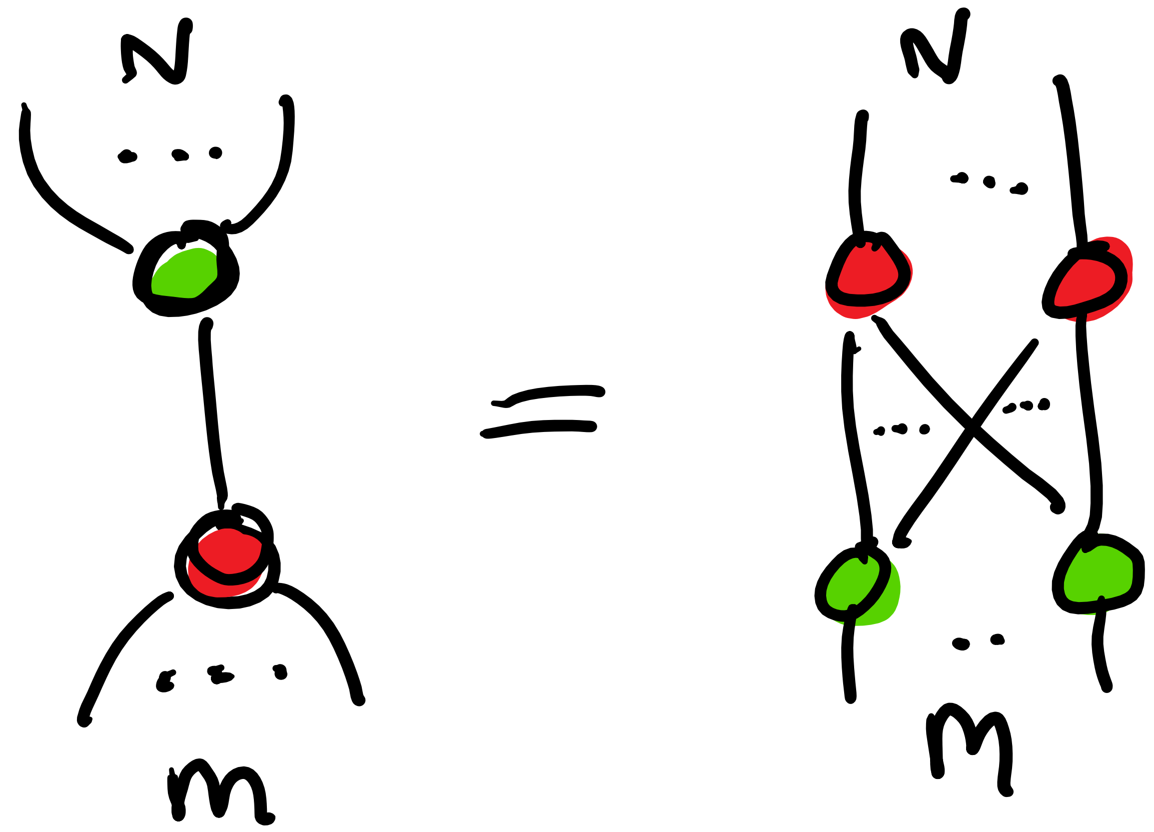


PHASE-FREE

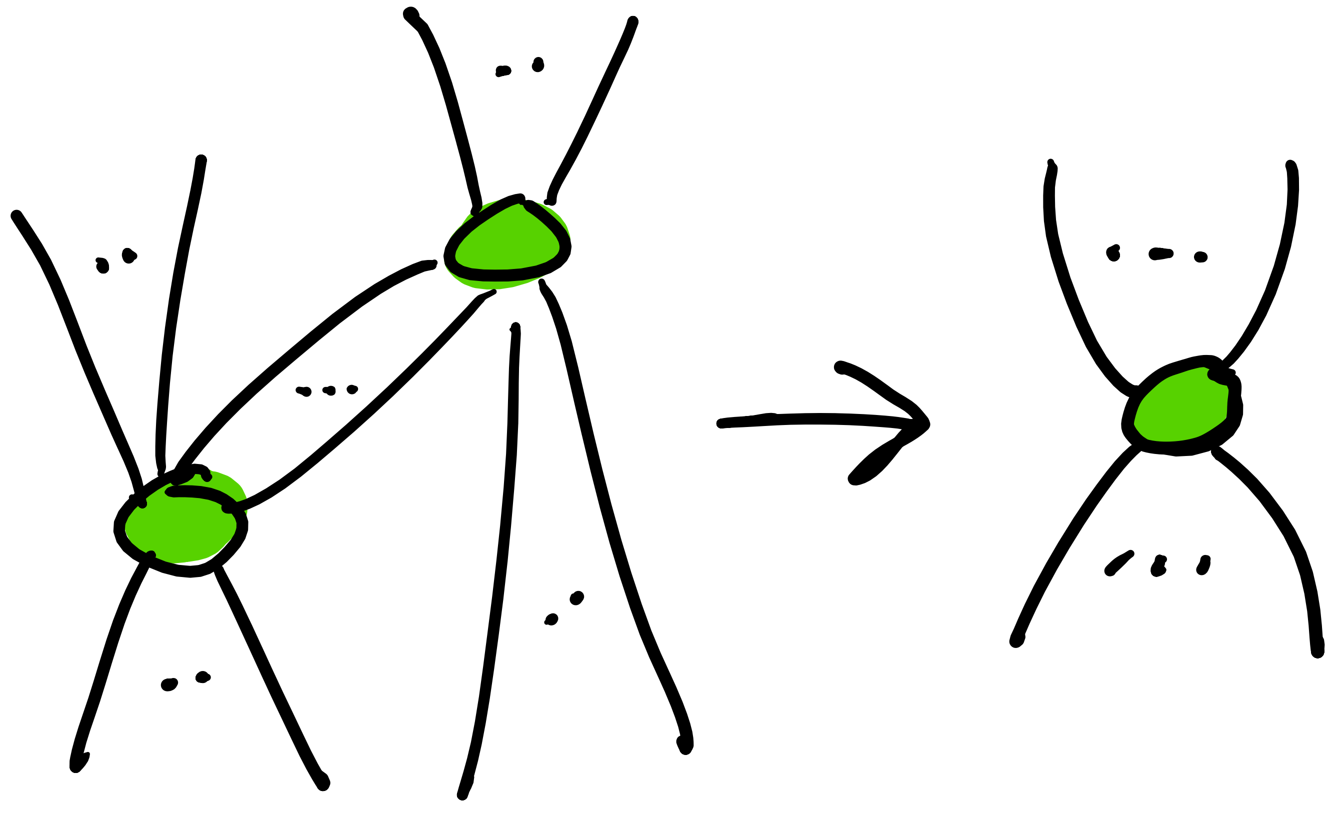
ZX-calculus

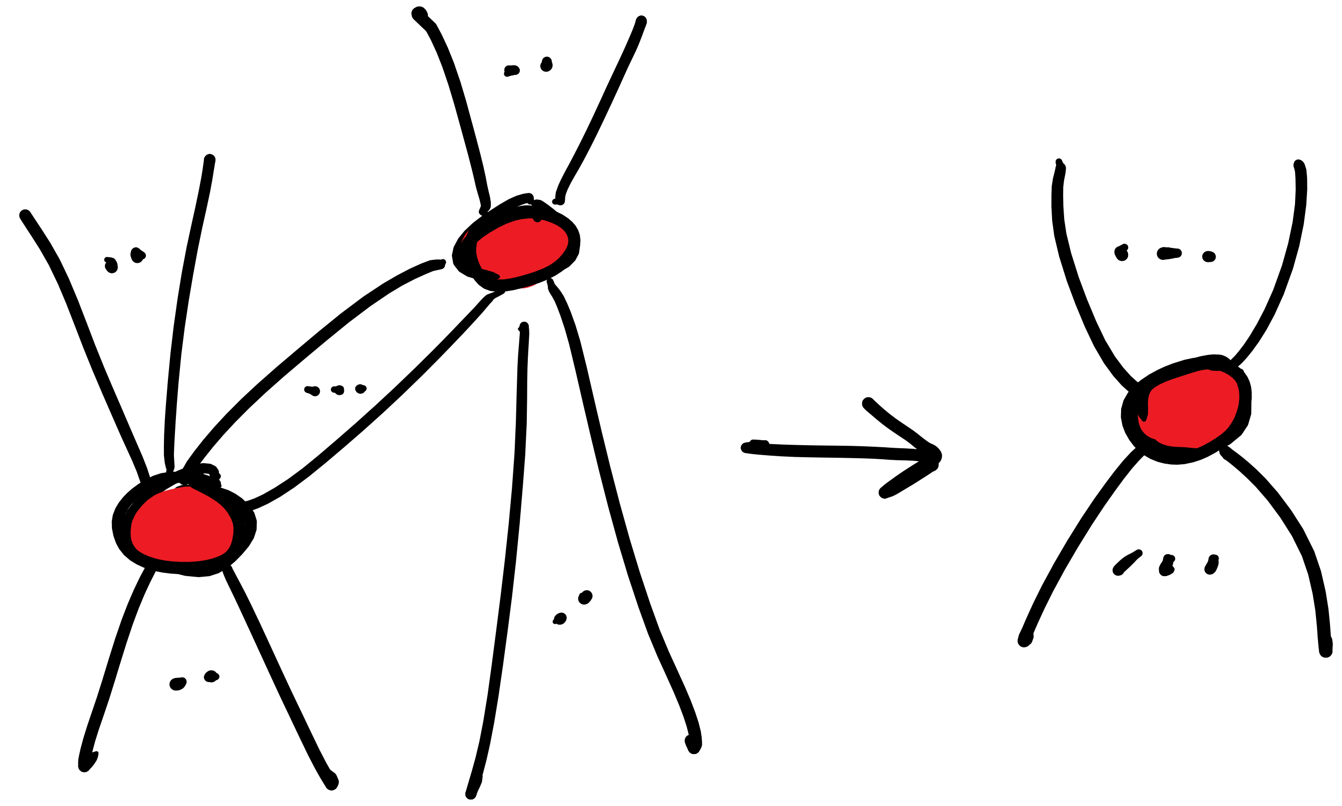


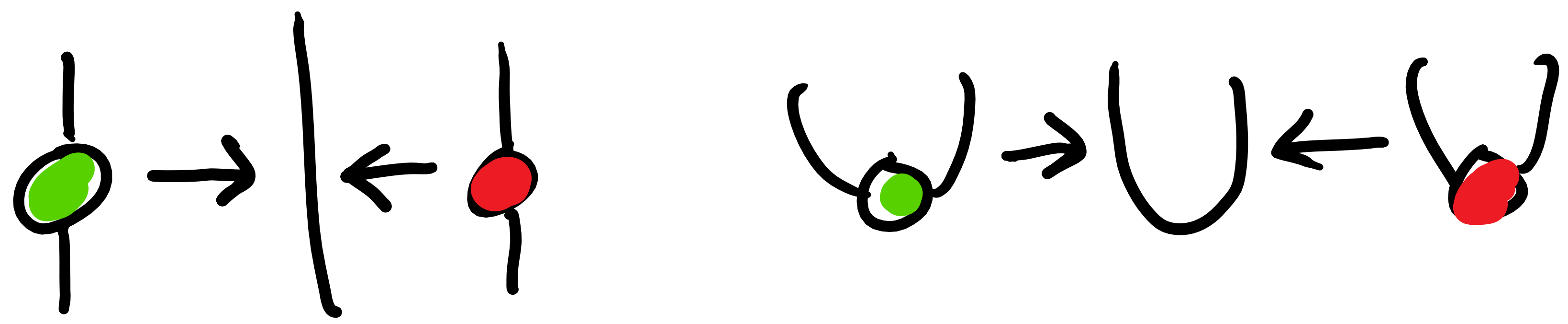


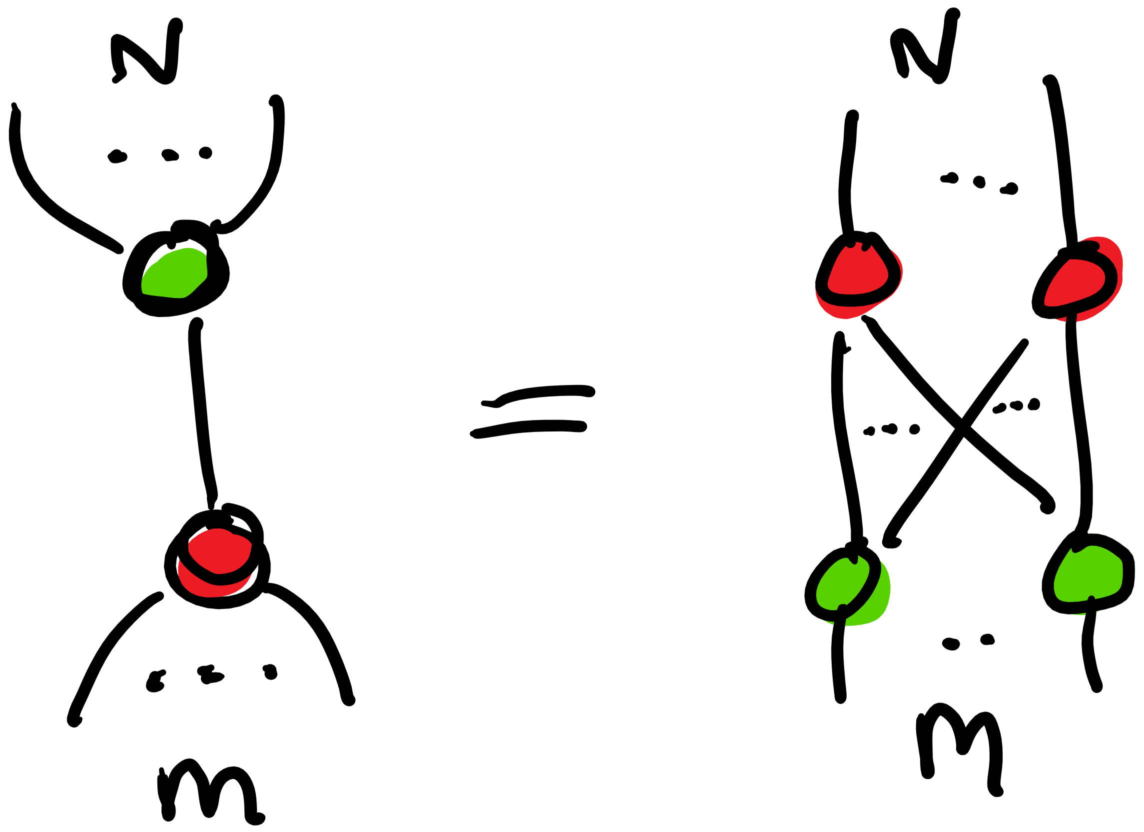


PHASE-FREE ZX-calculus









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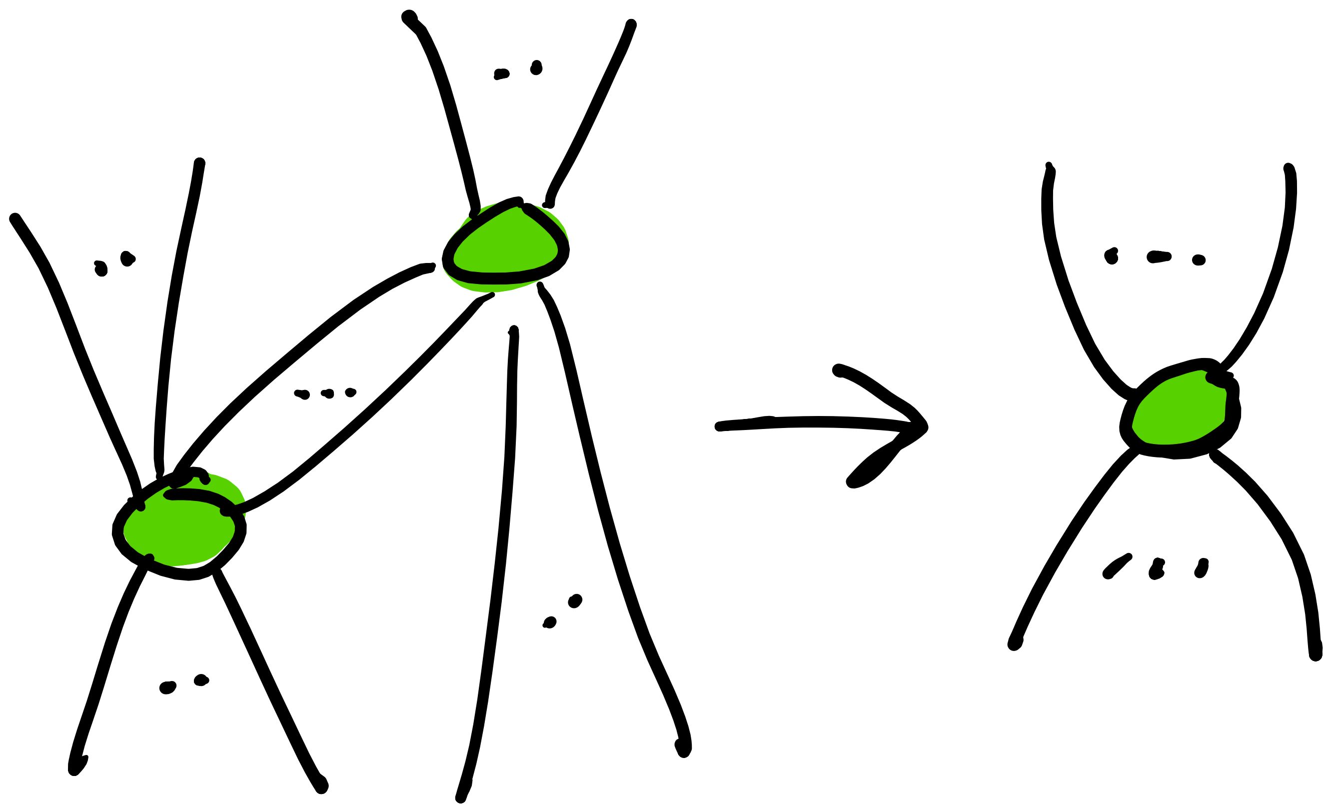
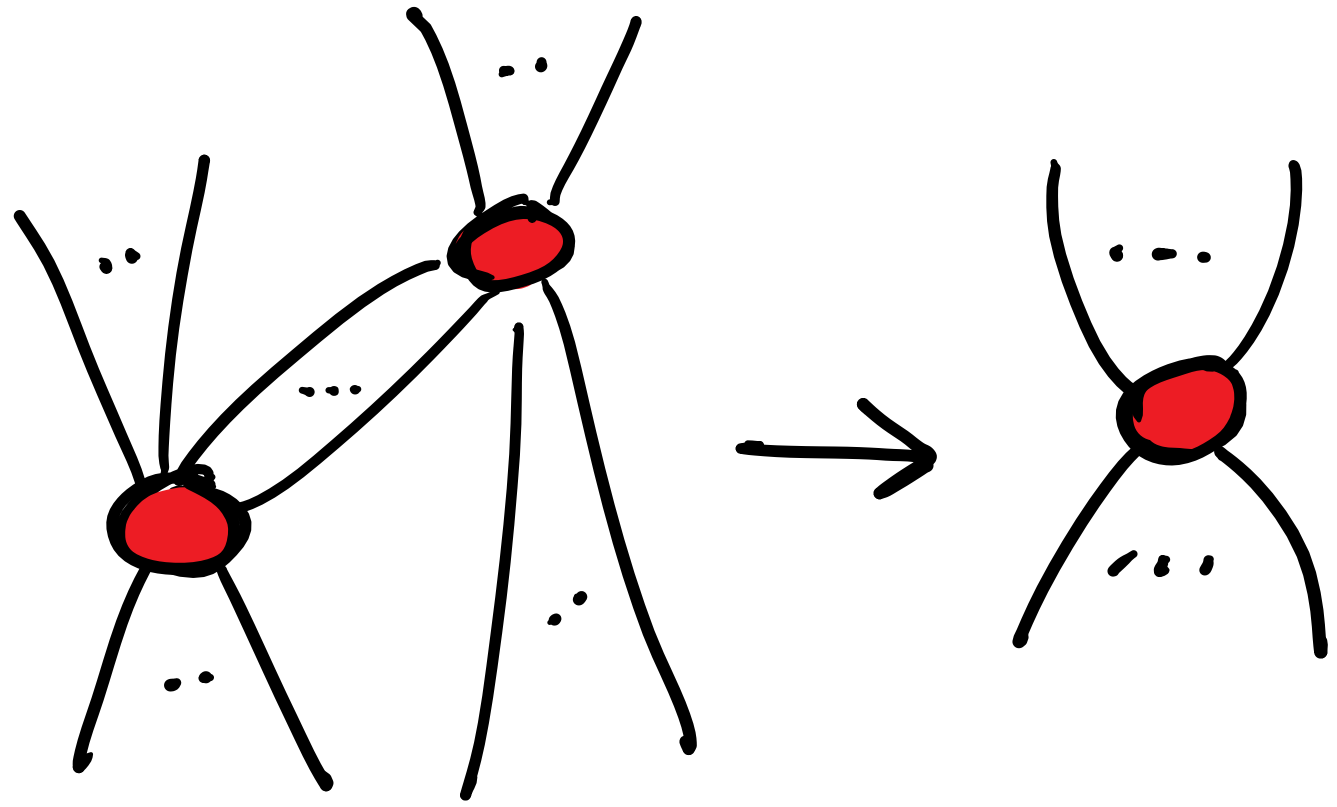
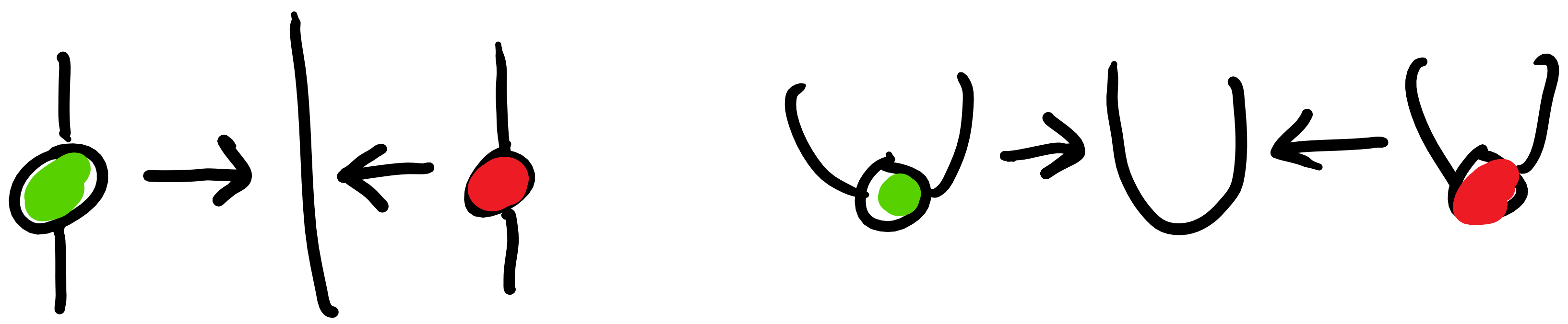
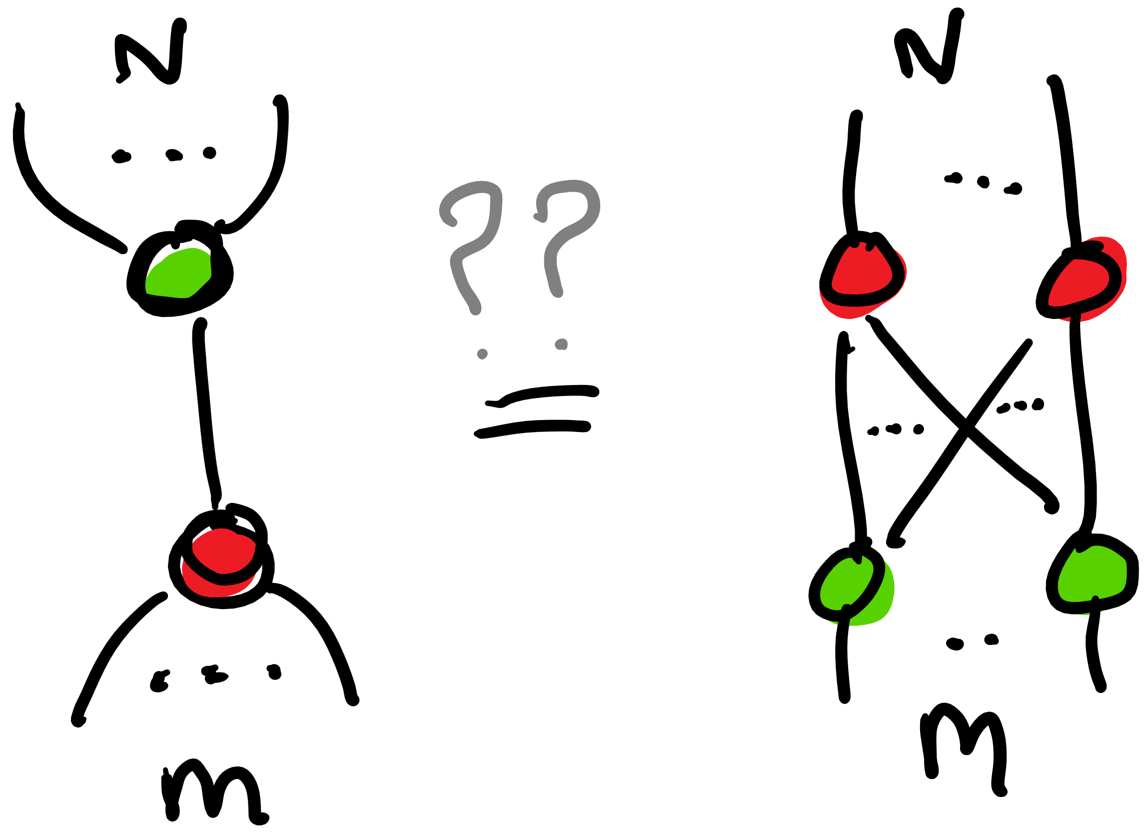
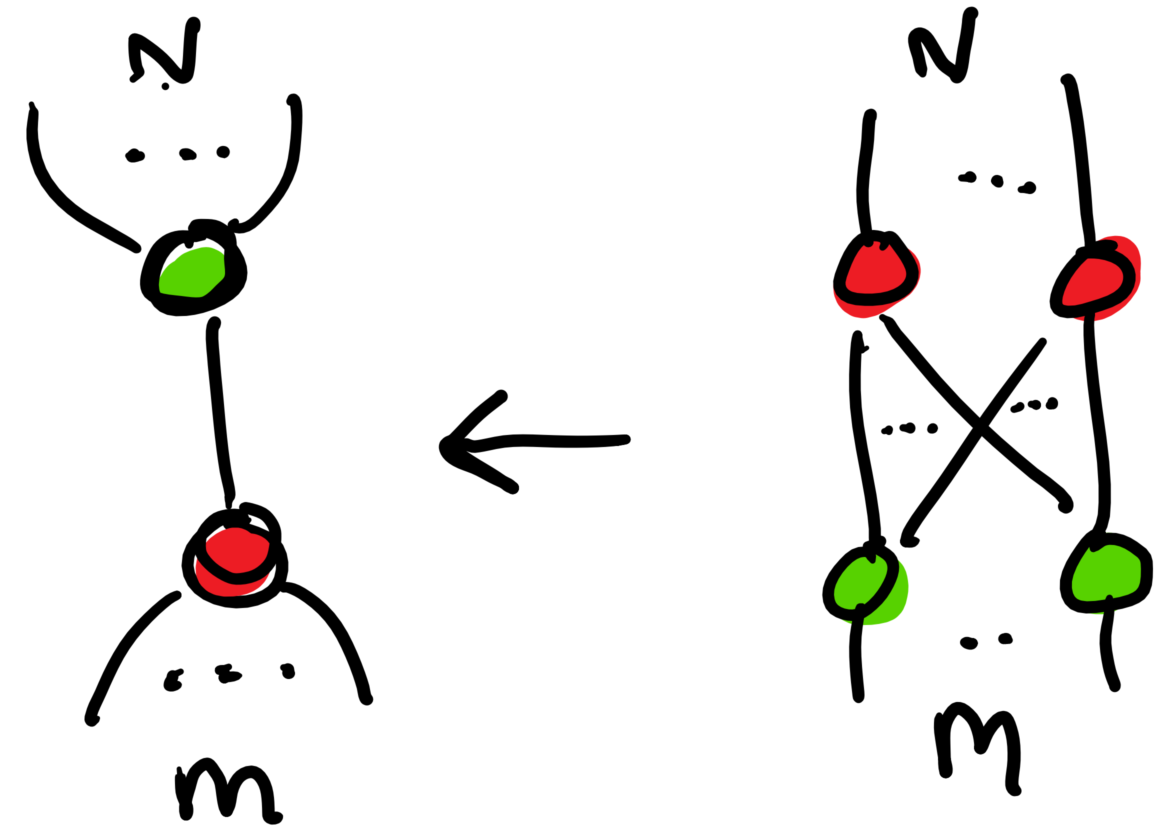
ZX-calculus











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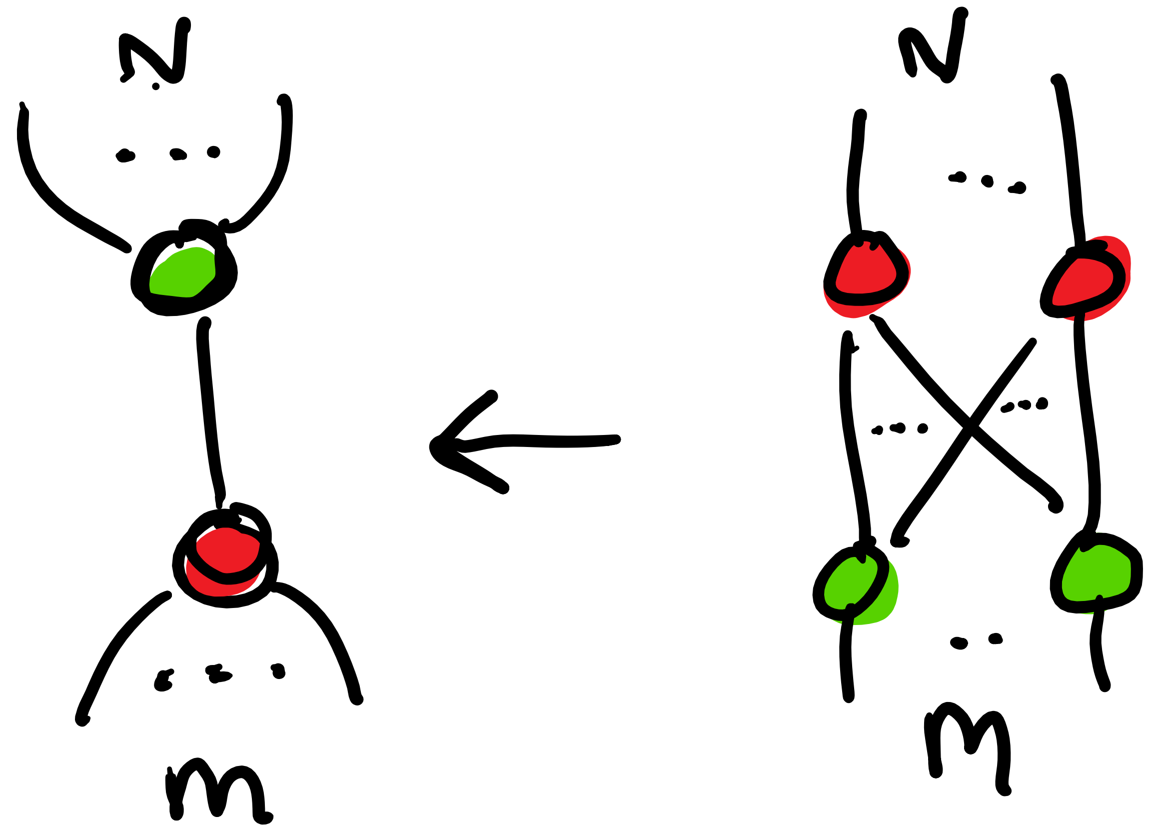

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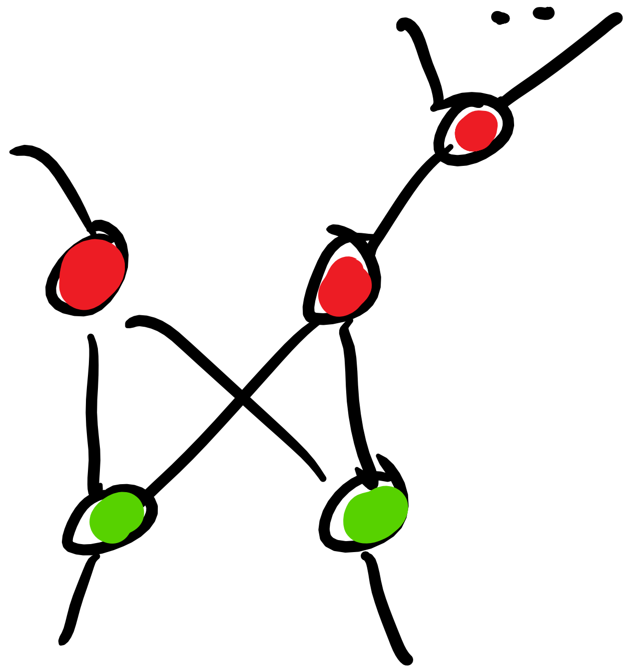

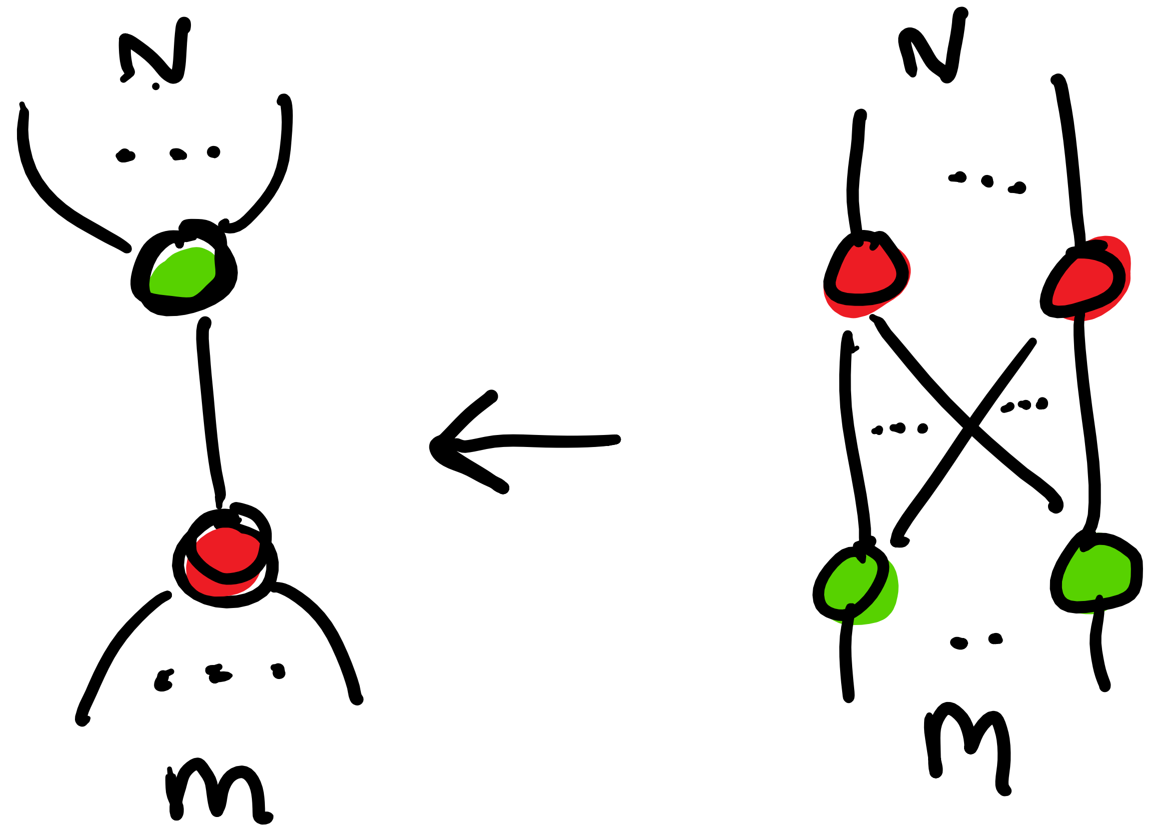
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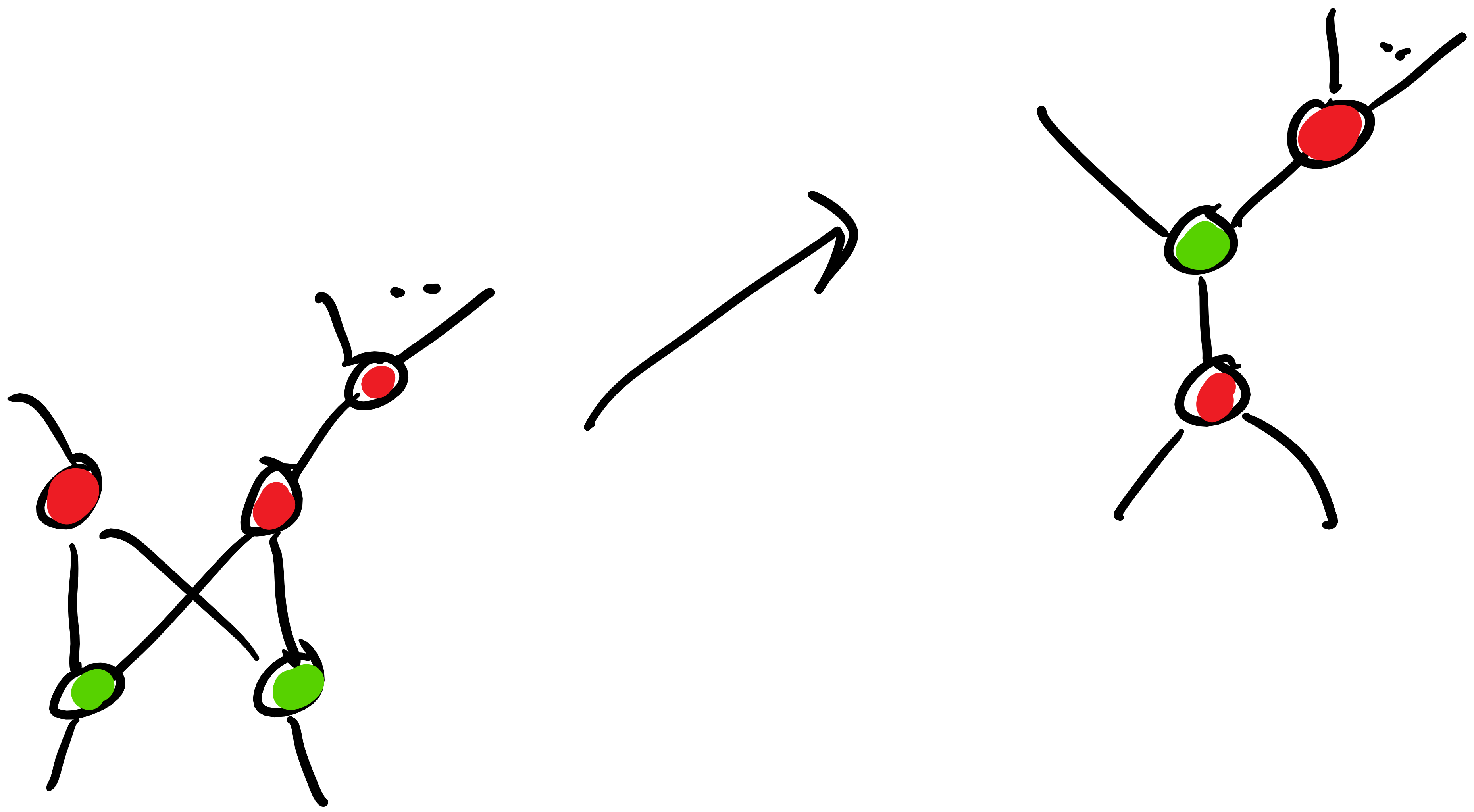

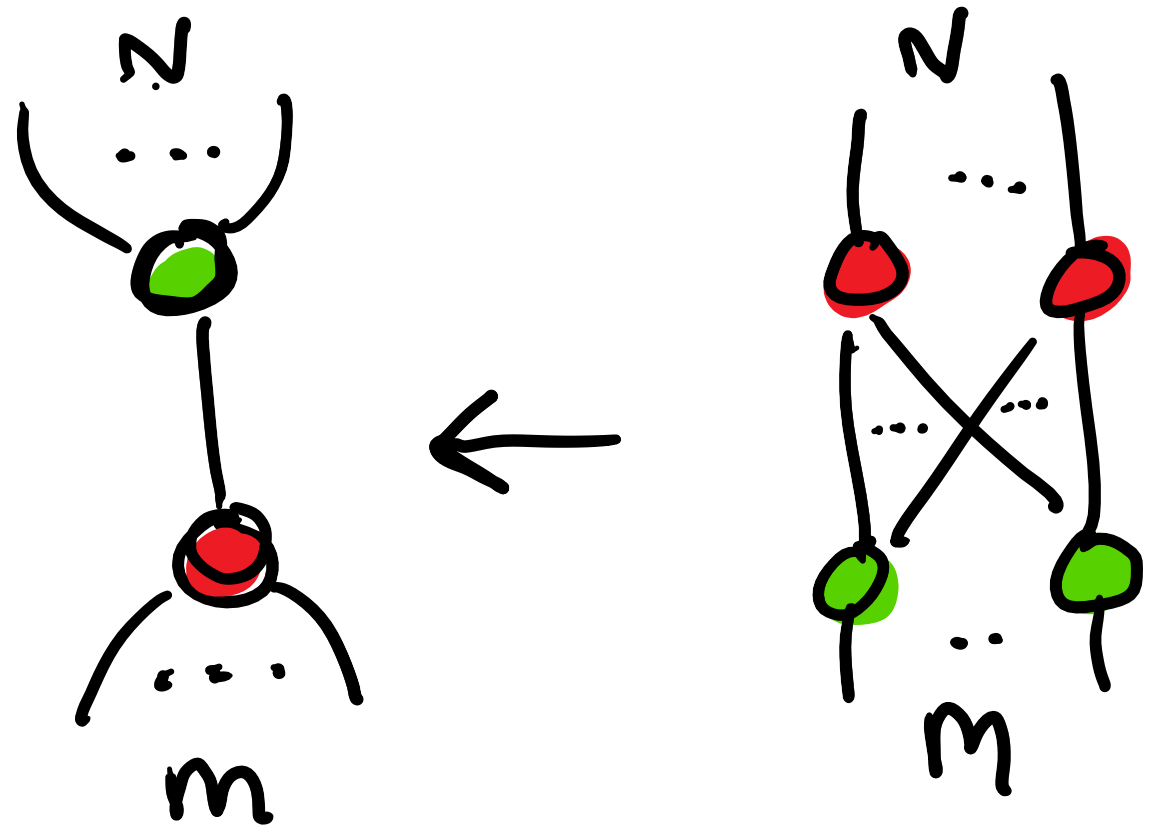
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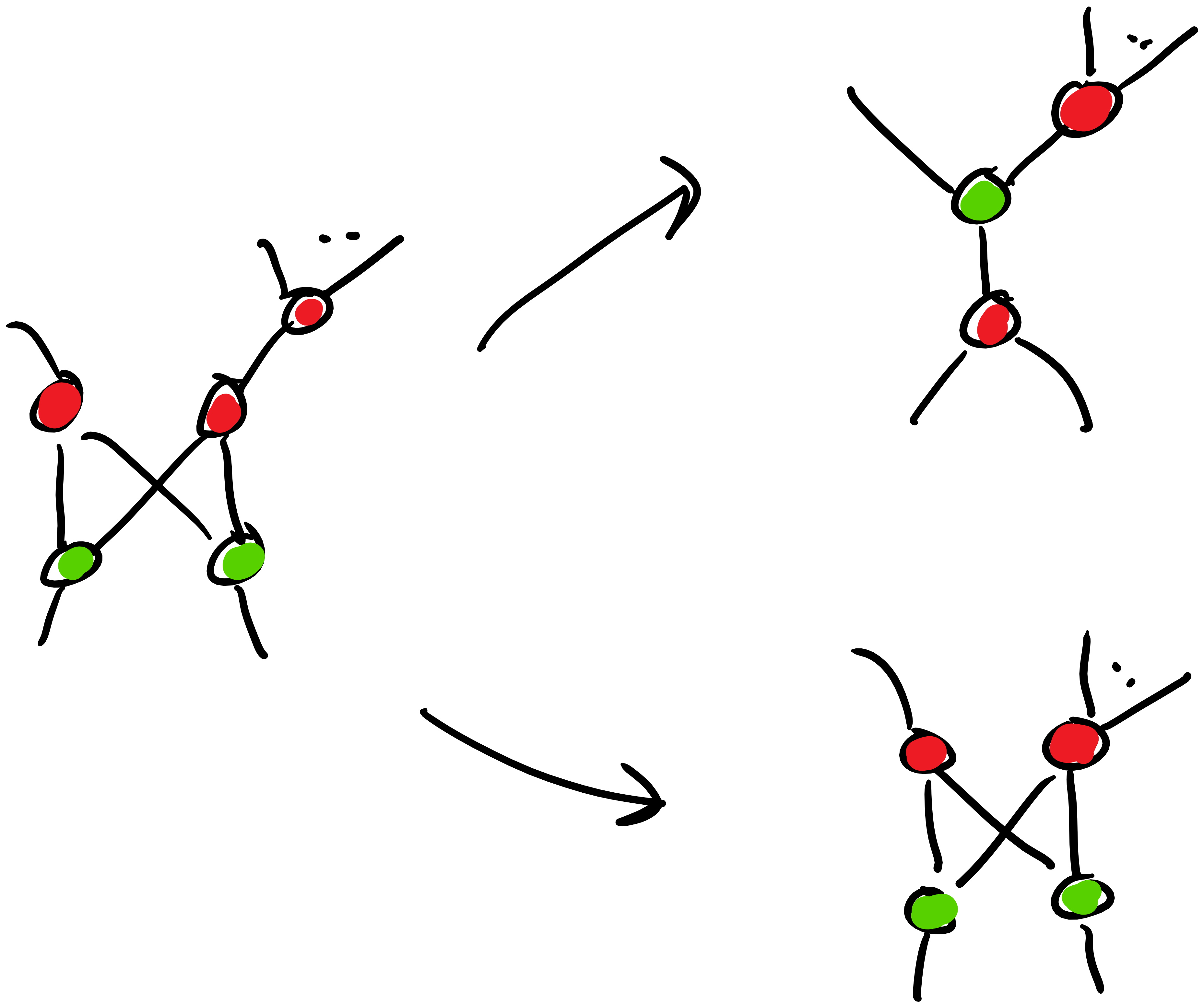

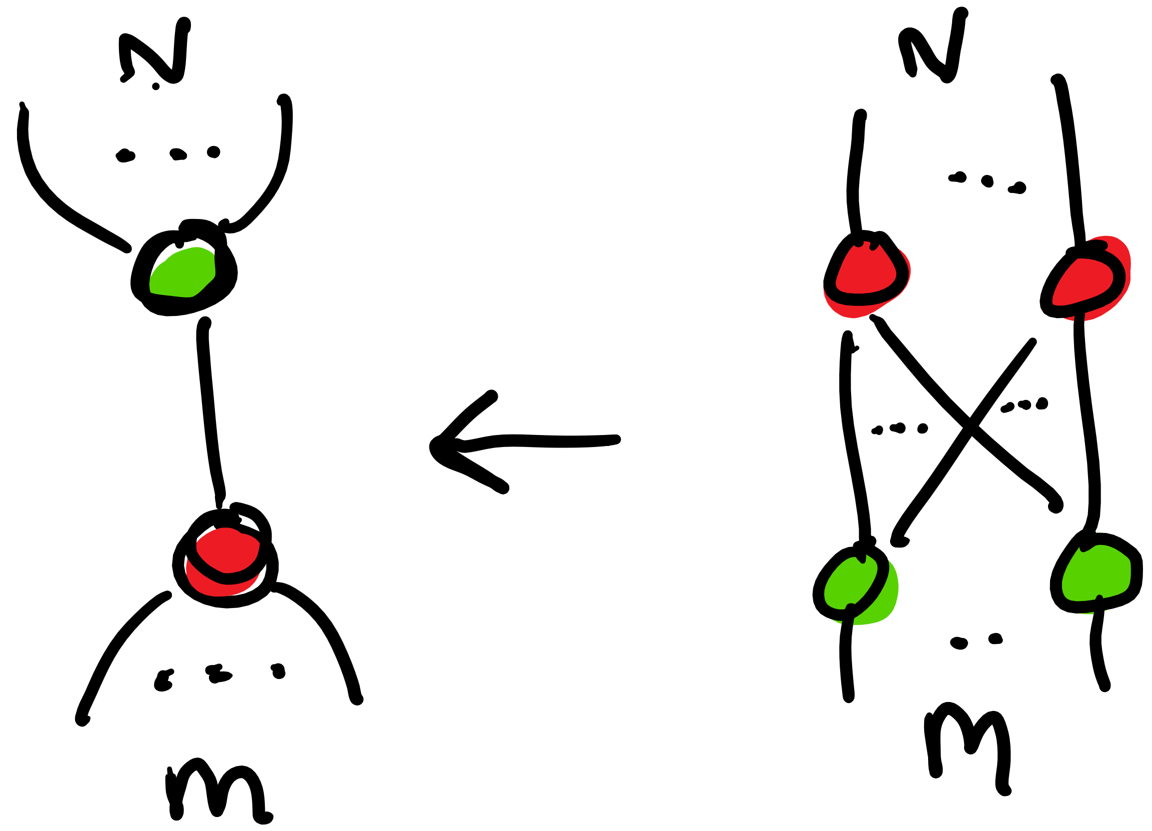
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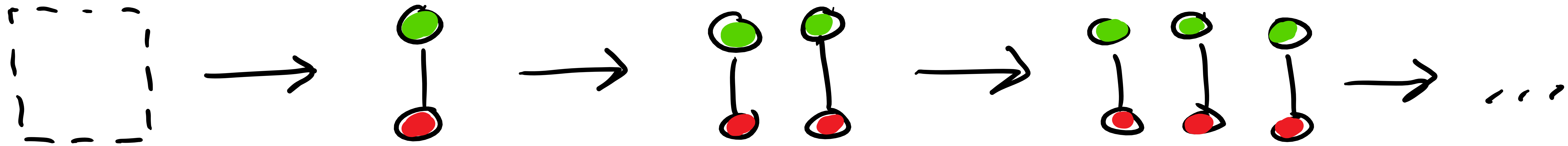
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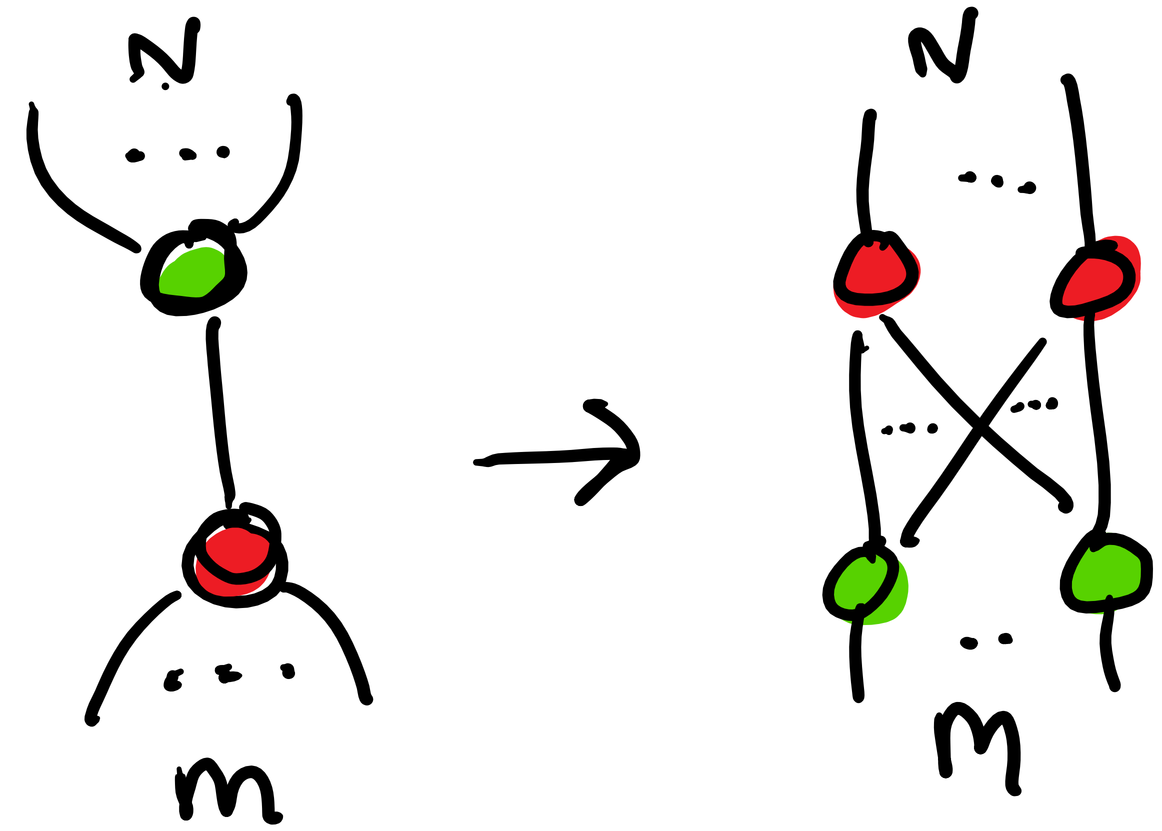
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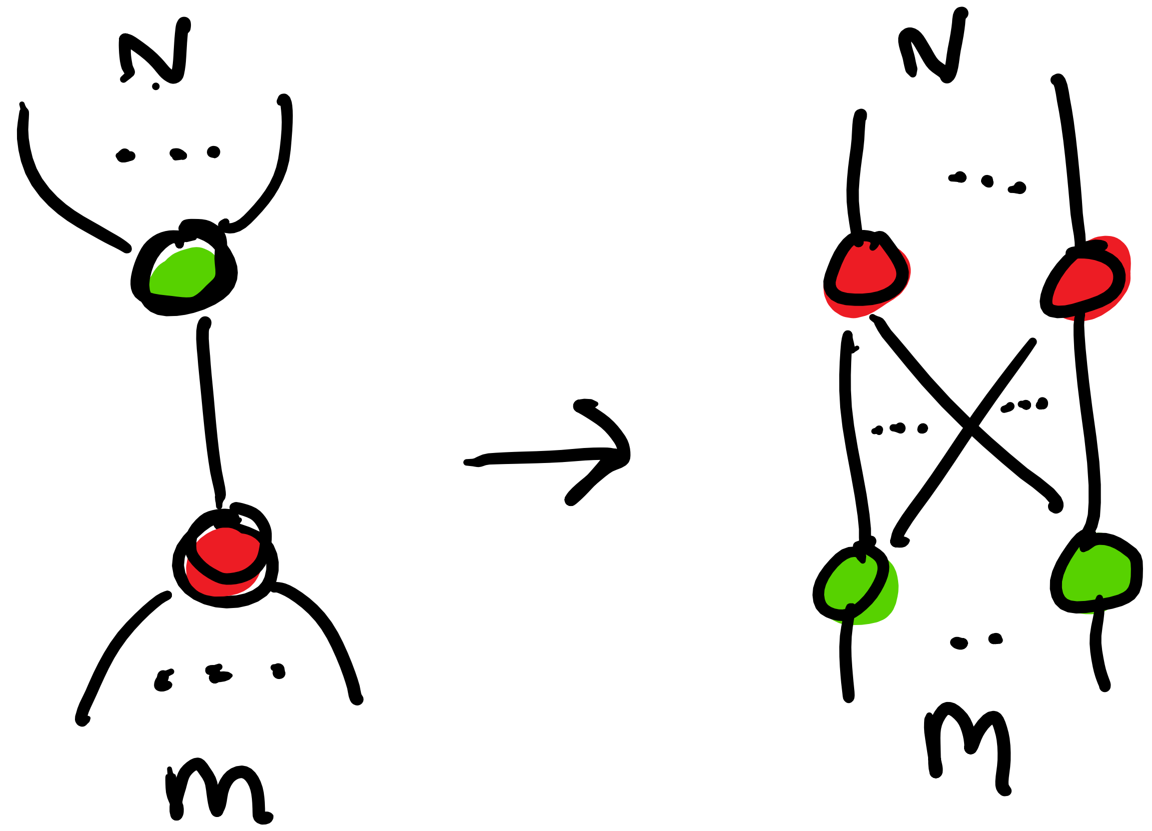
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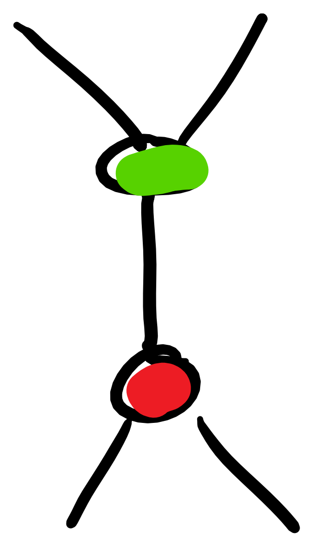

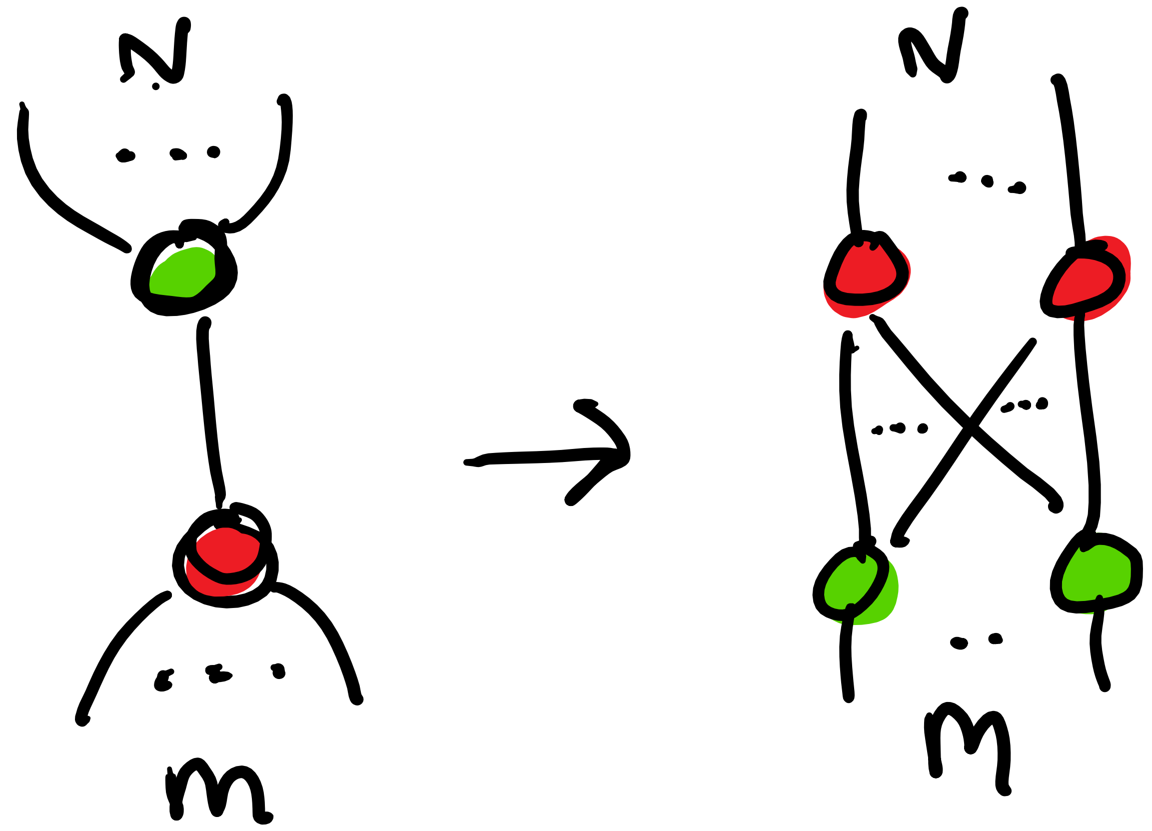
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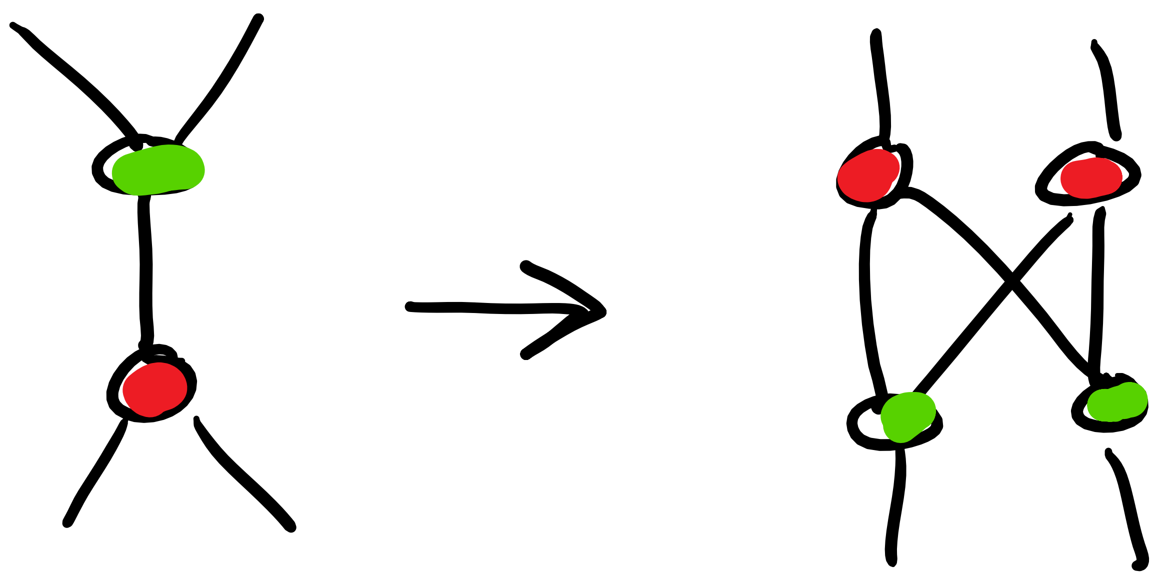

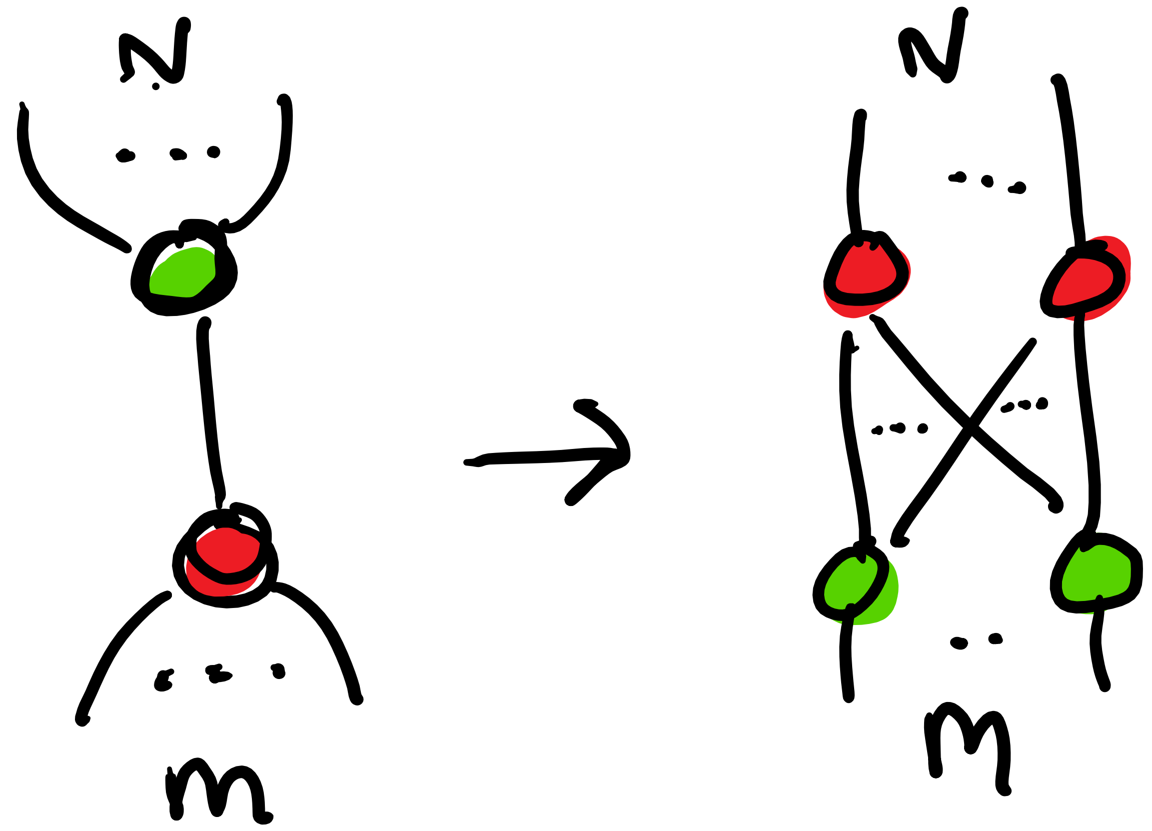
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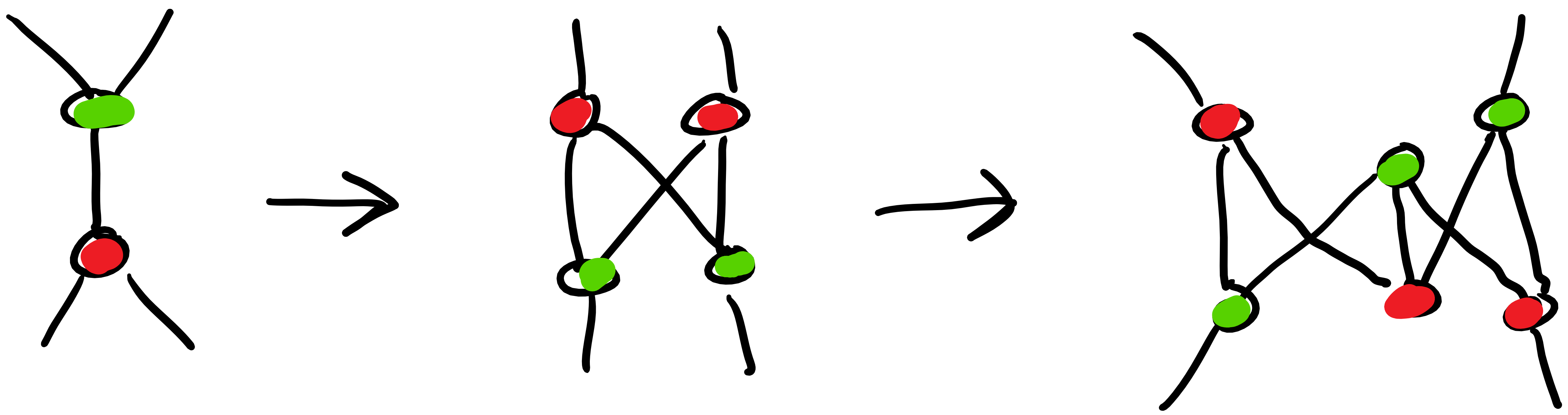

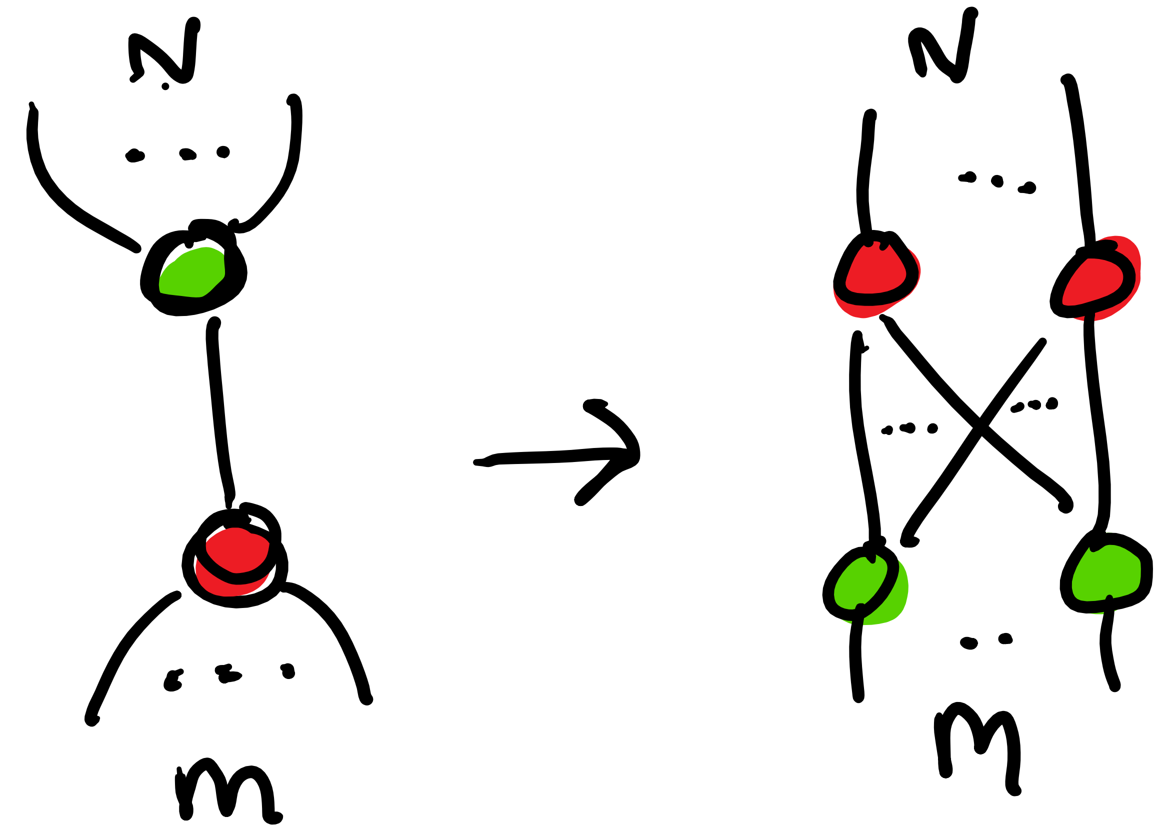
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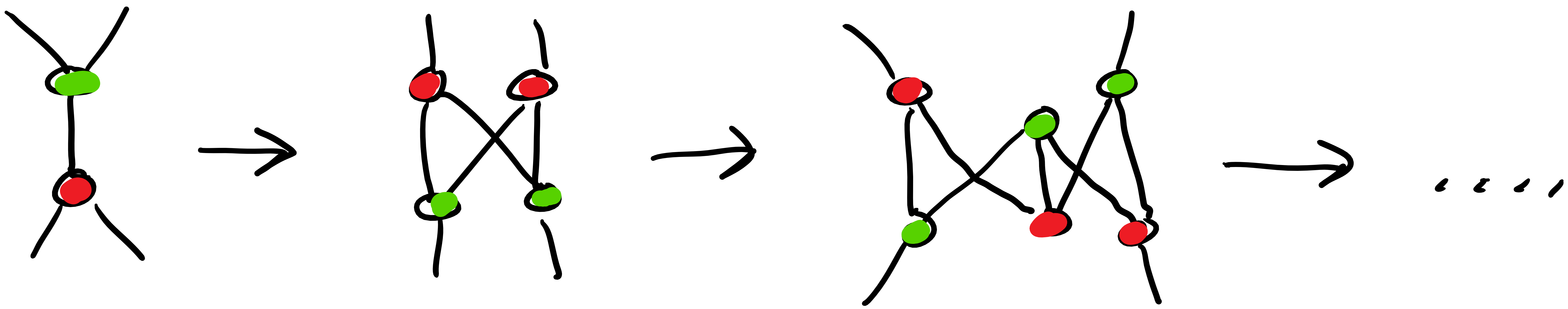
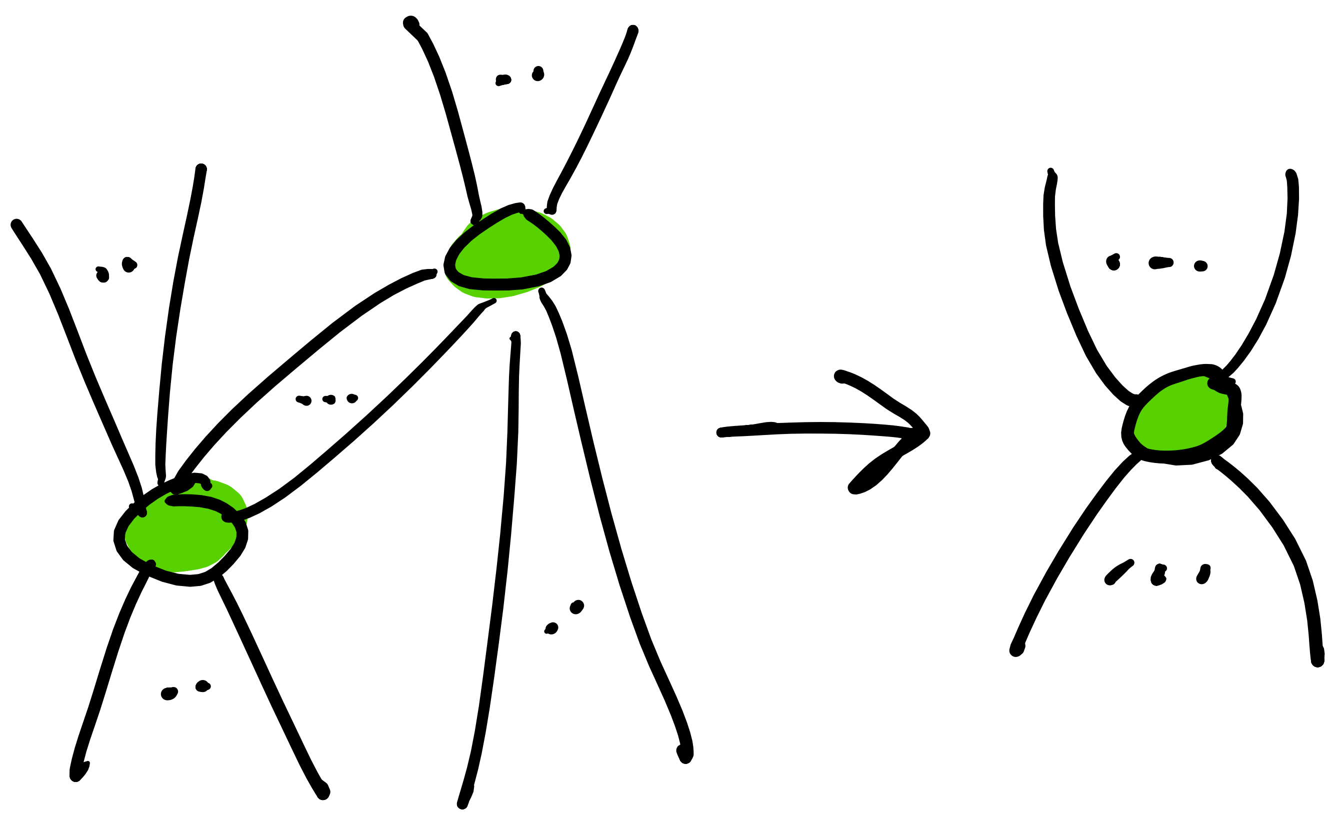
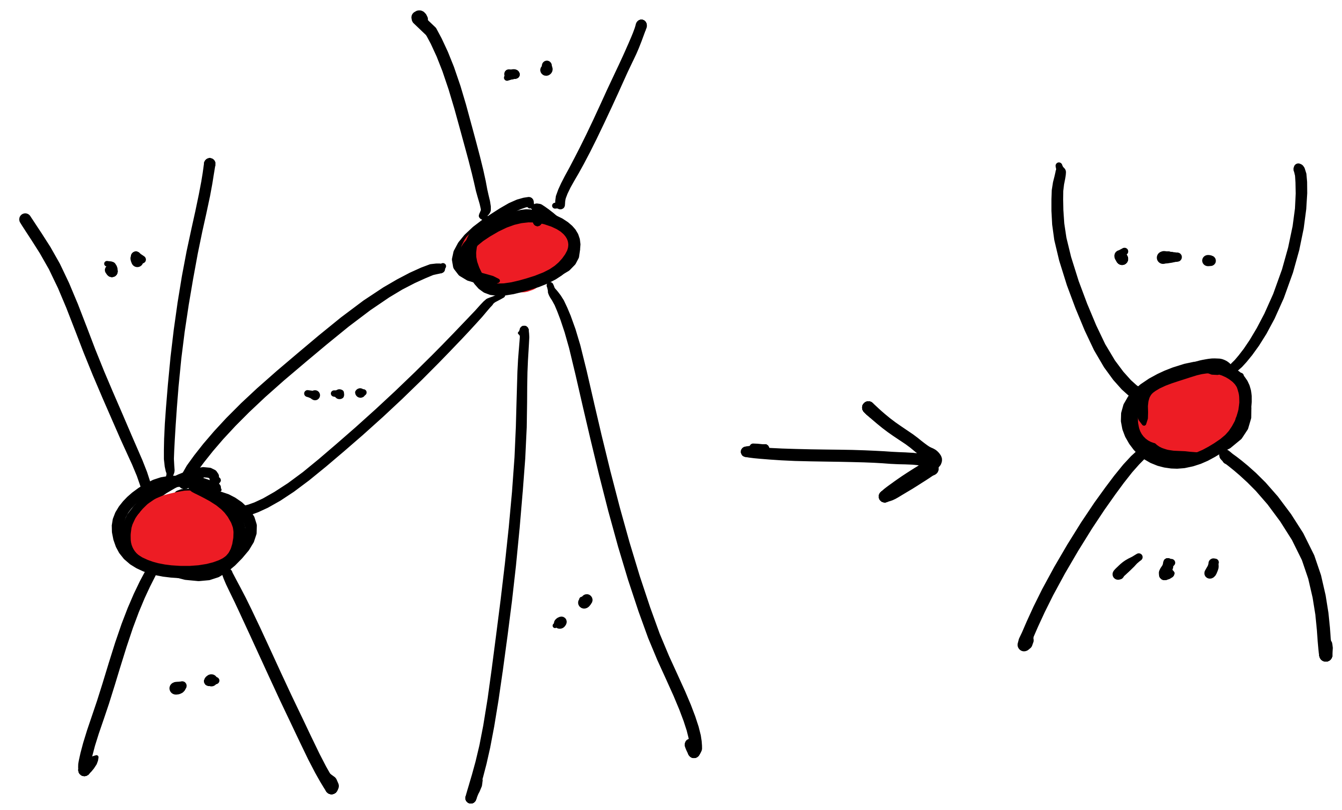
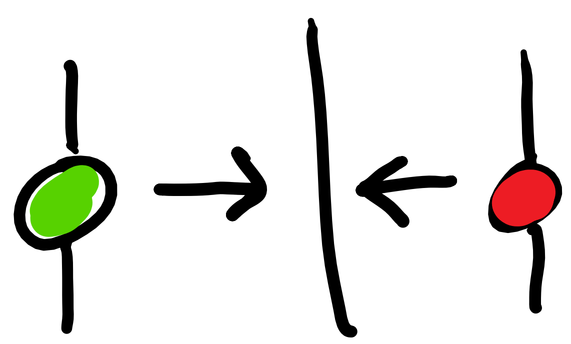


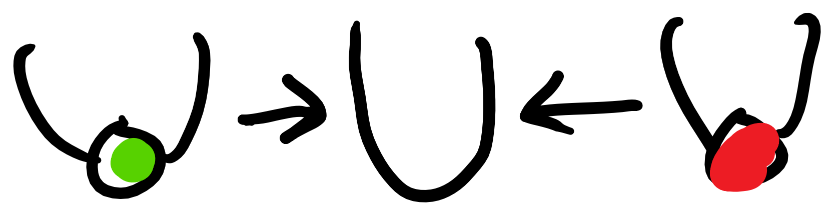
PHASE-FREE

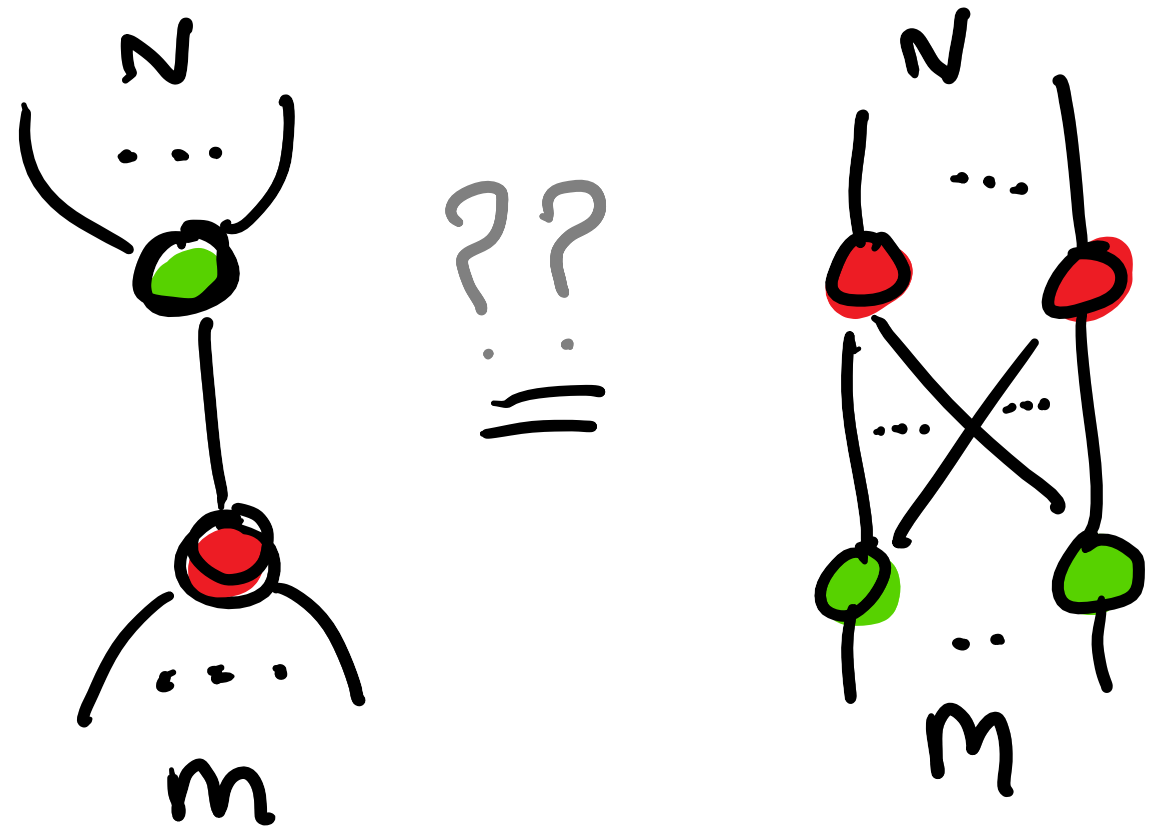
ZX-calculus





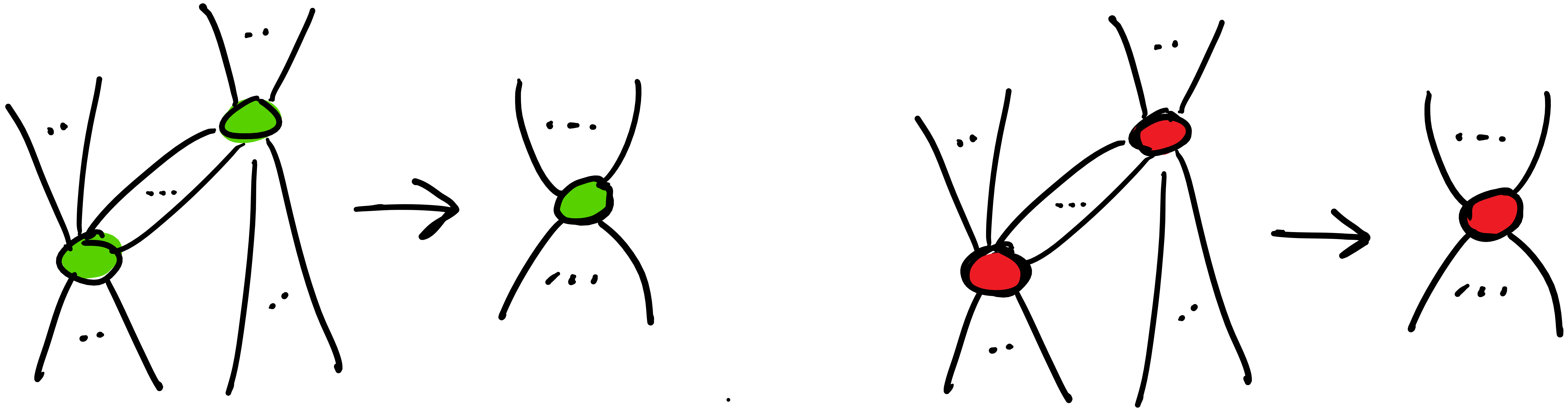


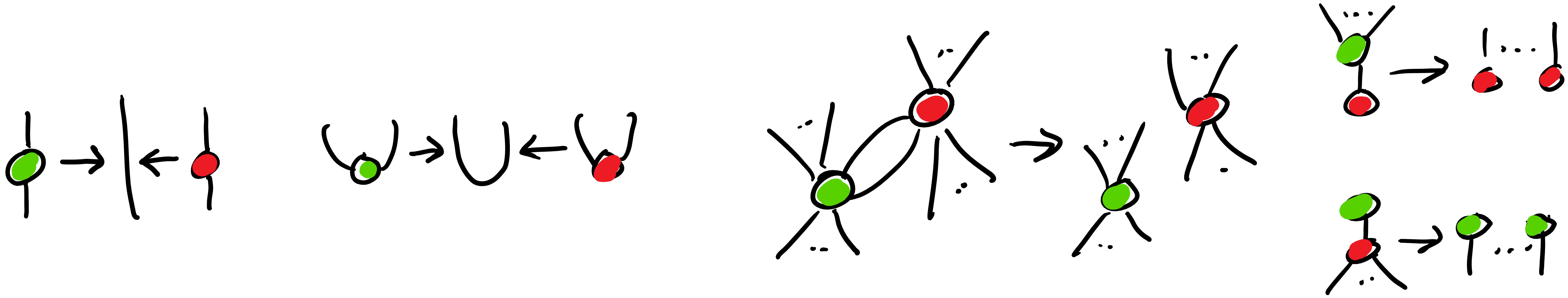




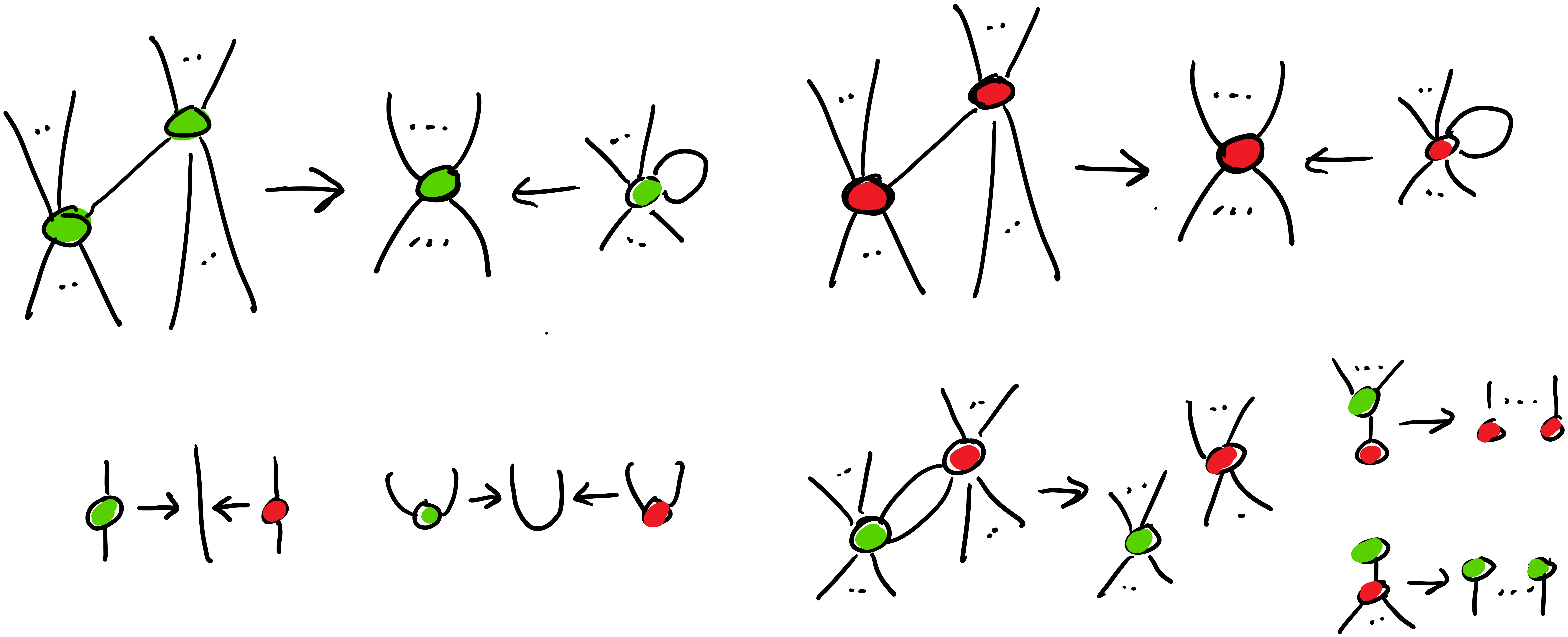
PHASE-FREE

ZX-calculus

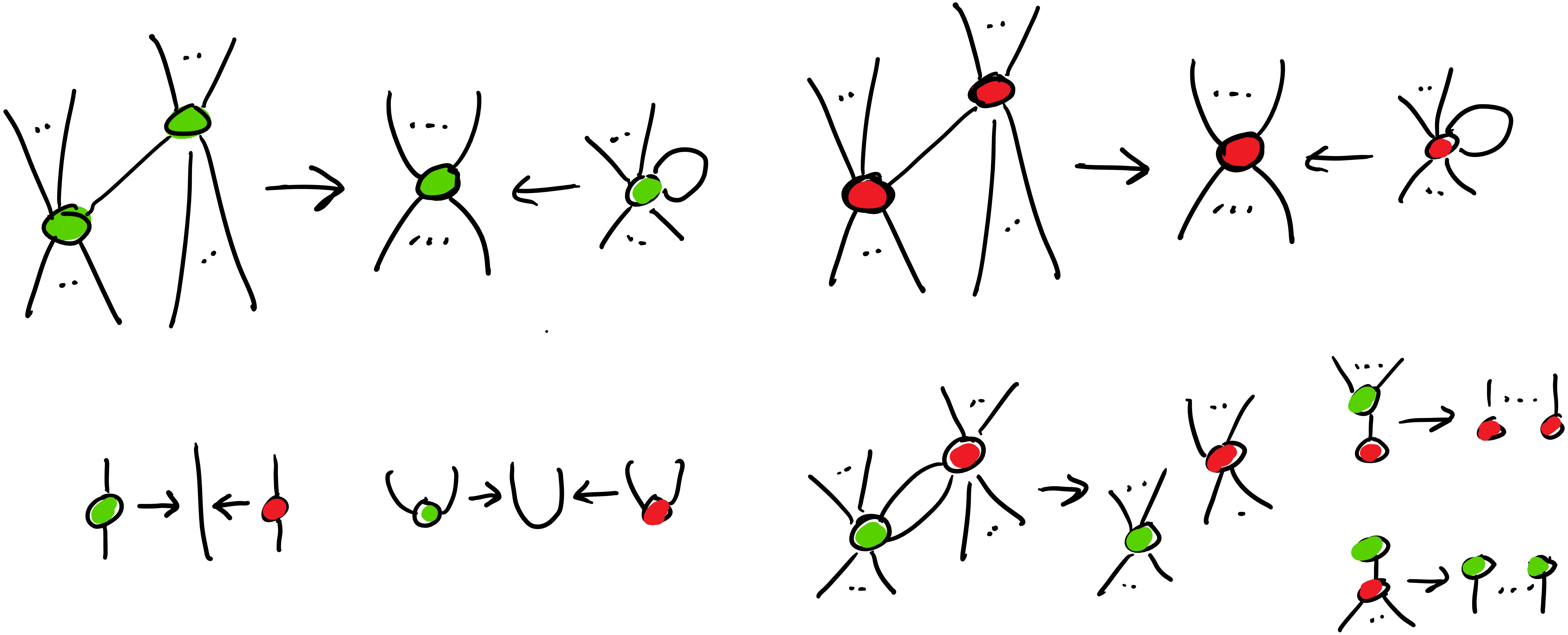




PHASE-FREE ZX-calculus



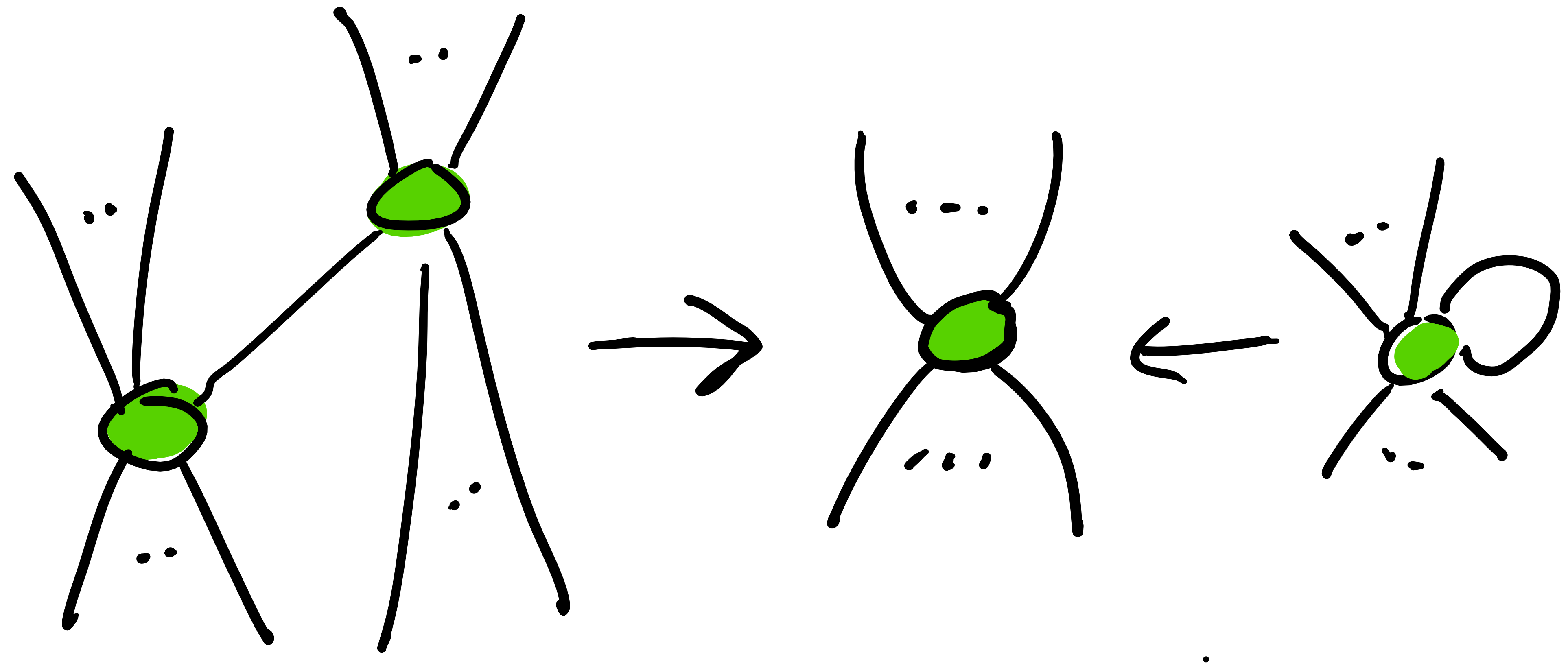
PHASE-FREE ZX-calculus

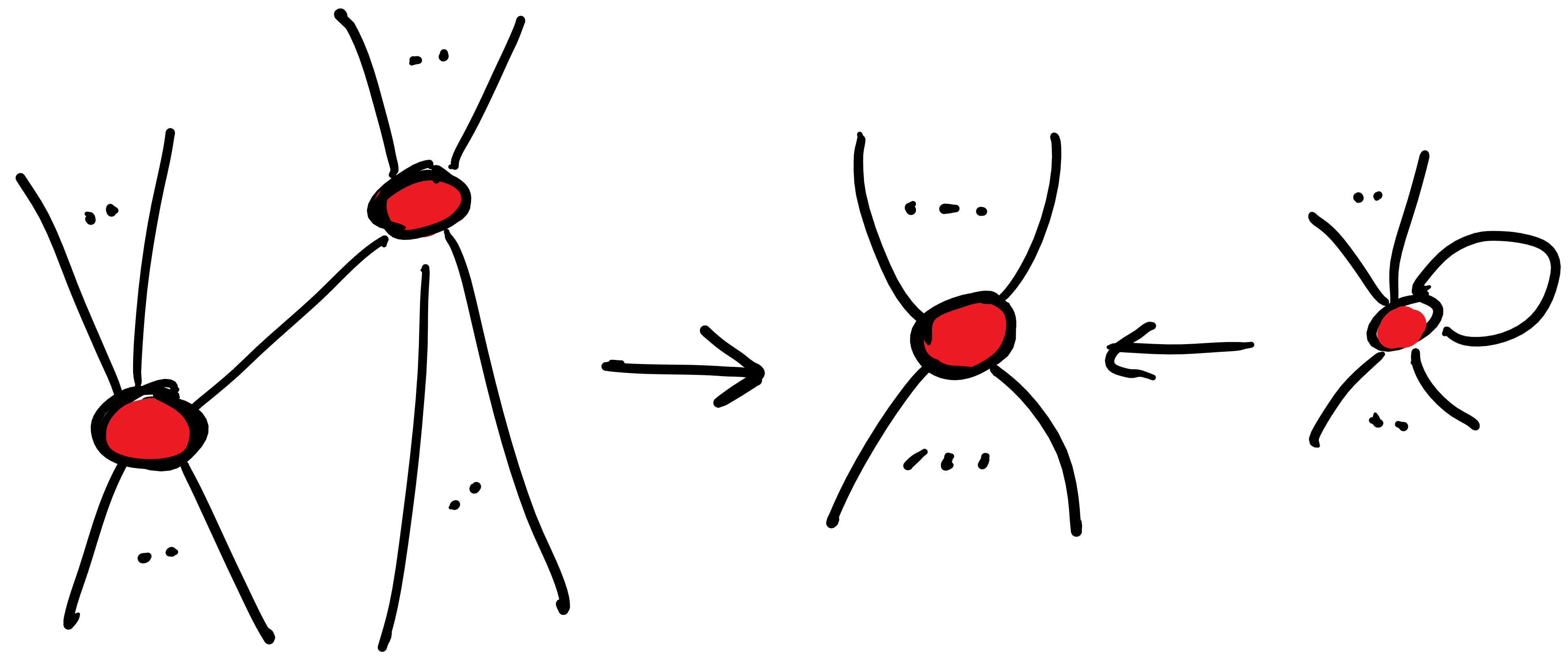


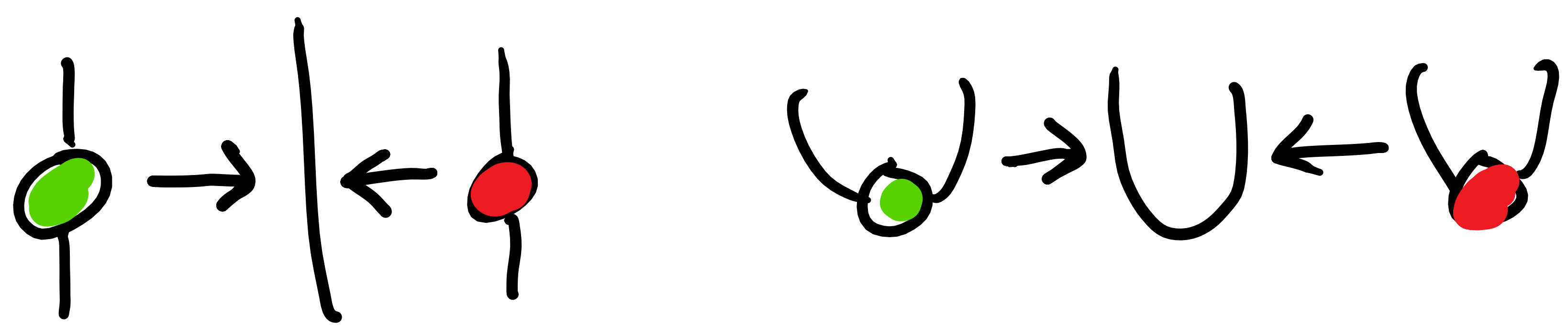
→ terminating and confluent.

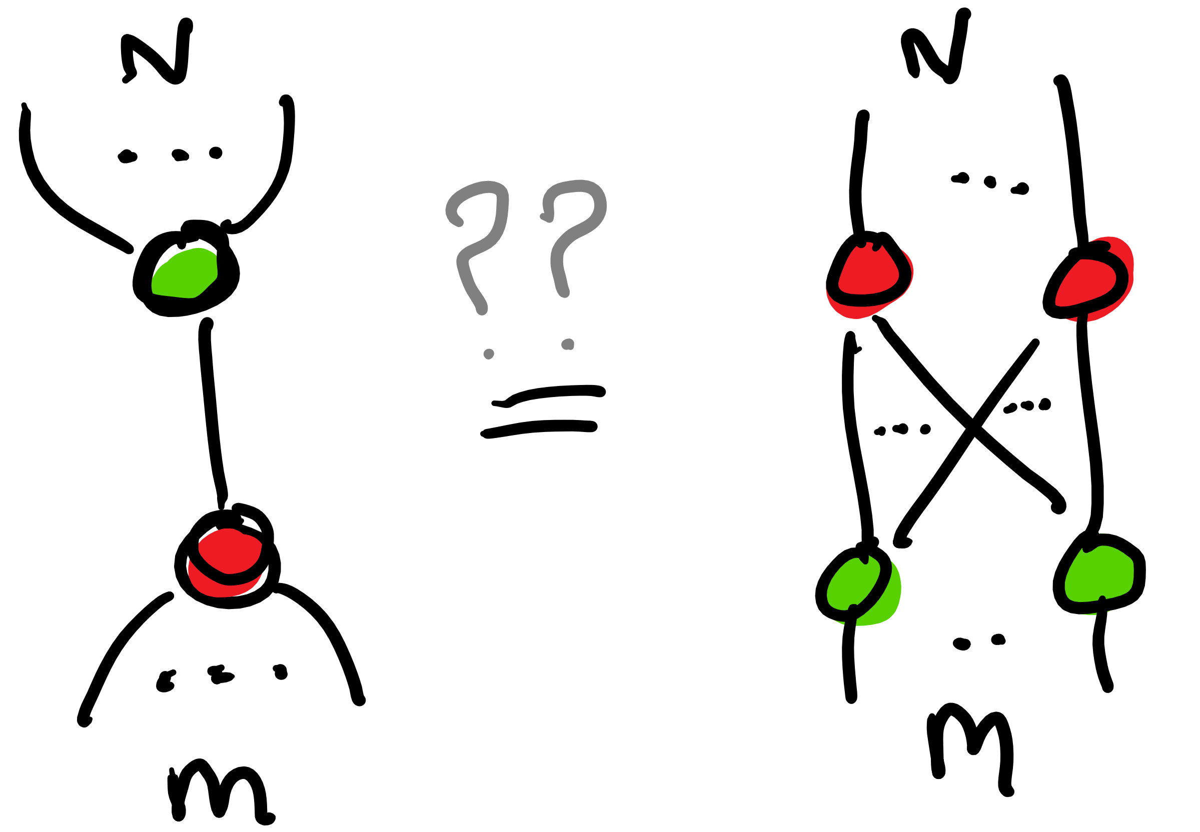
PHASE-FREE

ZX-calculus



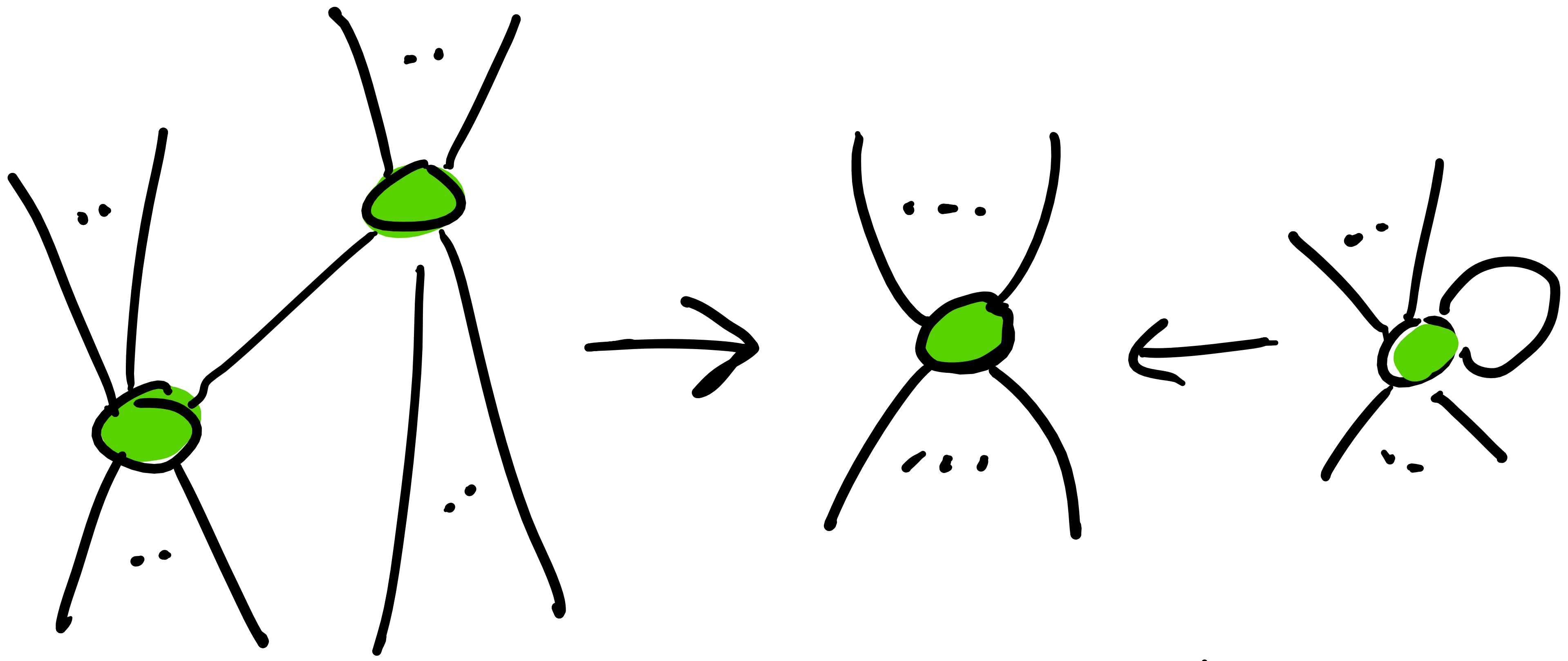


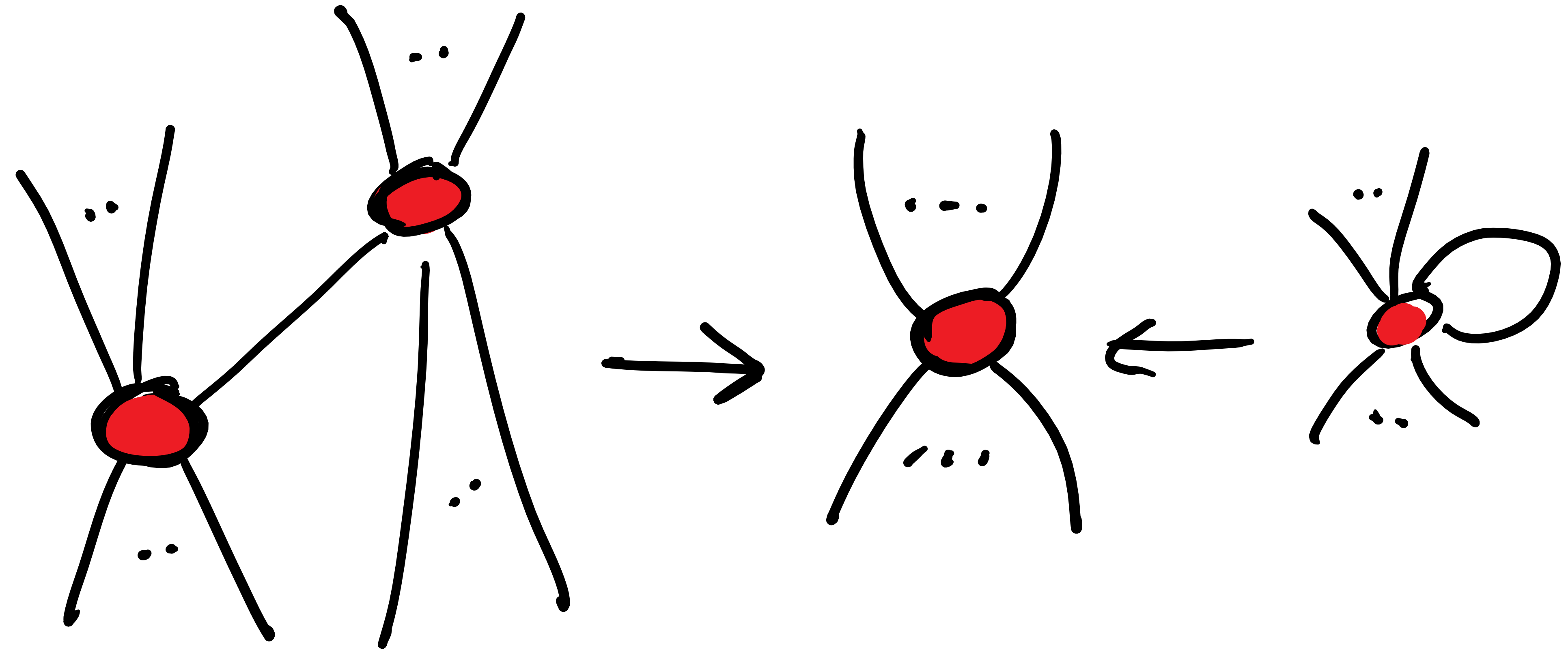


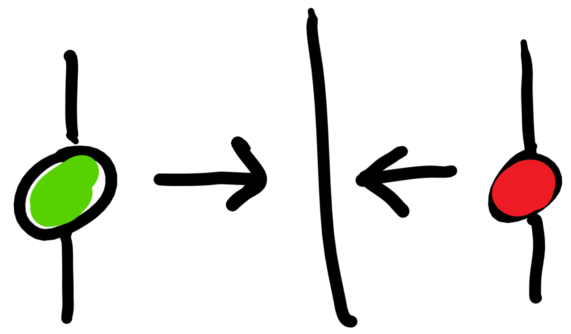


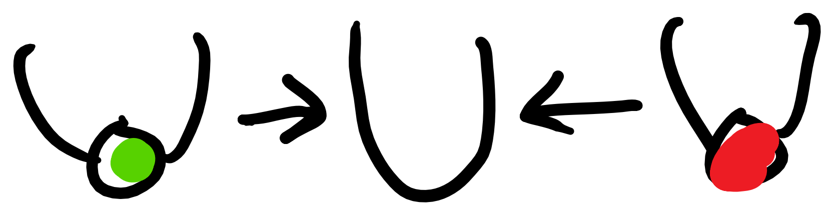
PHASE-FREE

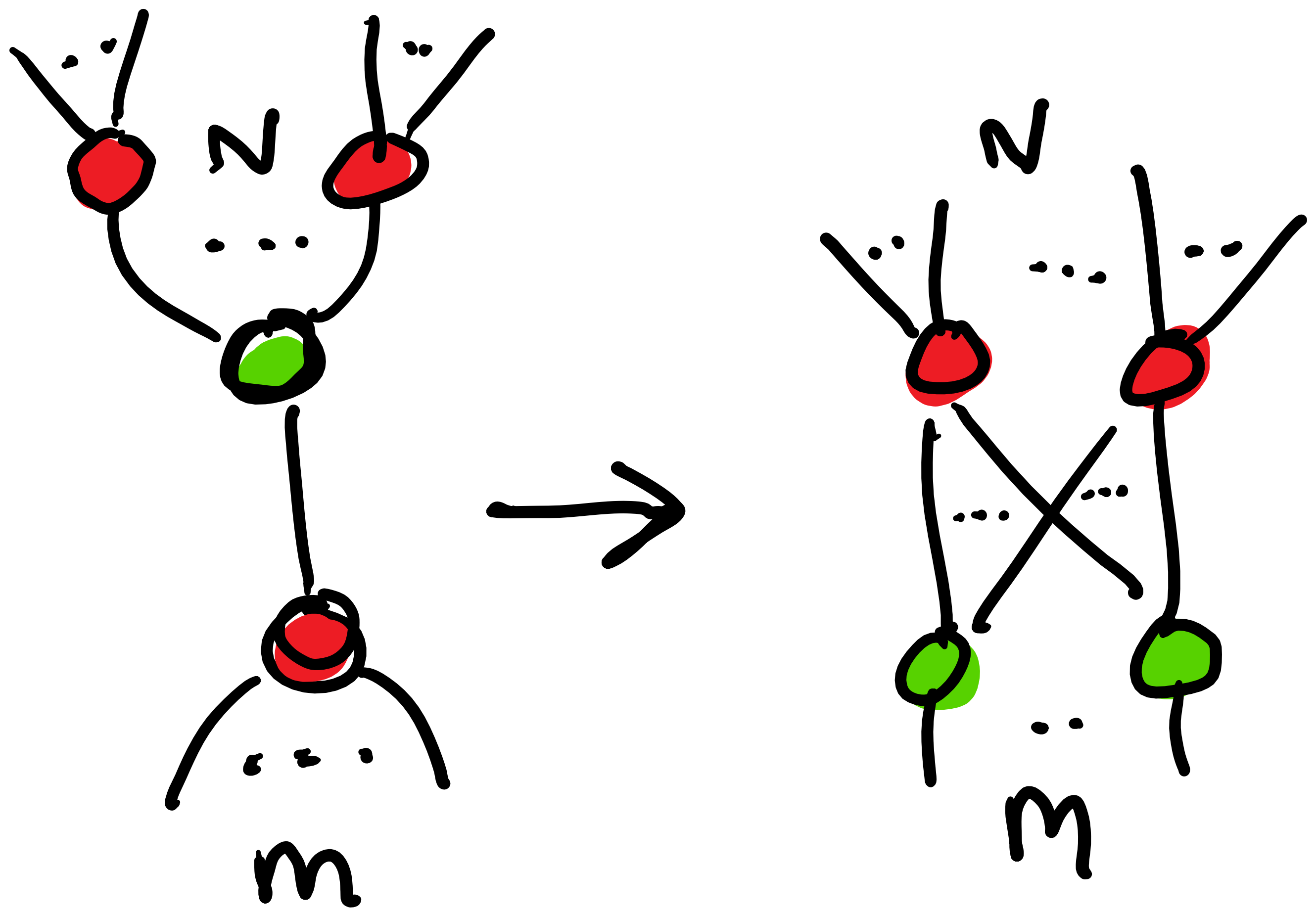
ZX-calculus



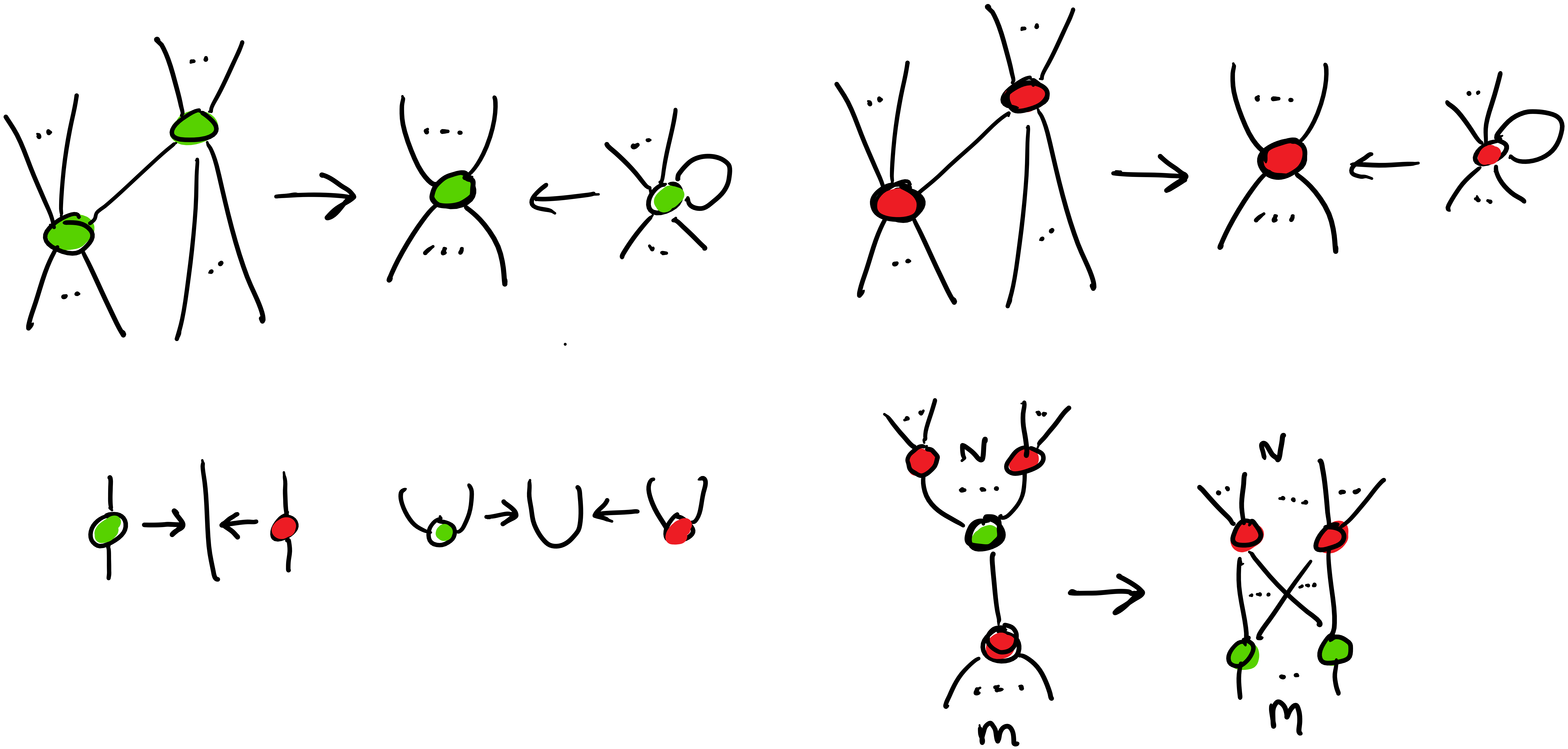








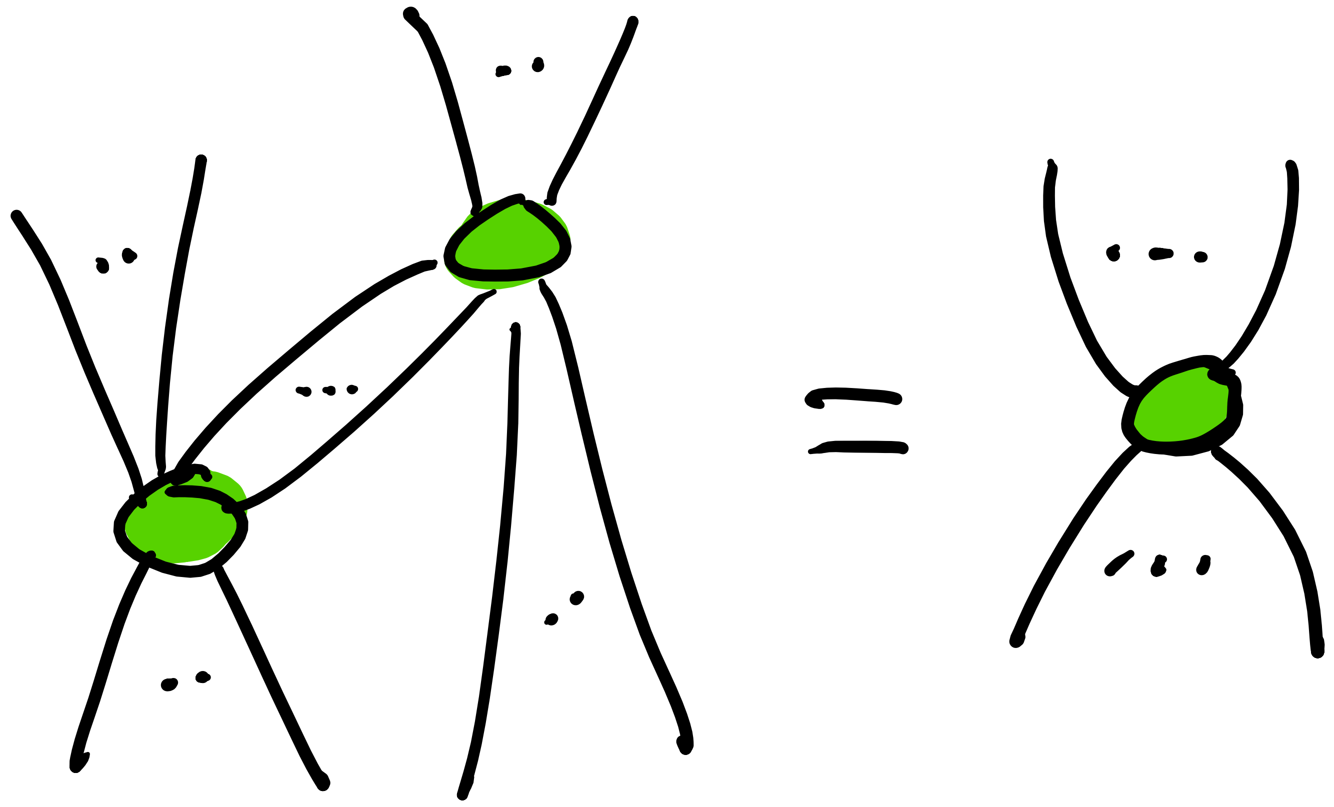
PHASE-FREE ZX-calculus

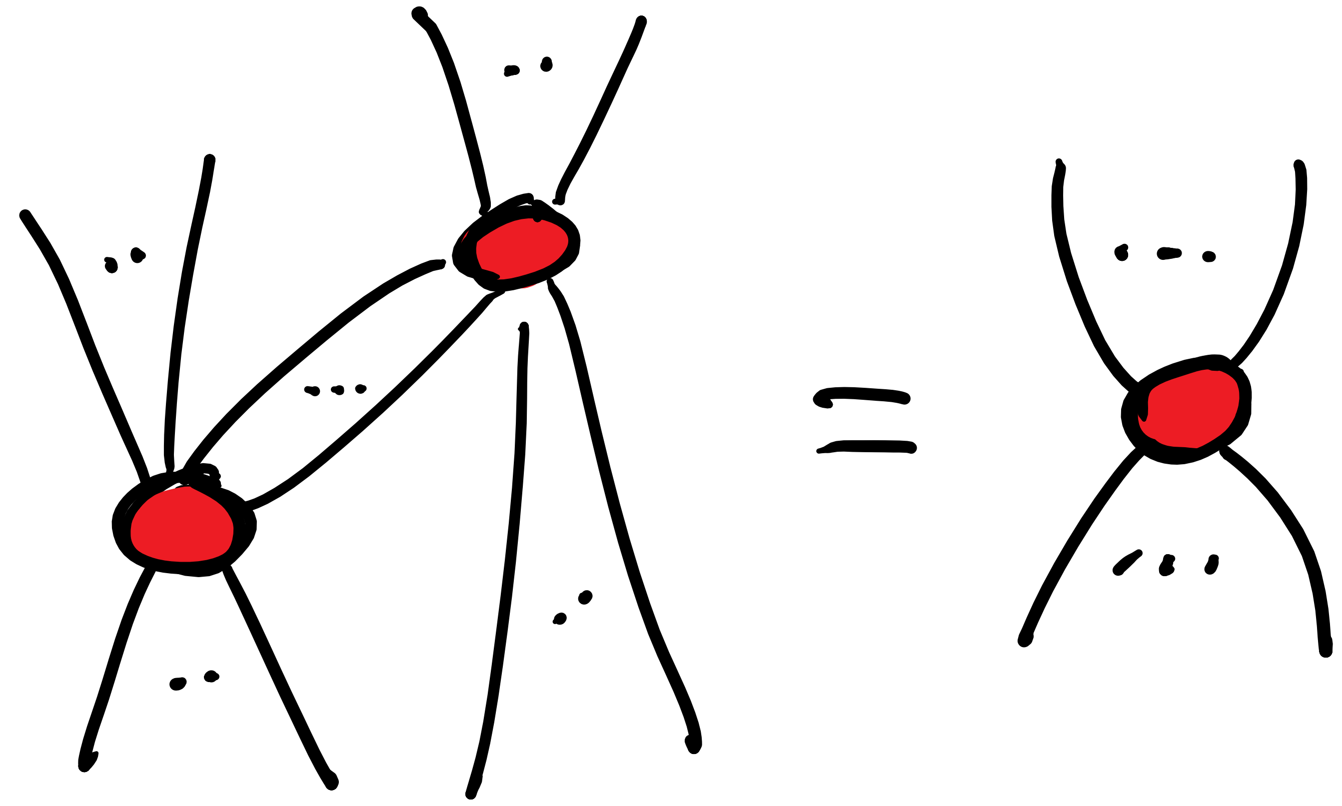


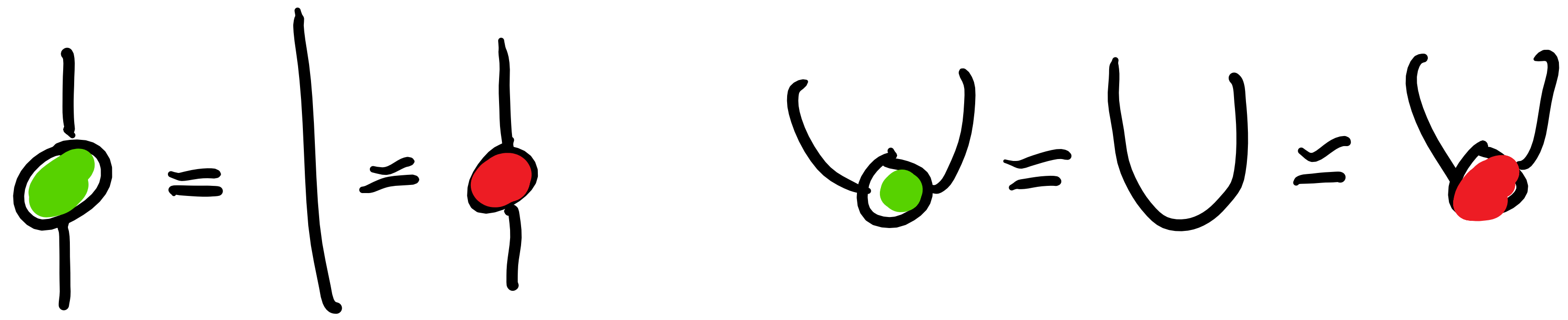
→ terminating with pseudo-NFs

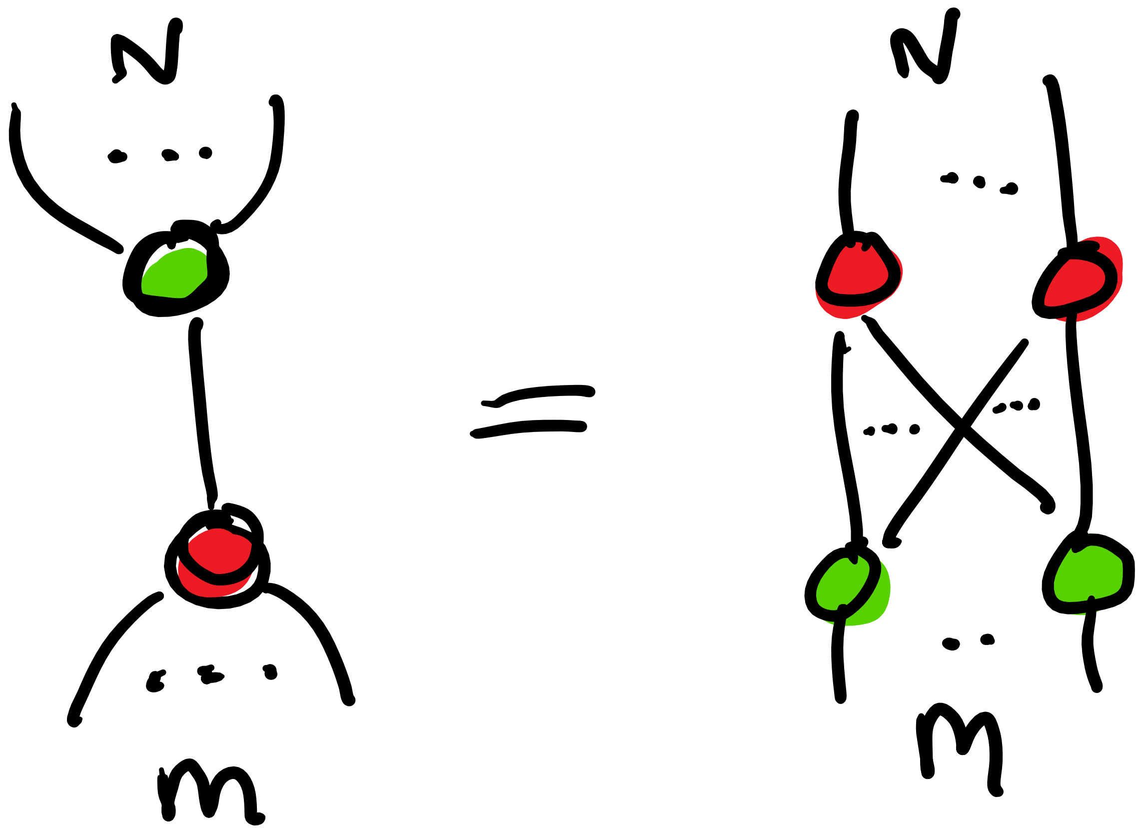
ZX-calculus

PHASE-FREE

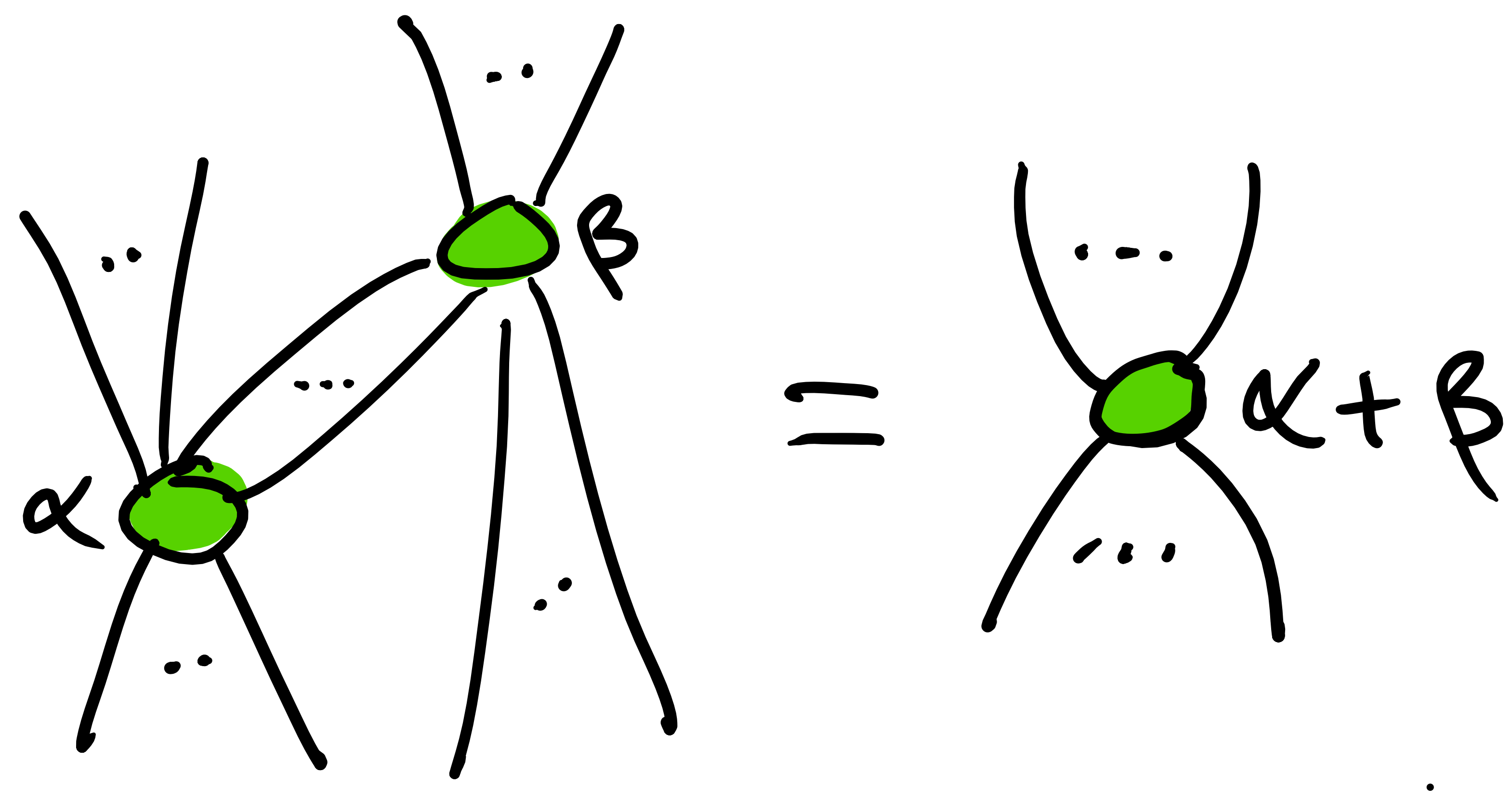


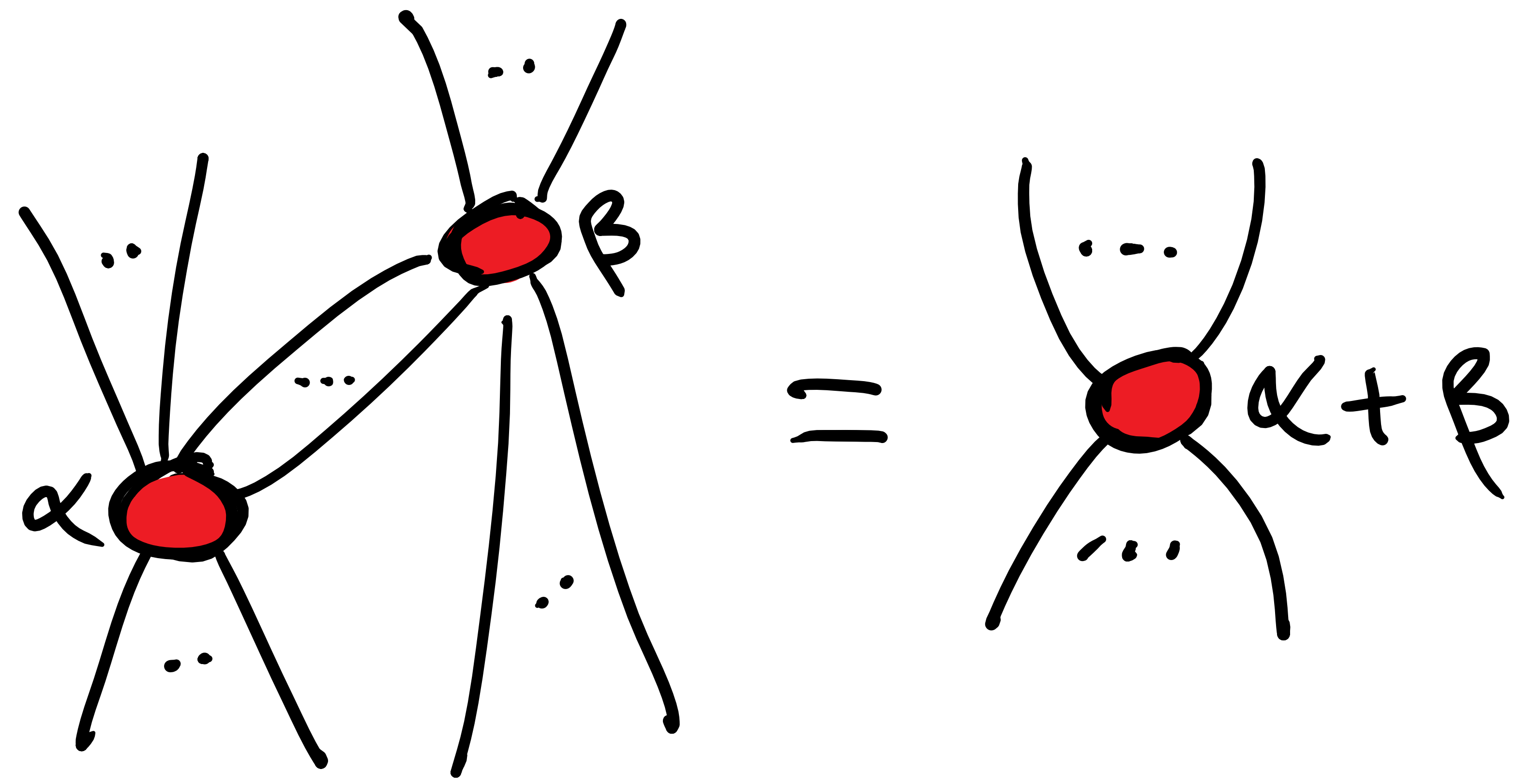


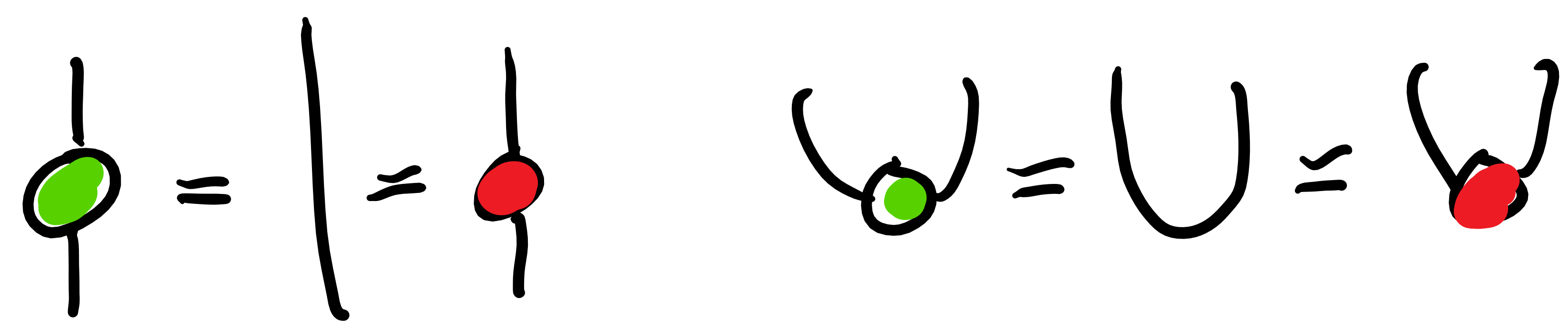


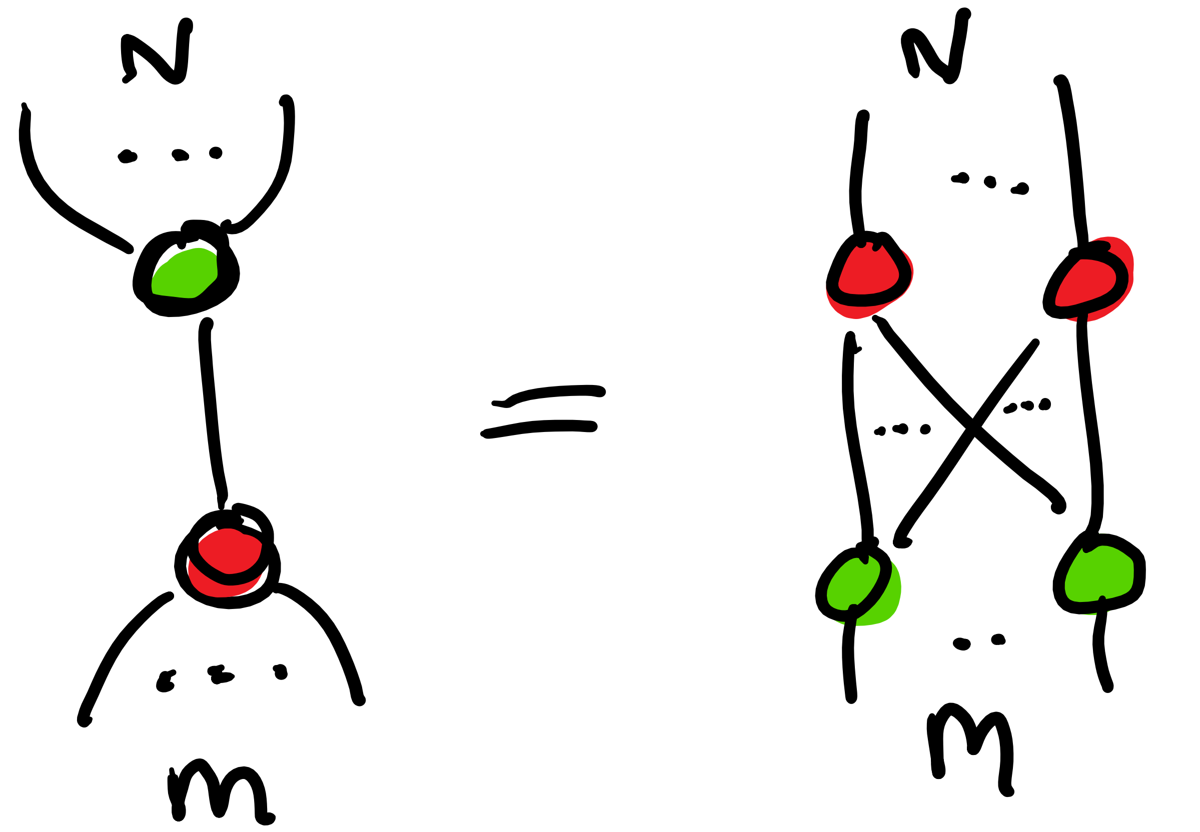


ZX-calculus

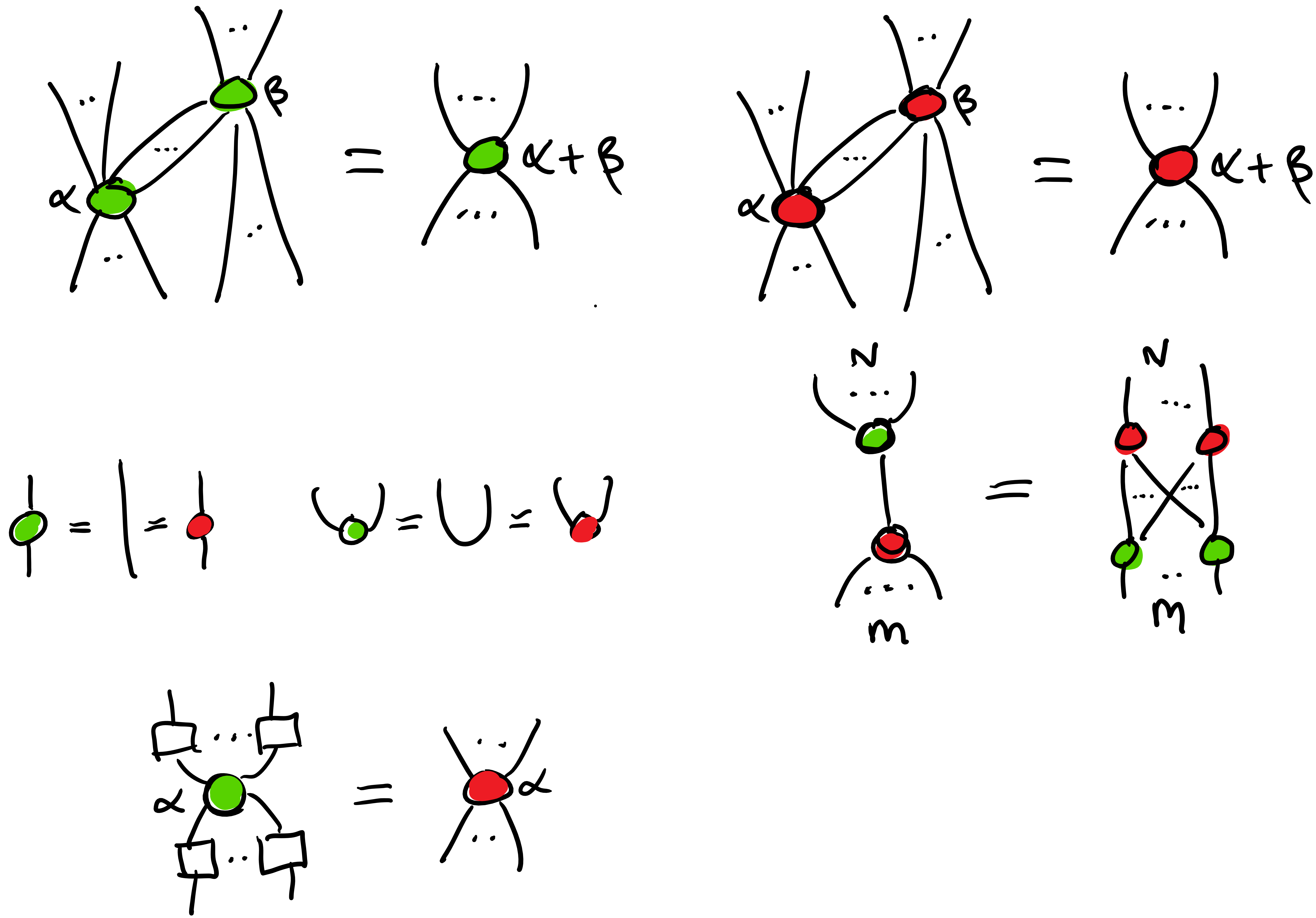




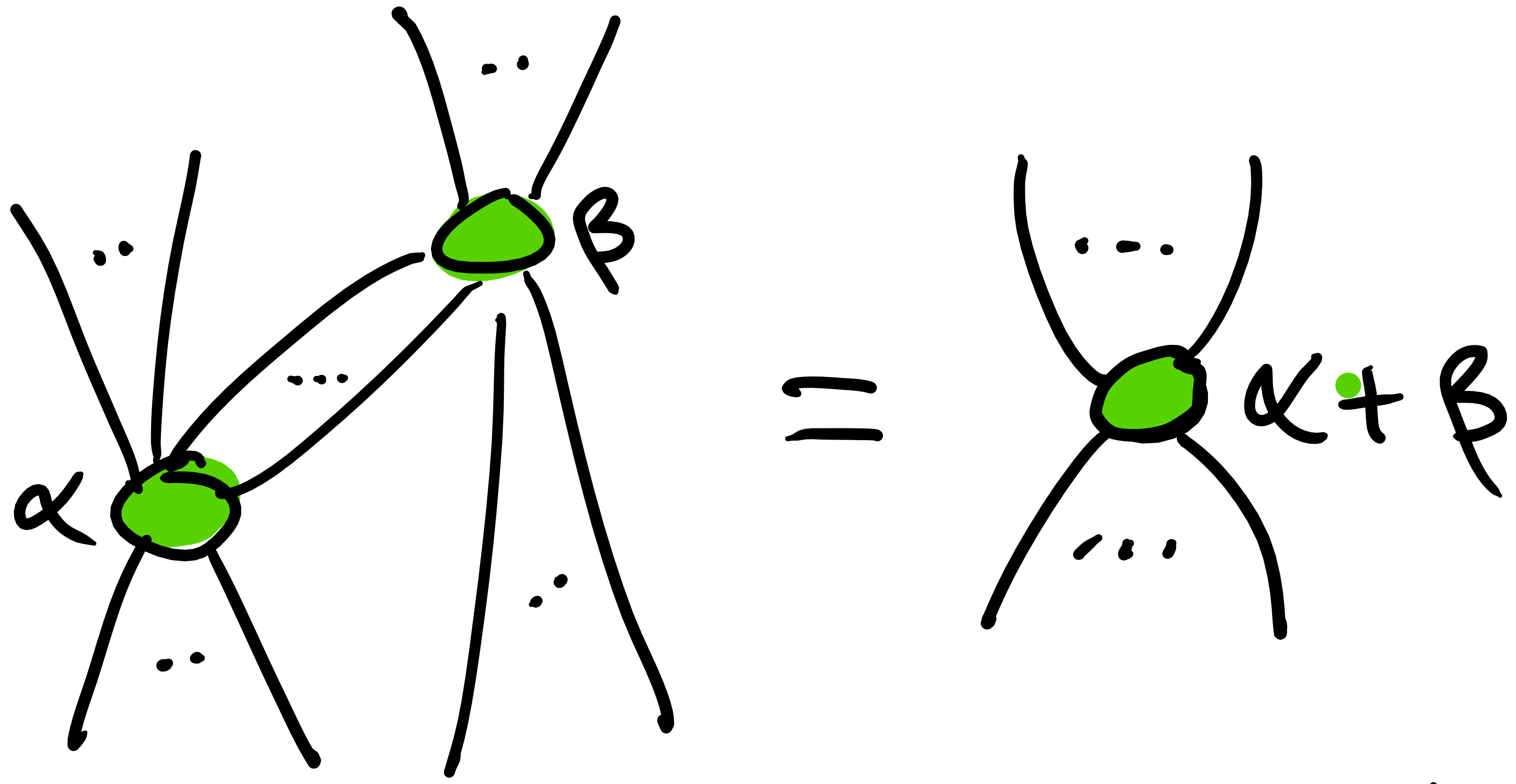


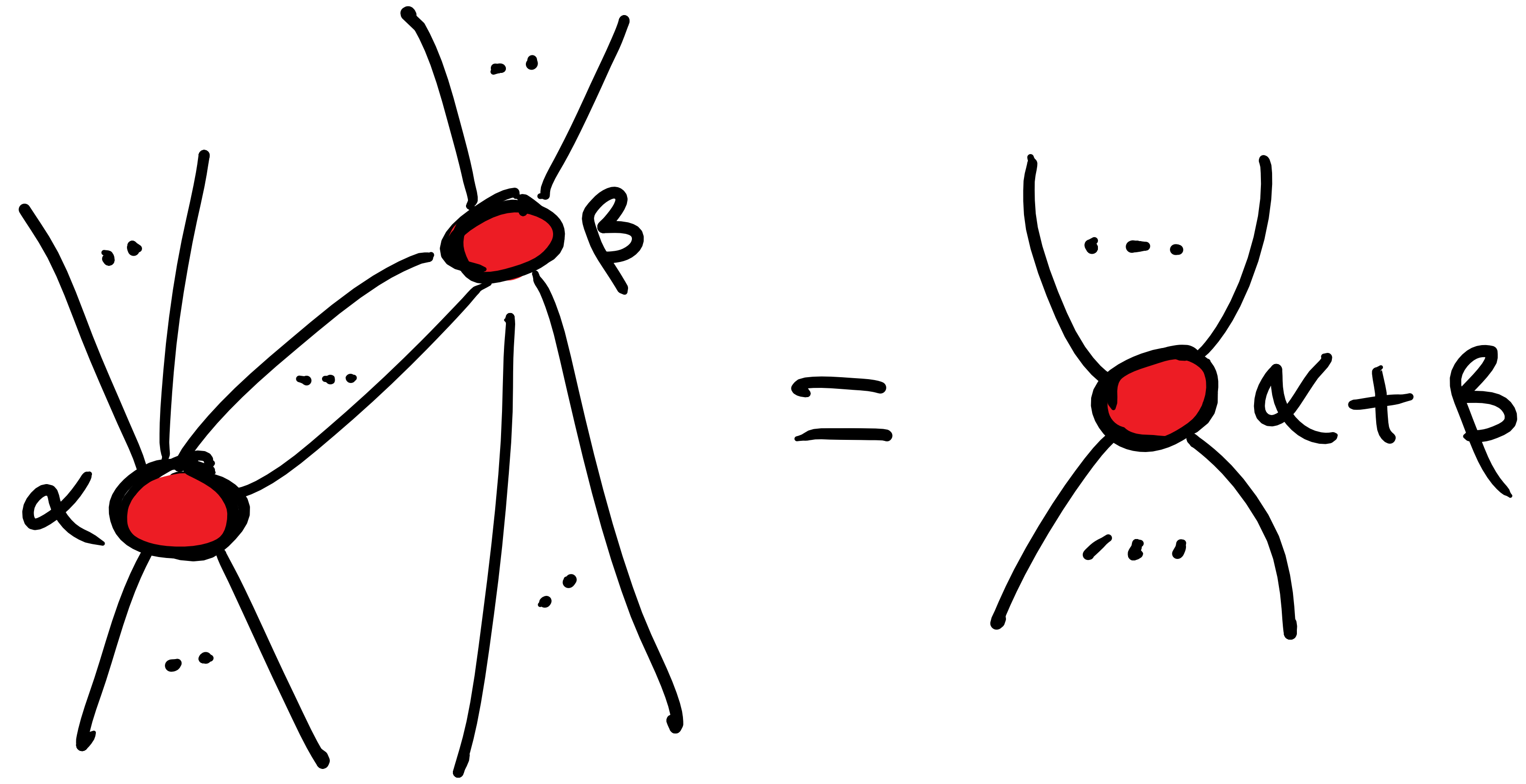


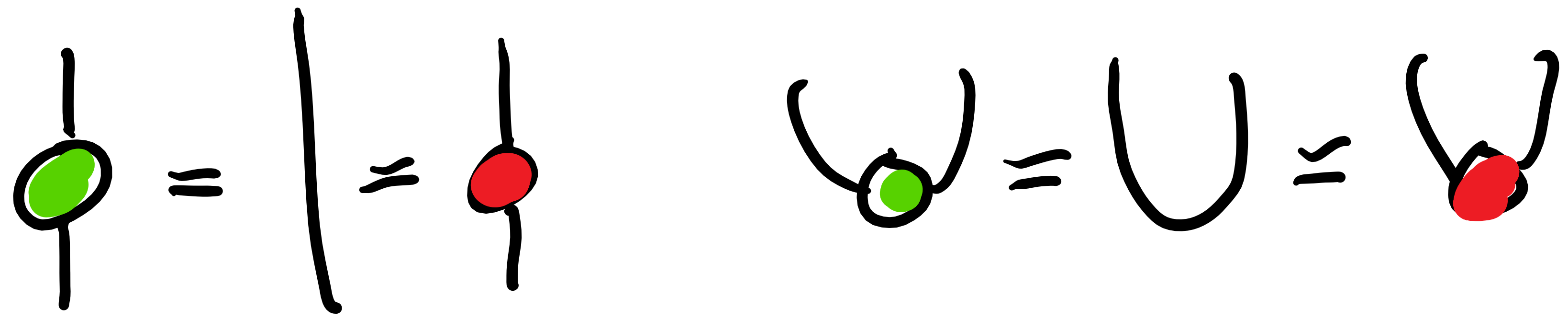
ZX-calculus

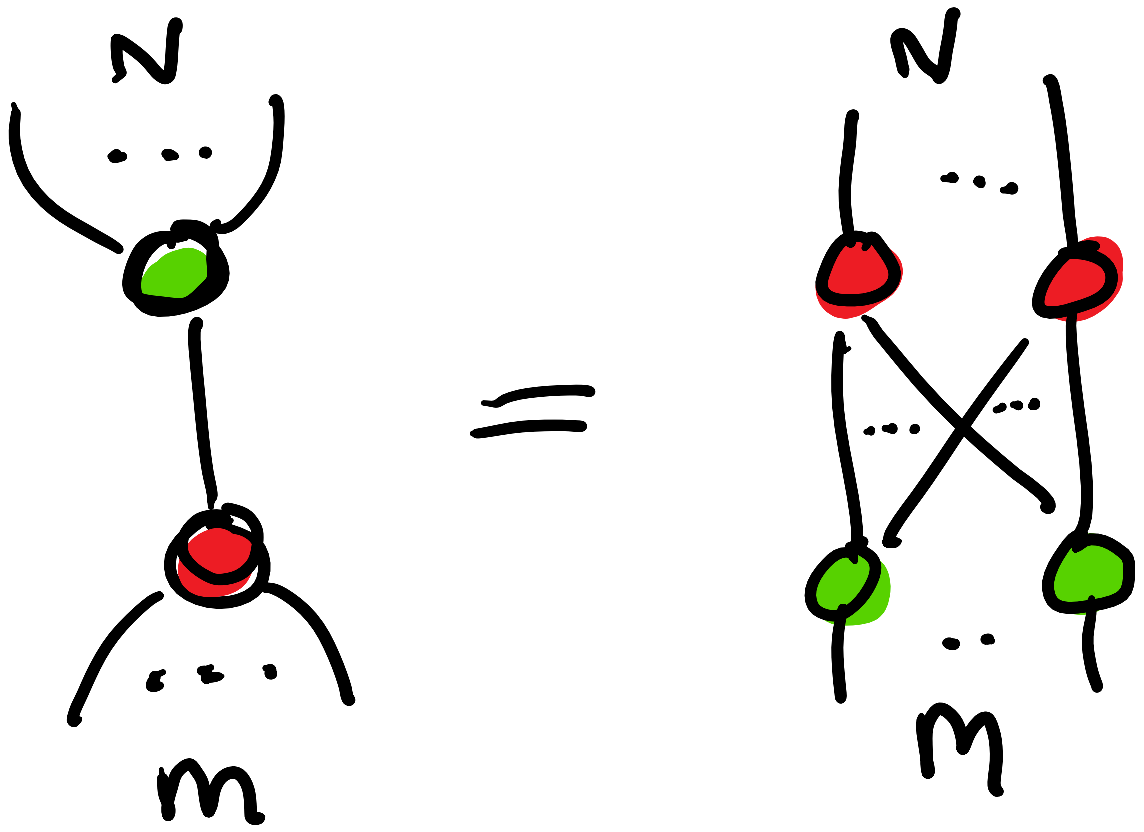


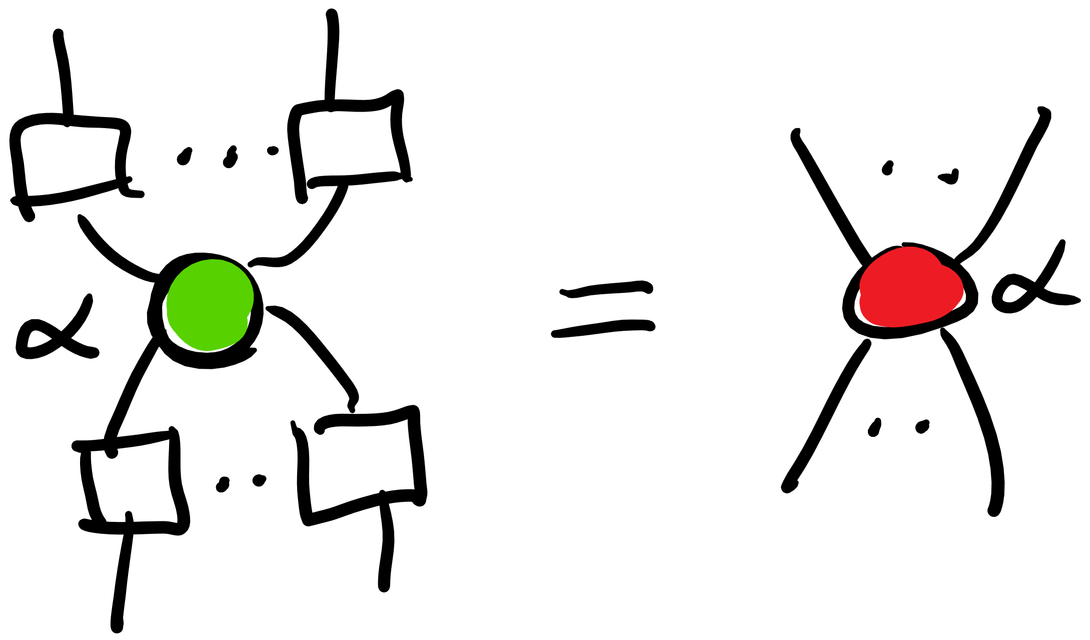
ZX-calculus

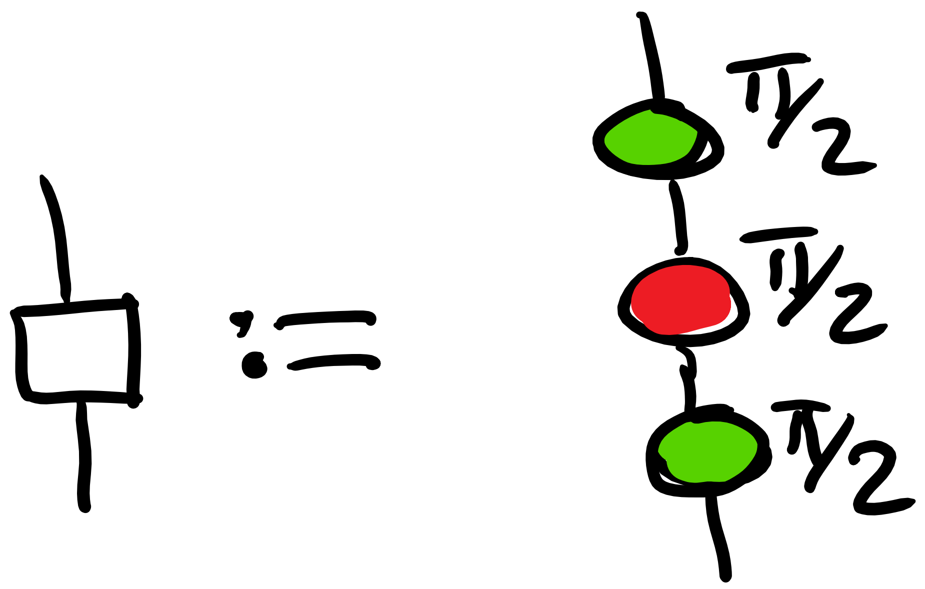






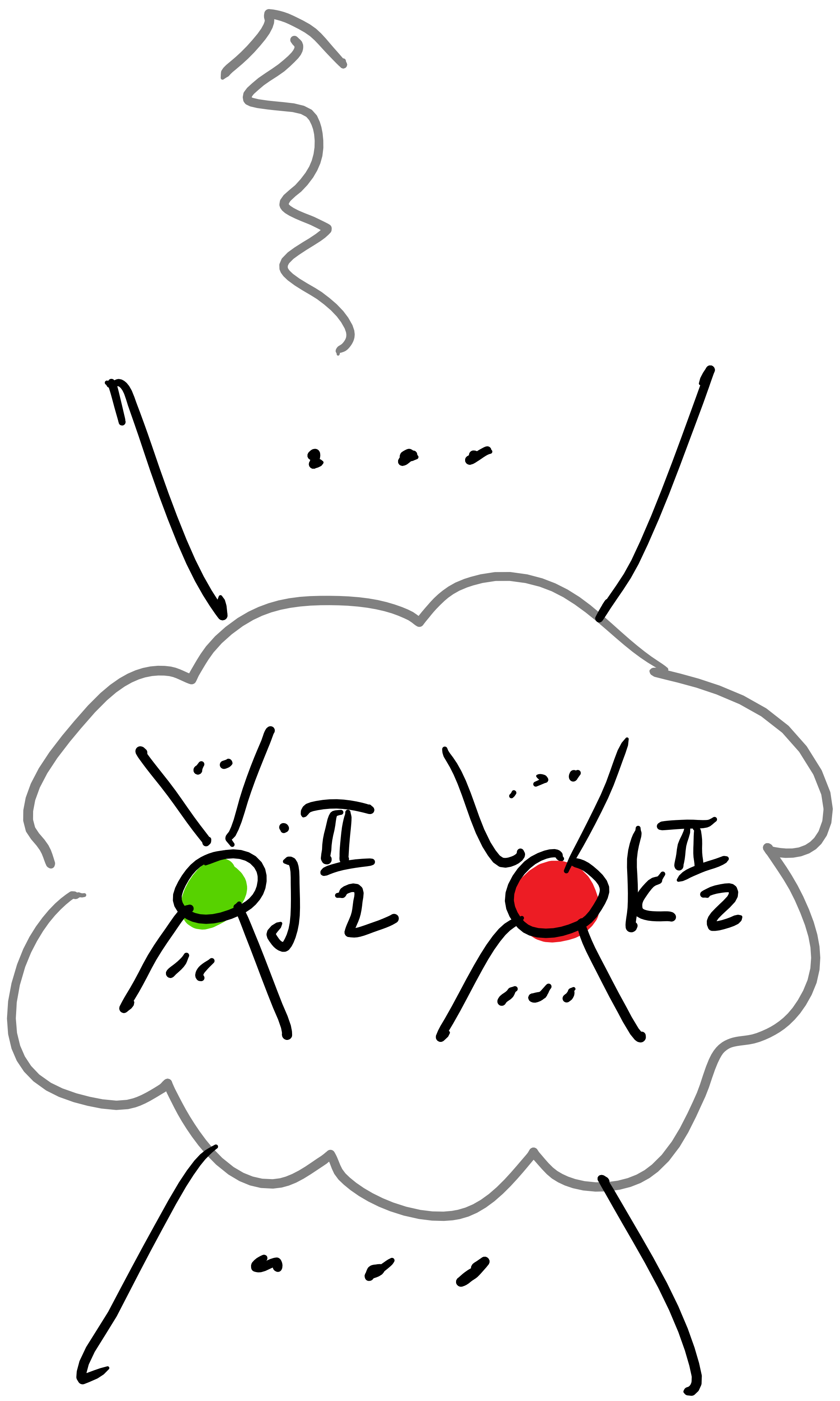






__THEOREM__ The ZX-calculus is __complete__ for __Clifford__ __ZX-diagrams__.

THEOREM The ZX-calculus is complete for Clifford ZX-diagrams.

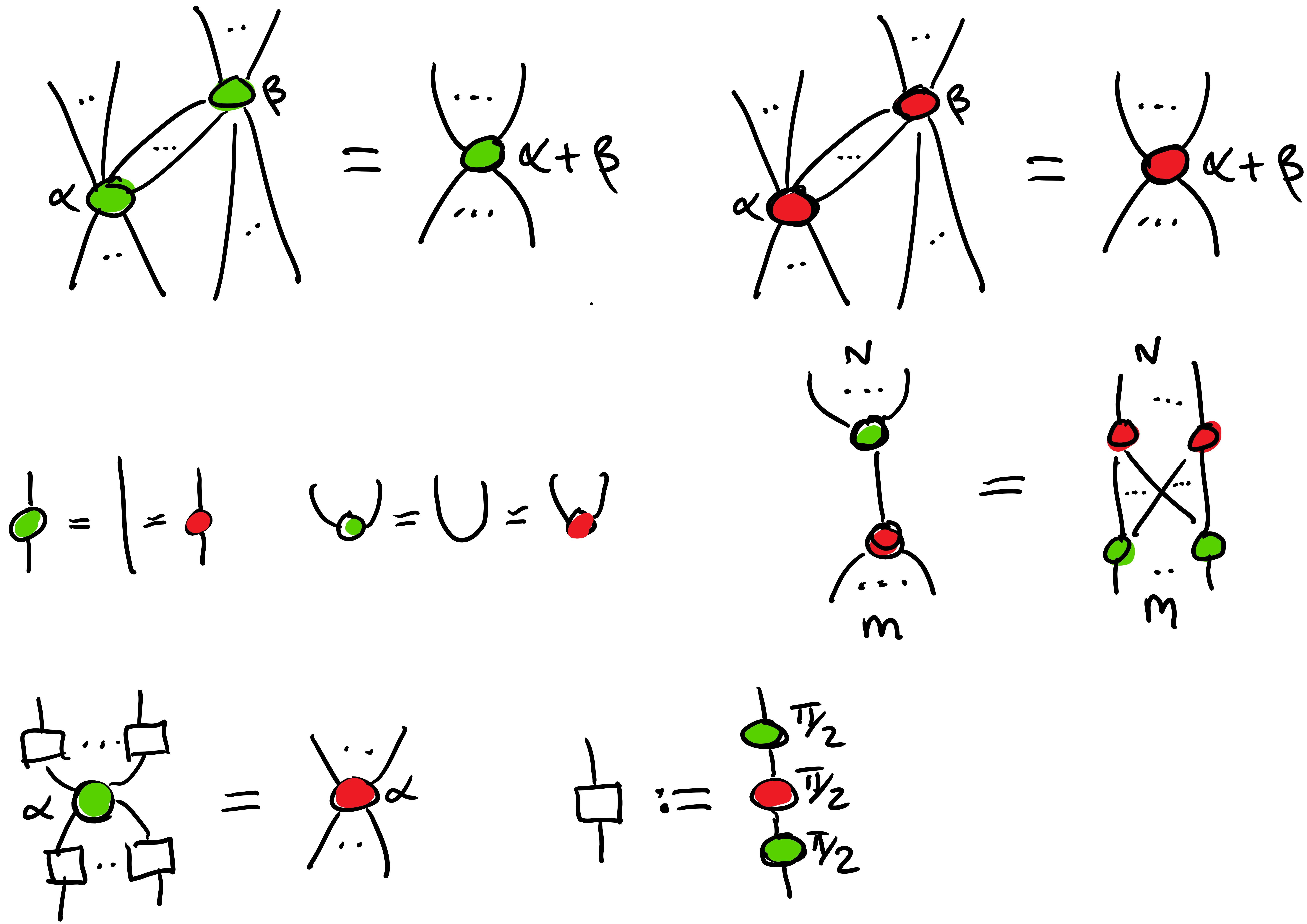


<u>THEOREM</u> Equality of Clifford ZX-diagrams is decidable in poly time.

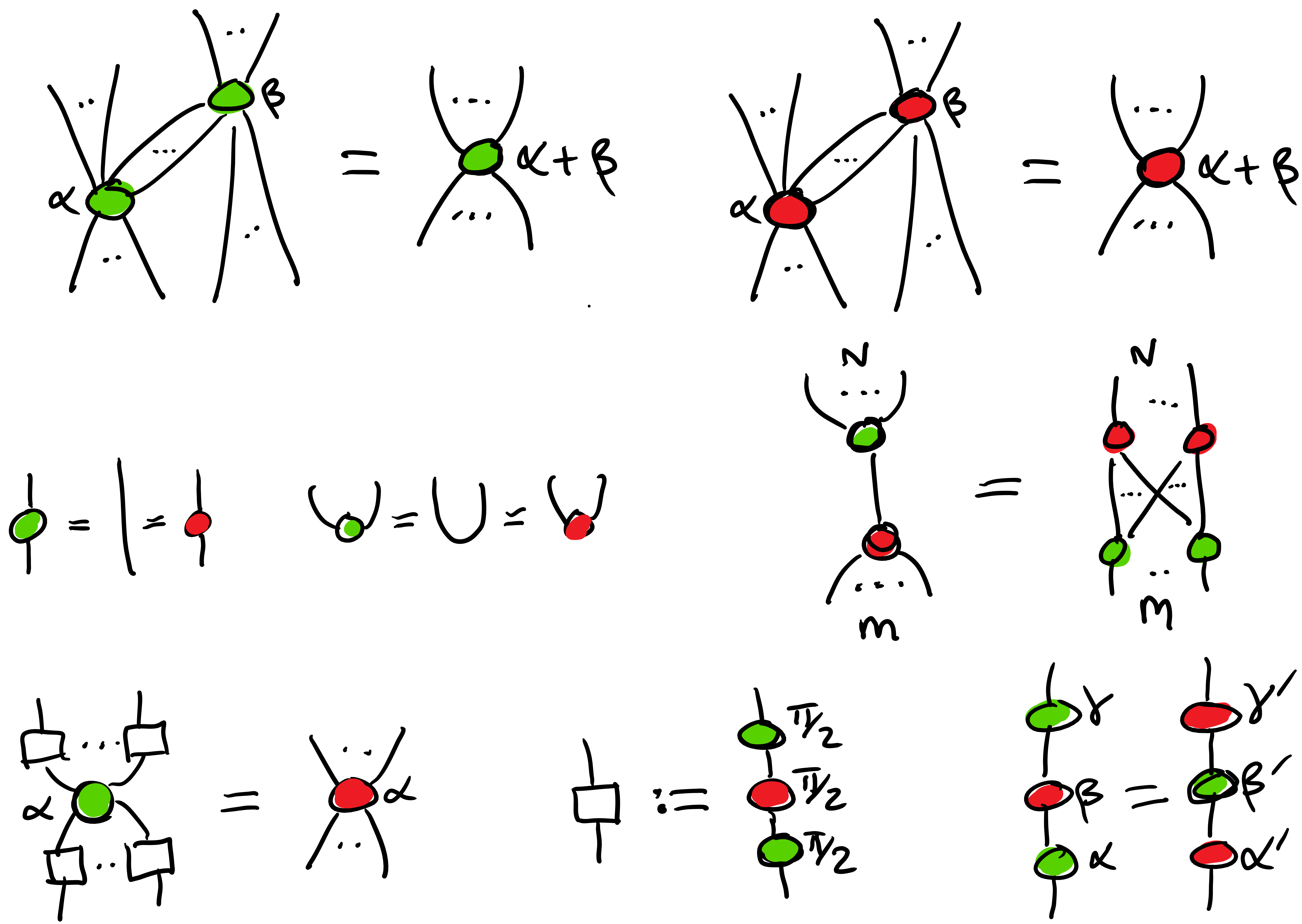
$\Rightarrow$ Quantum computations involving $\pi/2$ phases can be simulated efficiently on a classical computer.

* Gottesman-Knill theorem

ZX-calculus



UNIVERSAL ZX-calculus



<u>THEOREM</u> The universal ZX-calculus is complete for <u>all</u> ZX-diagrams.

THEOREM The universal ZX-calculus is complete for all ZX-diagrams.

* Wang & Ng 2017
* PRESENTATION Vilmart 2018

→ equality is decidable, but NOT efficient

$\rightarrow$ equality is decidable, but NOT efficient

(unless $BQP \subseteq P$)

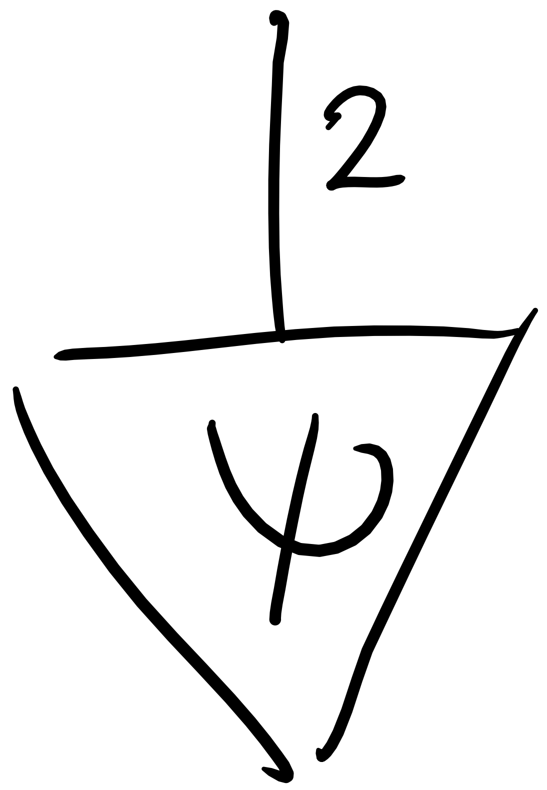
quantum efficient

QUANTUM THEORY

qubit

QUANTUM THEORY

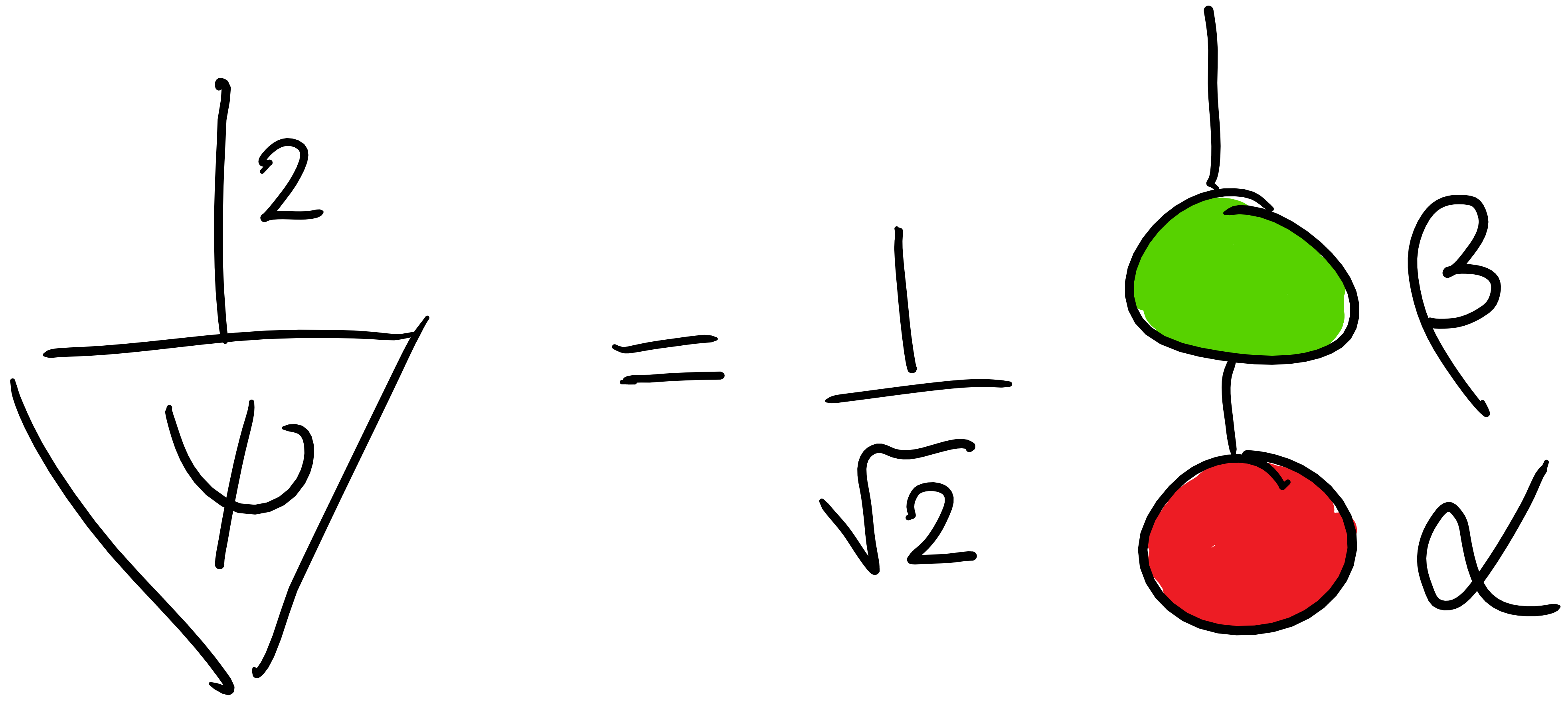
QUANTUM STATES



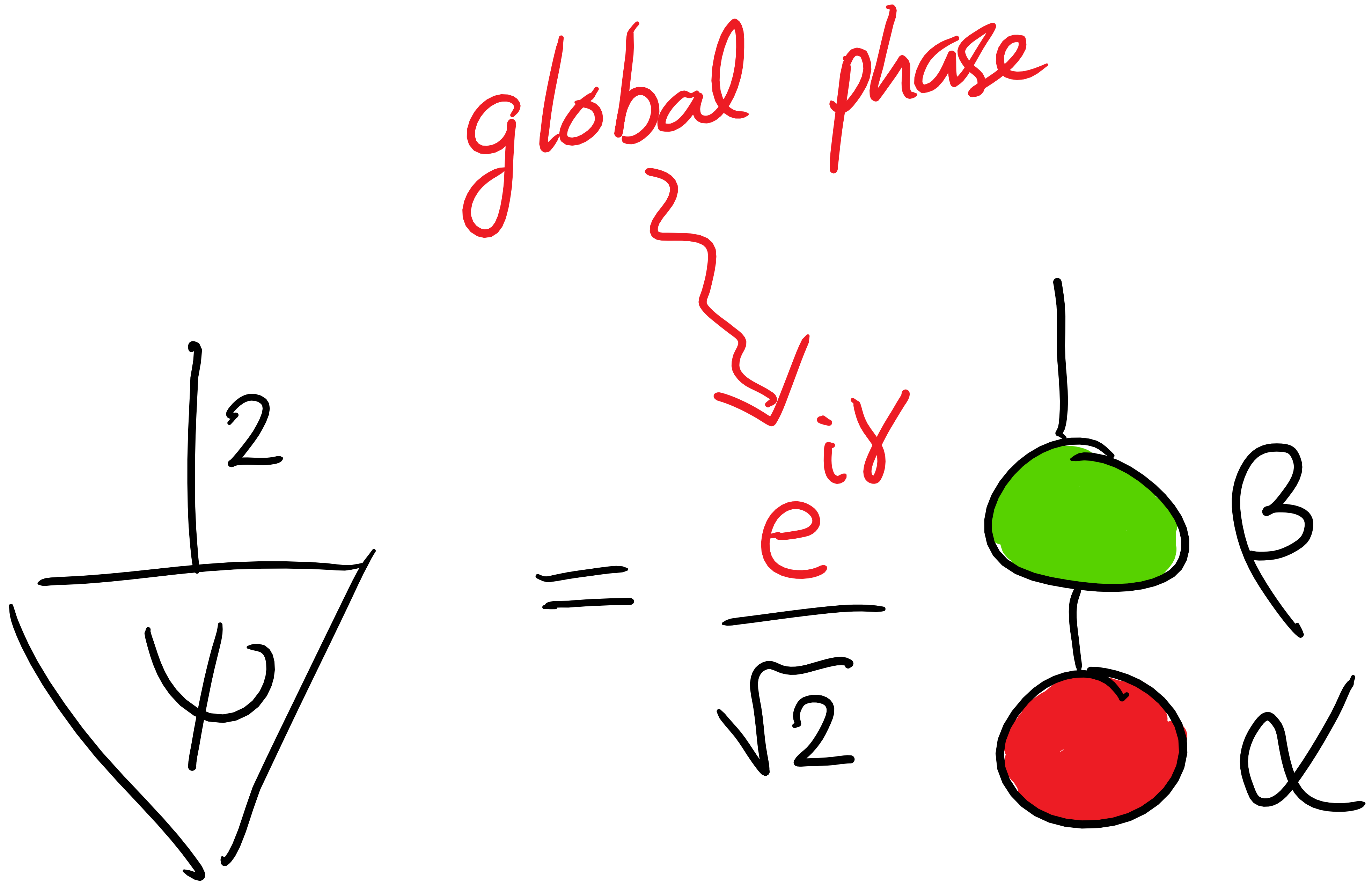
$$= \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

normalised

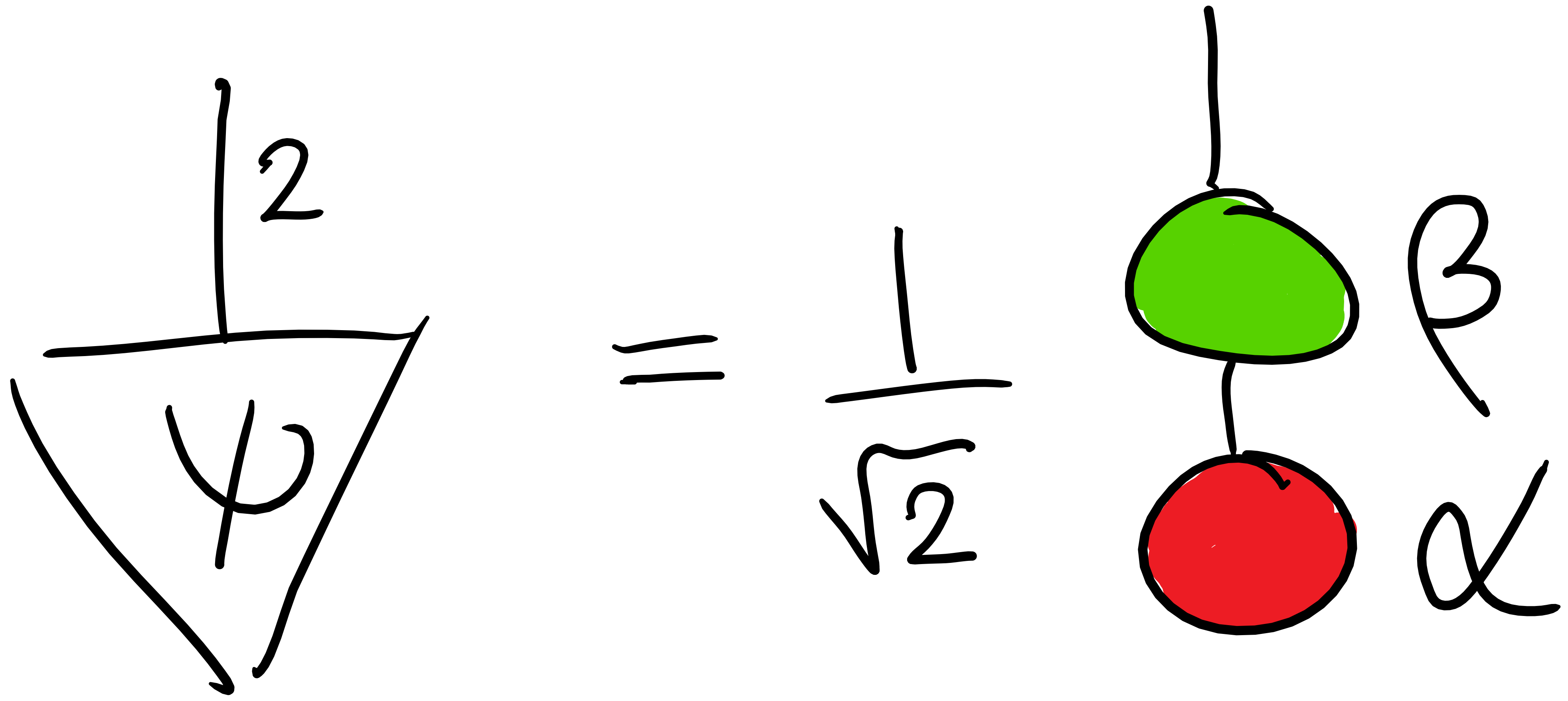
Quantum States



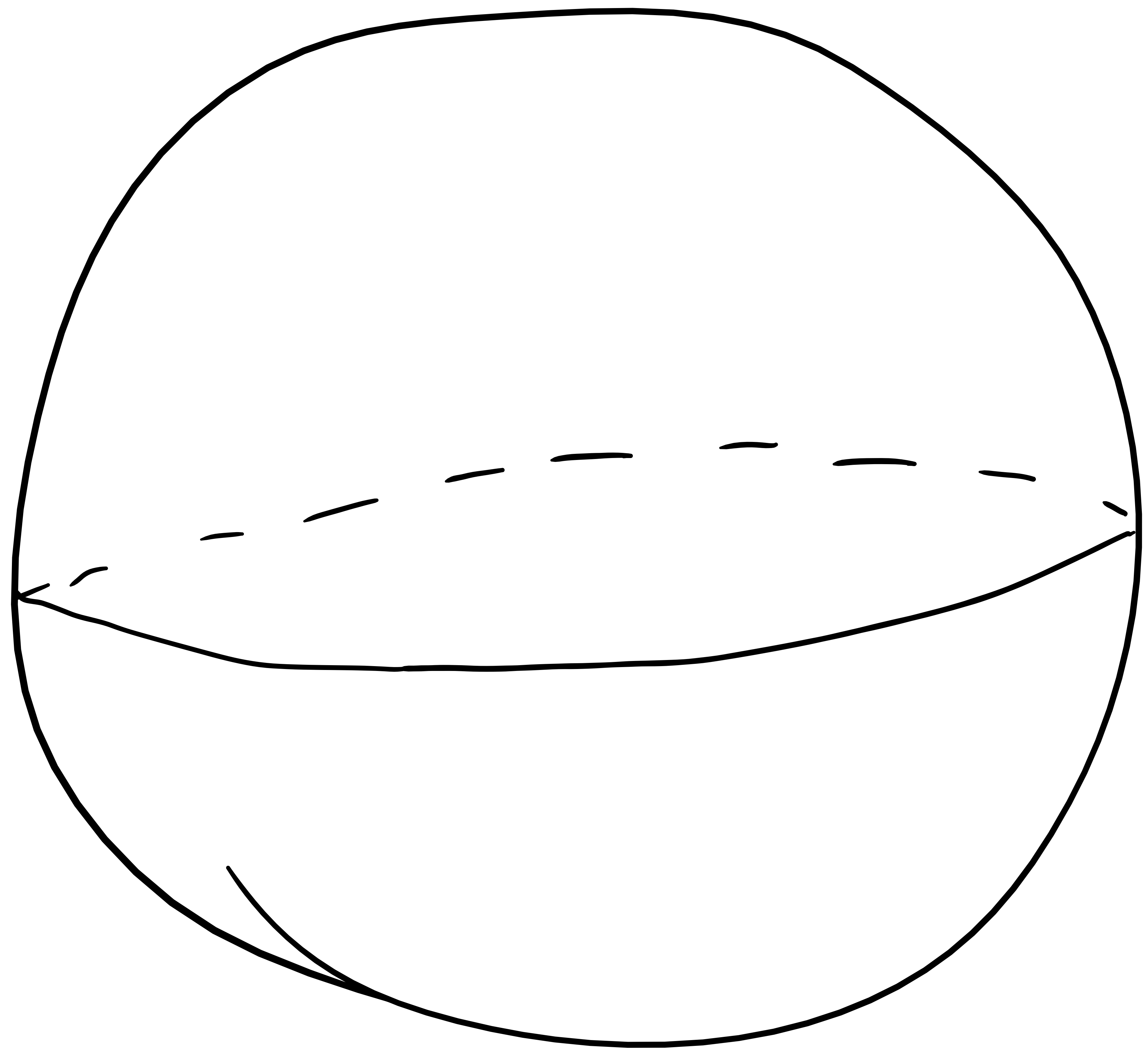
QUANTUM STATES



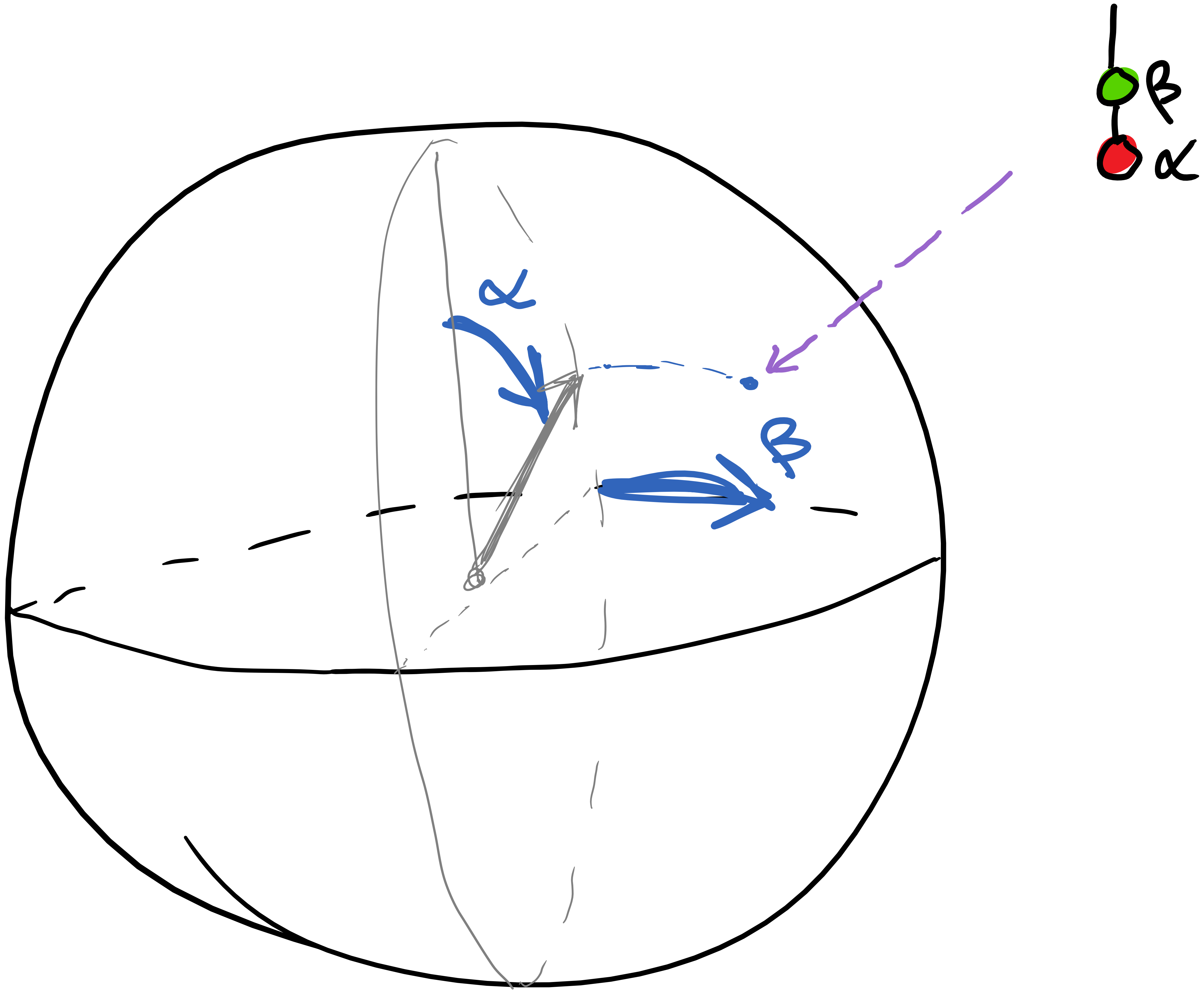
Quantum States



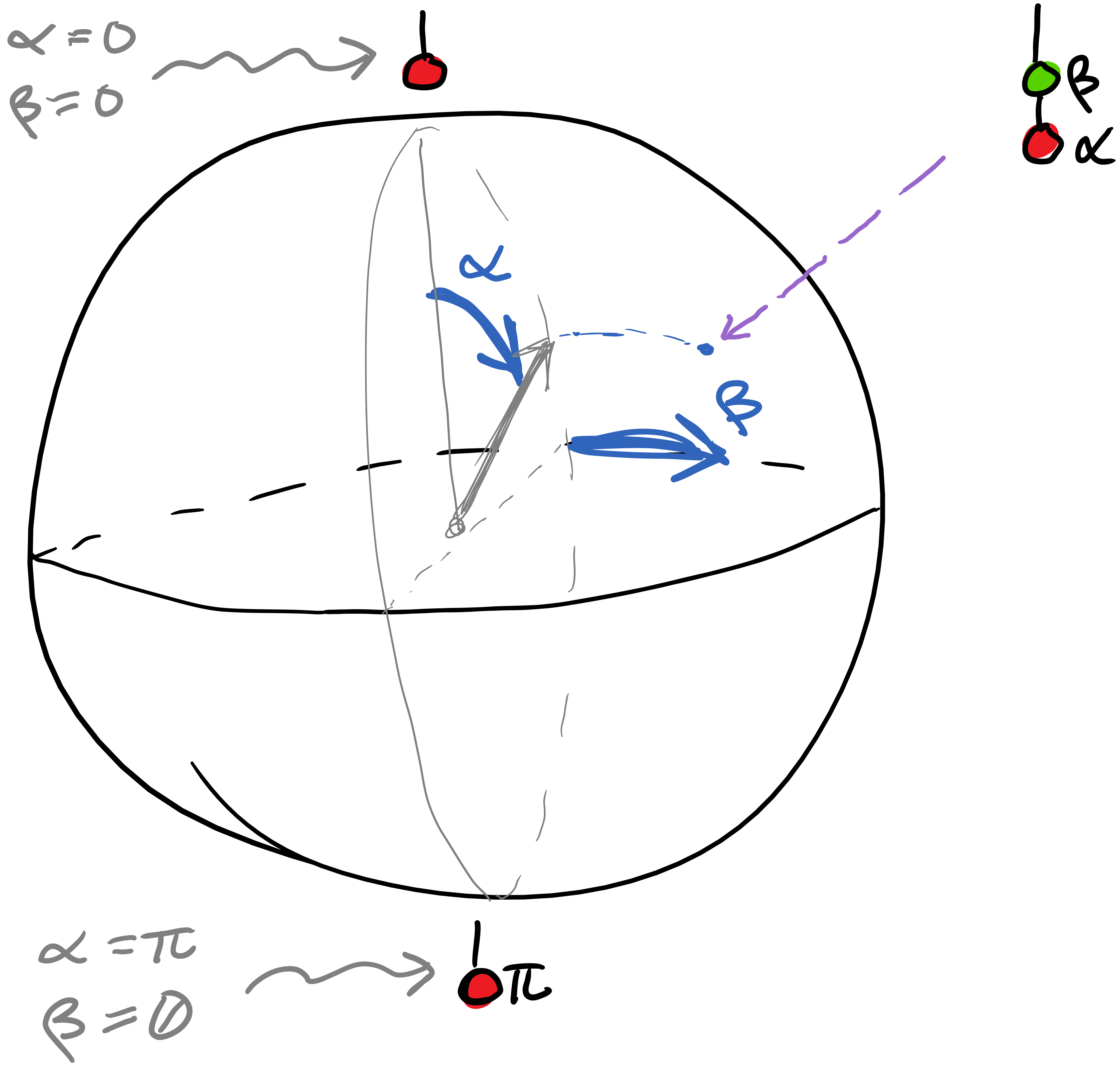
Bloch sphere



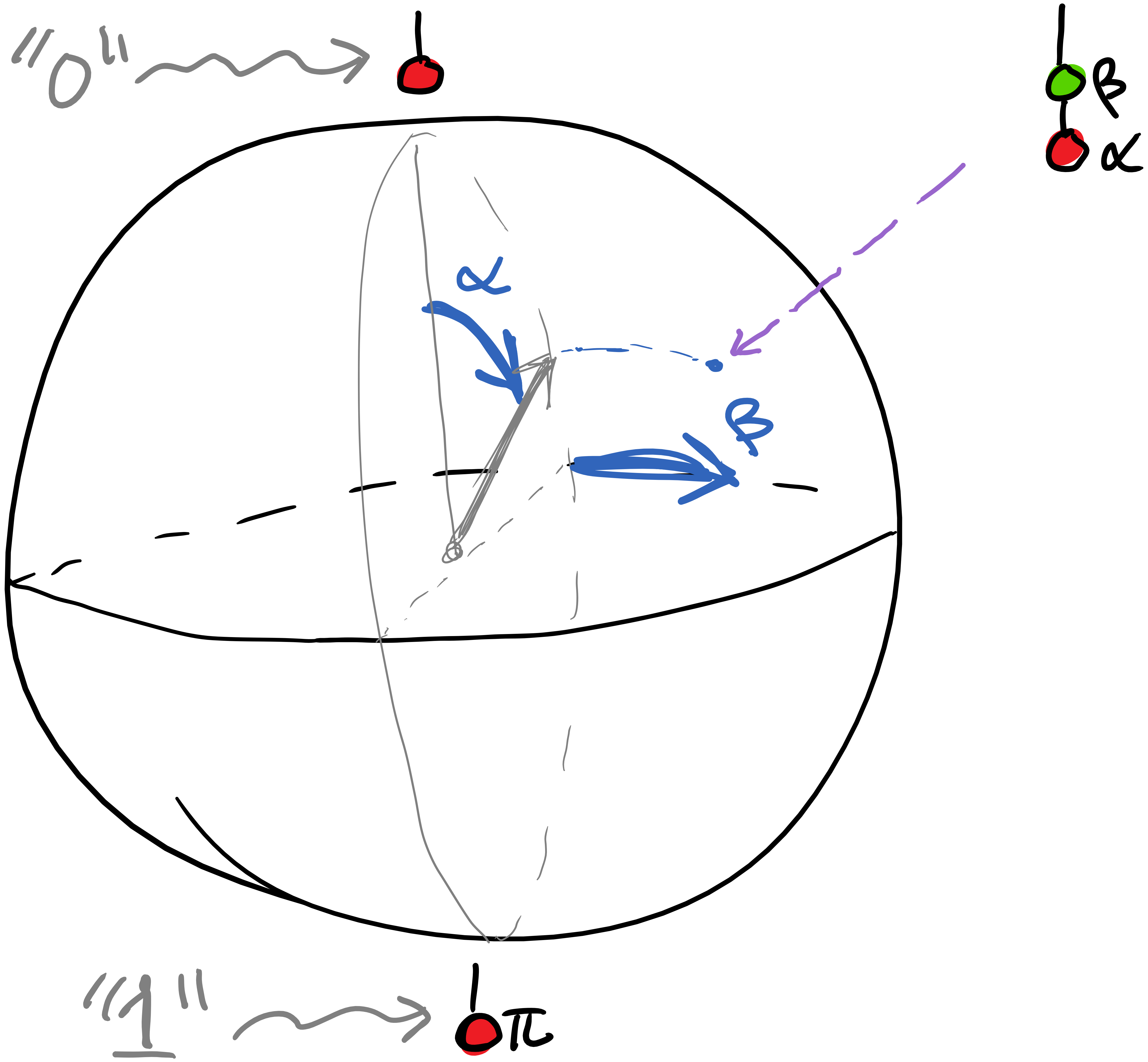
Bloch Sphere



Bloch Sphere



Bloch Sphere



Bits

Qubits

BITS	QUBITS
2 States: ∘ 0 ∘ 1	

Bits

2 States:

0

1

Qubits

a whole sphere of states:

Bits

2 States:

0

1

logic gates :=

ANY function $\{0,1\} \to \{0,1\}$

Qubits

a whole sphere of states:

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Qubits

a whole sphere of states:

quantum logic gates :=

unitary matrices $\leftrightarrow$ rotations

BITS

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CAN BE READ AT **ANY** TIME

QUBITS

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CAN ONLY BE READ BY **QUANTUM MEASUREMENT**

BITS

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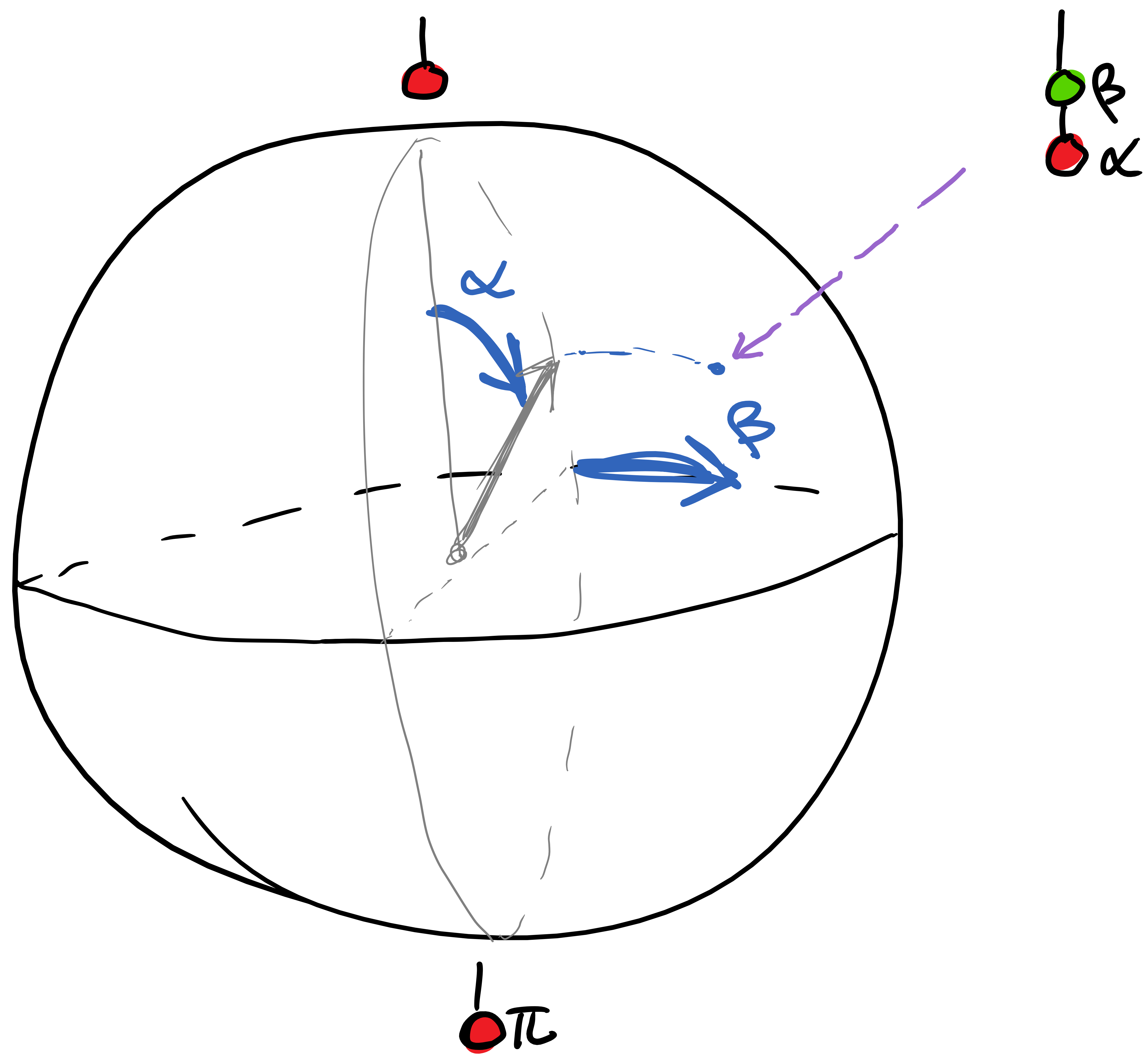
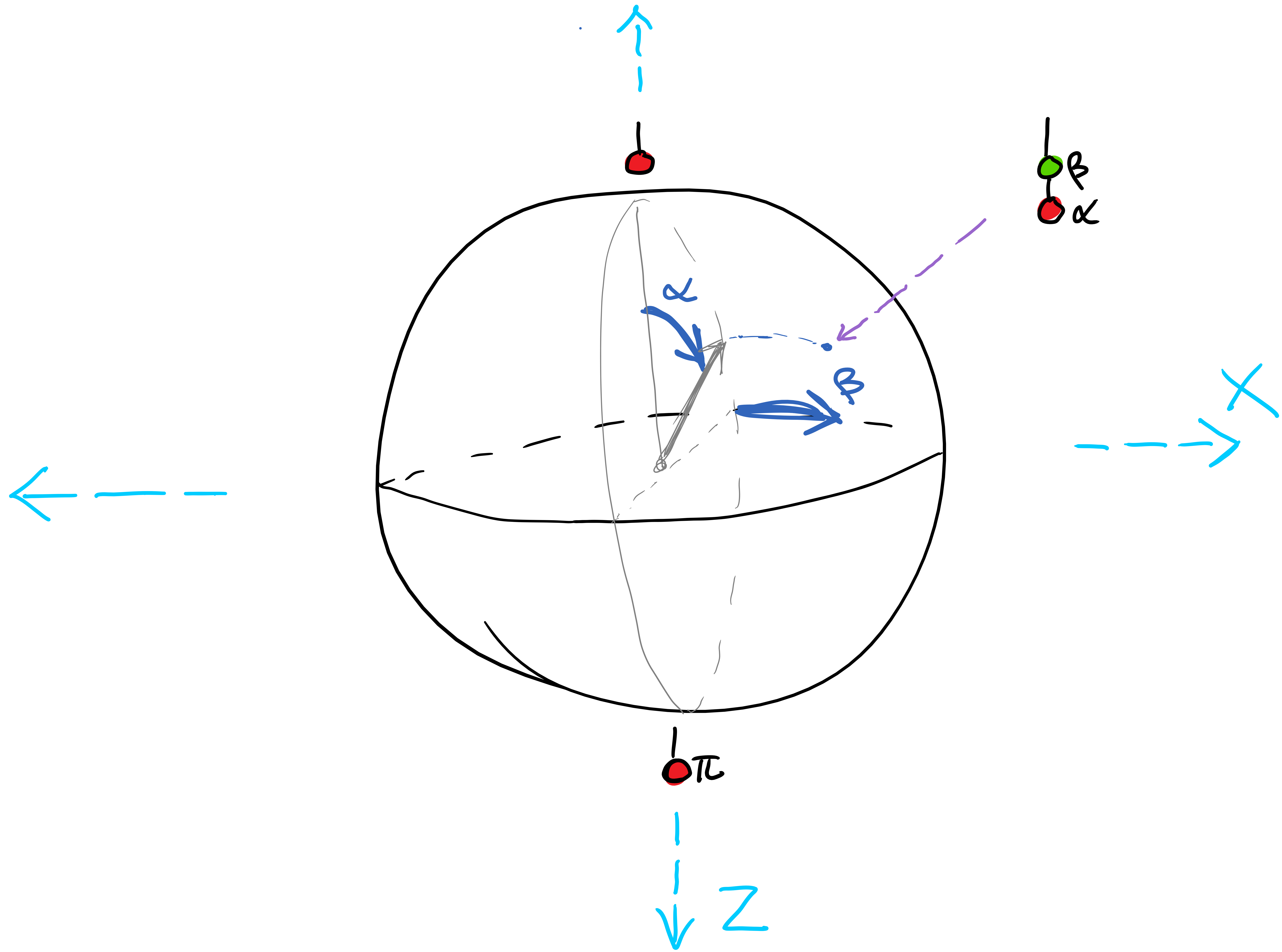
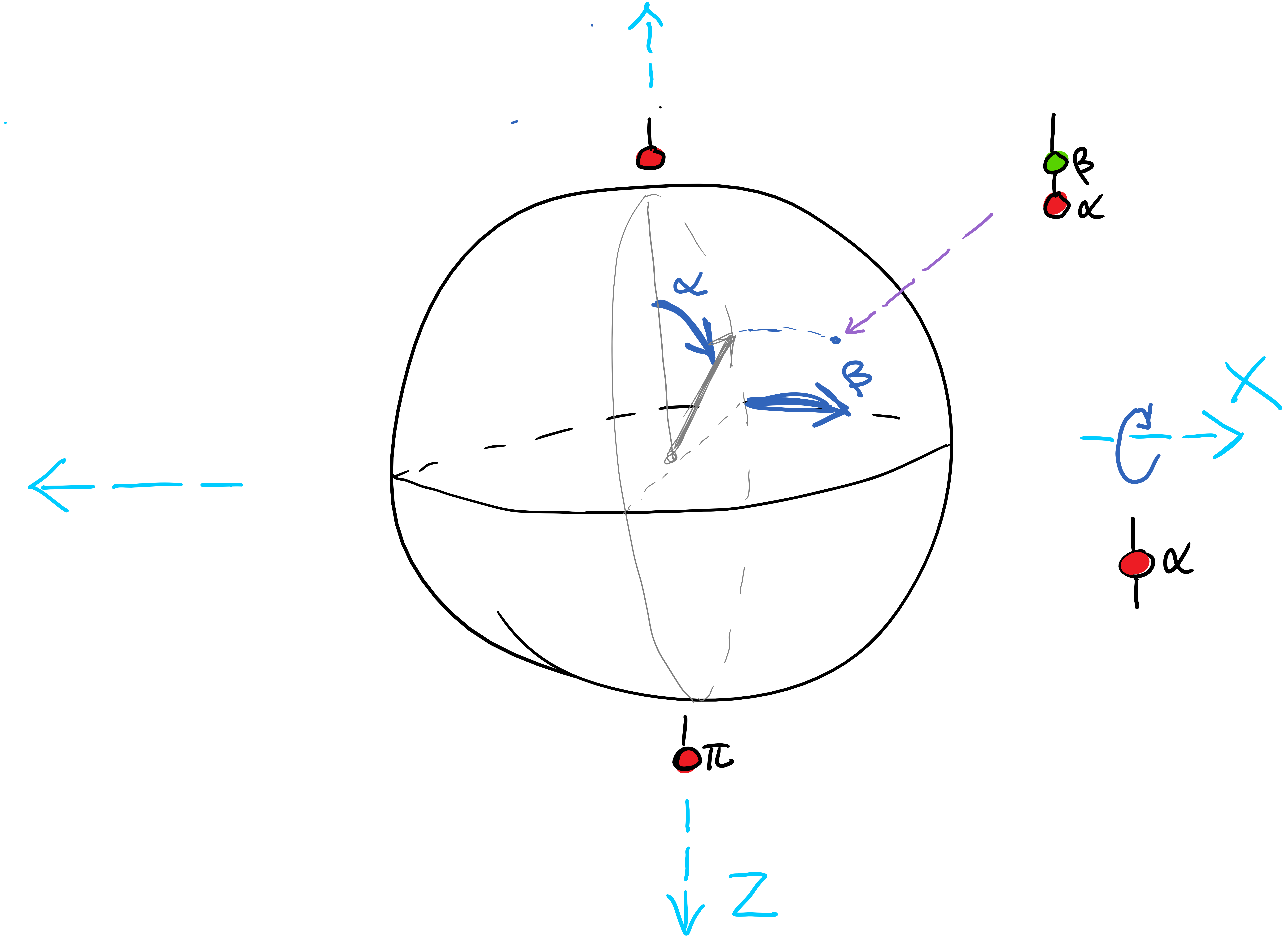
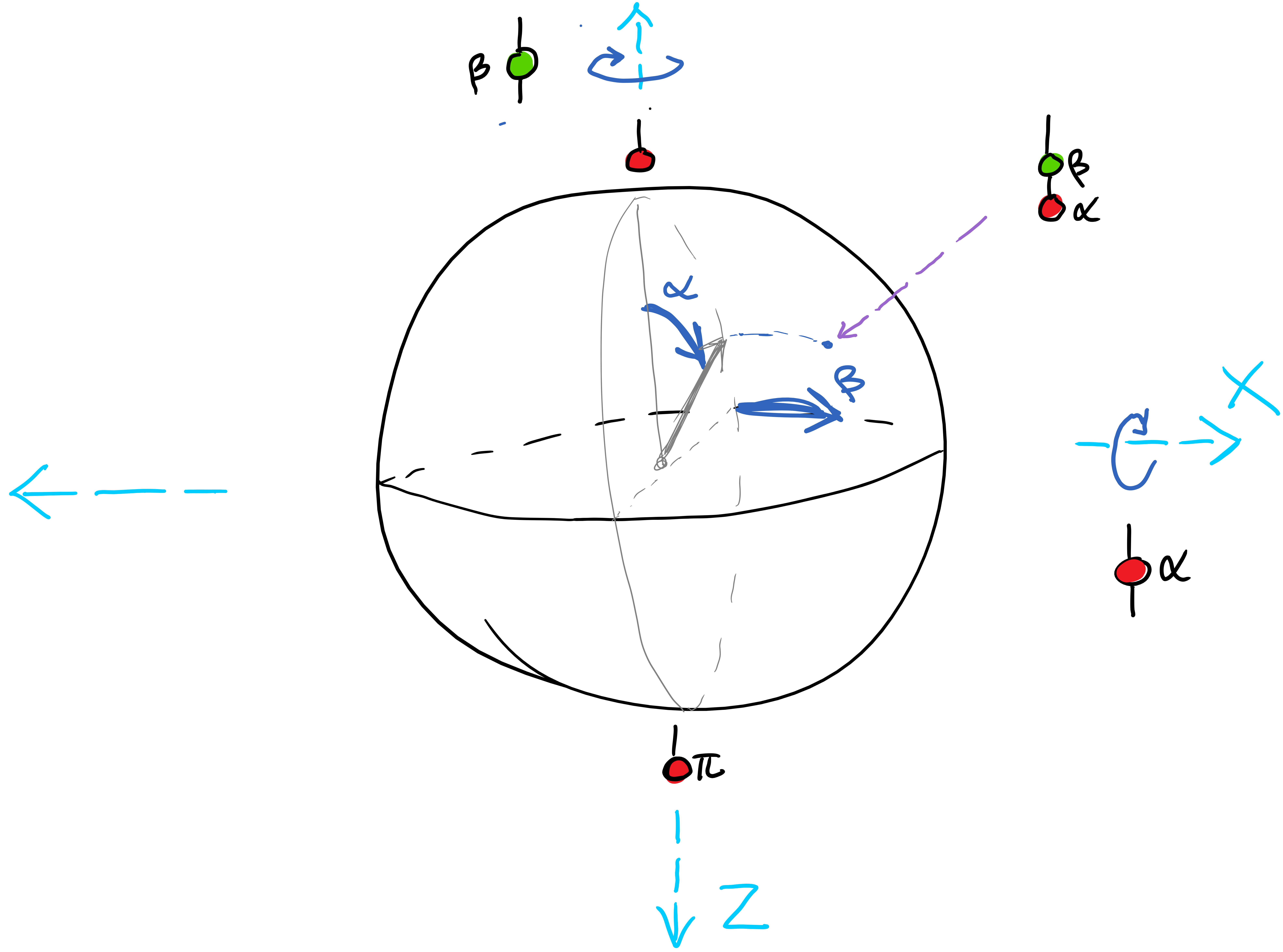

β
α
α
β
π



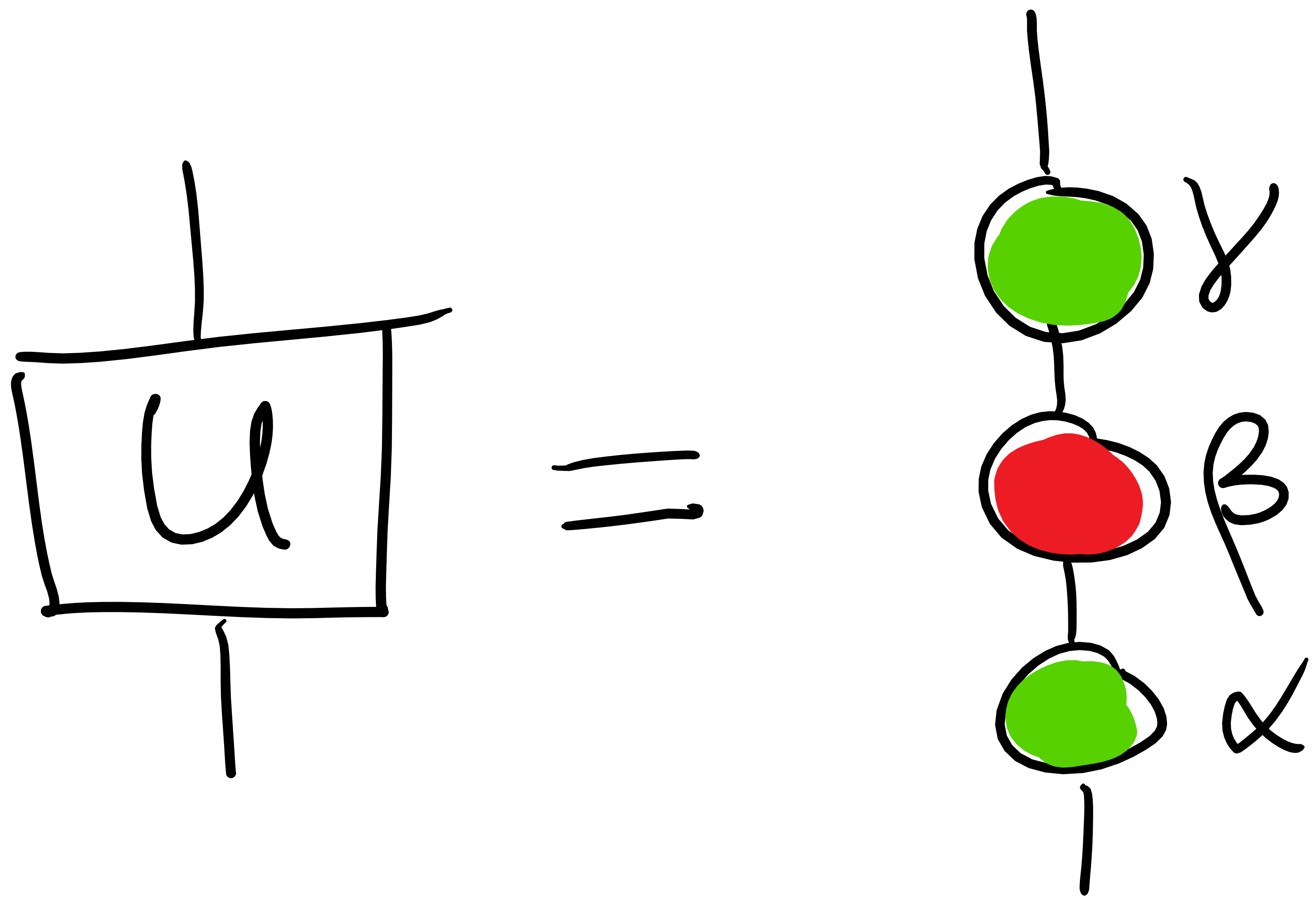
β
α
α
β
X
π
Z

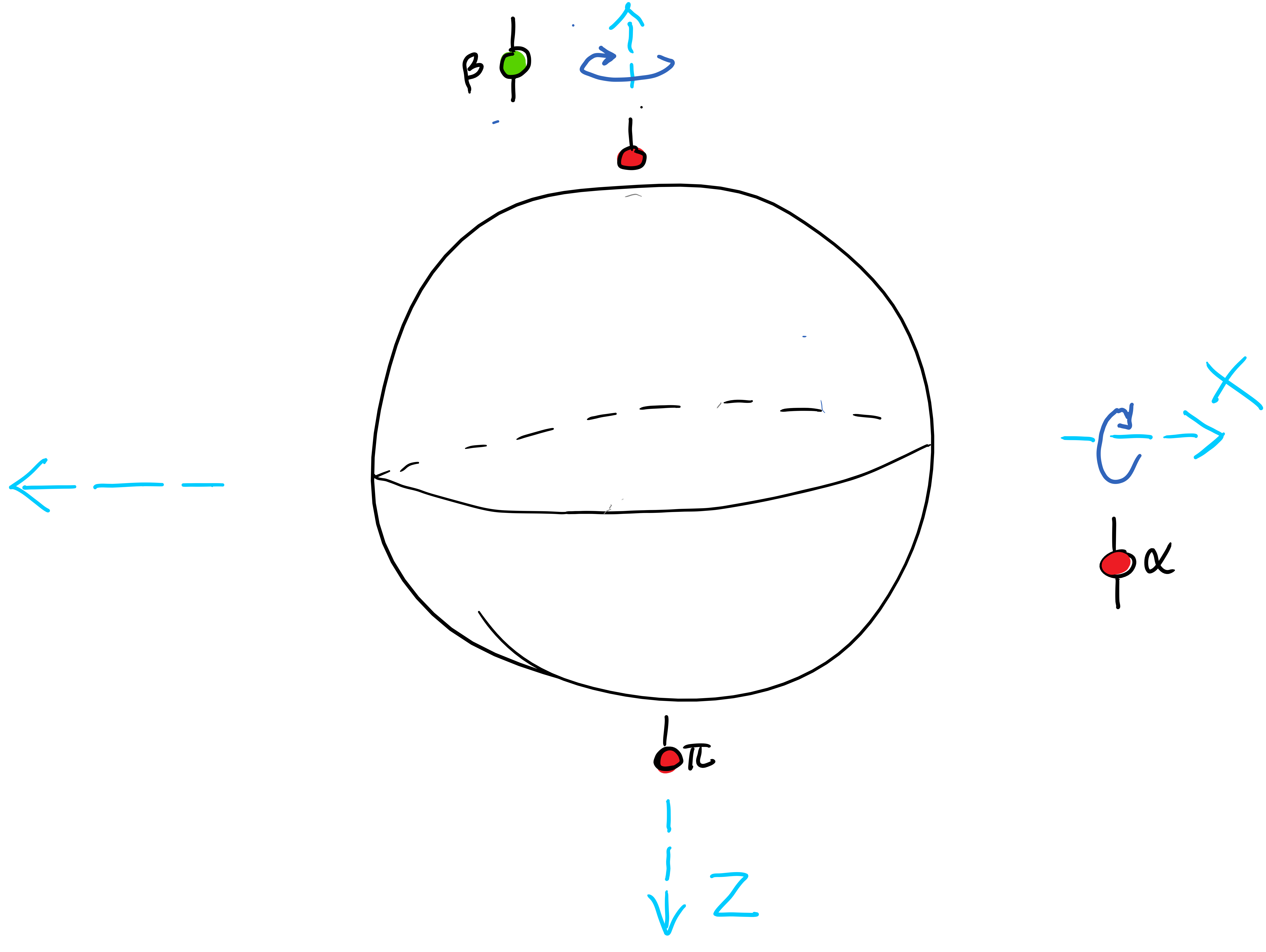


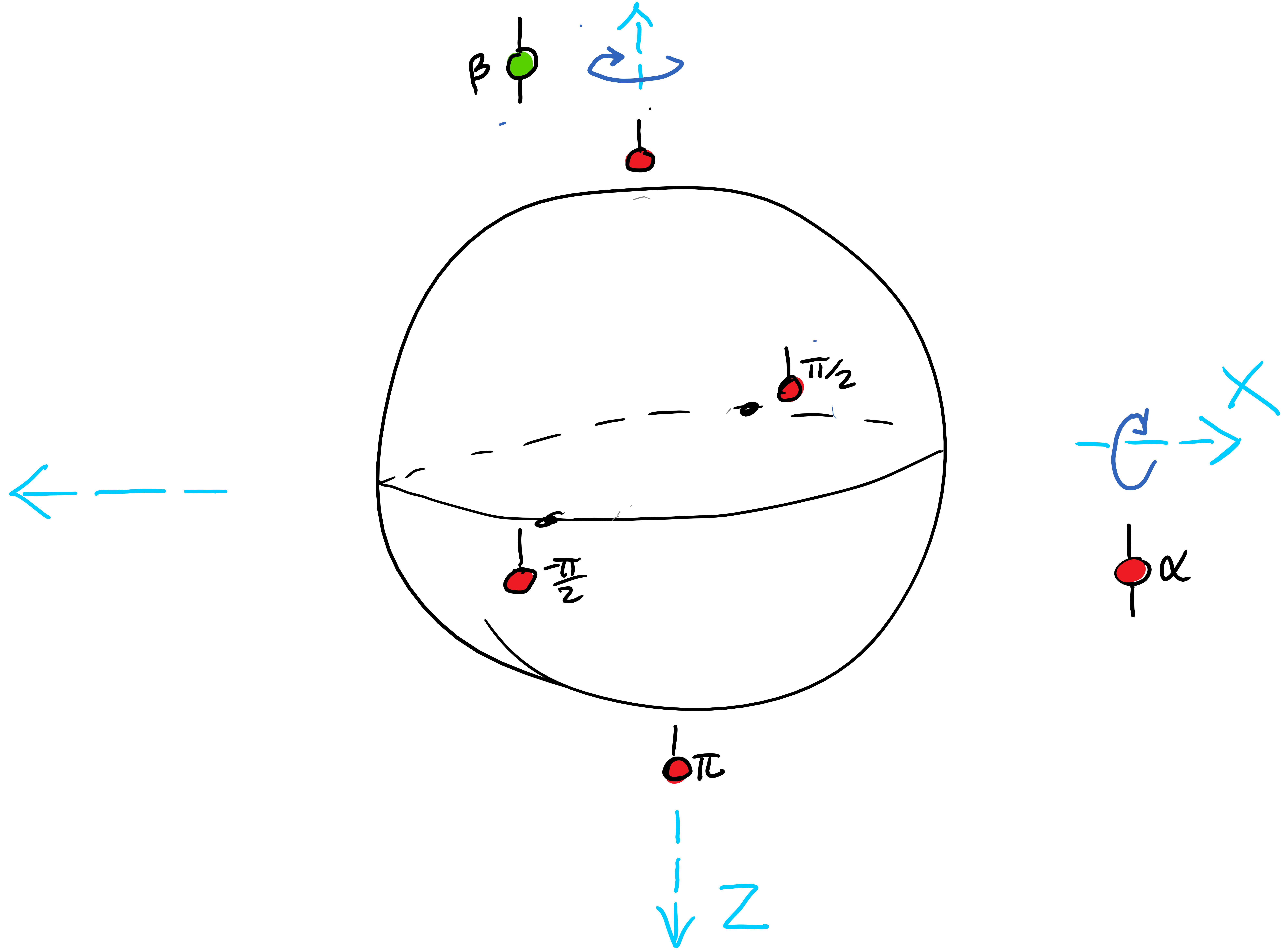
α
β
β
α
X
α
π
Z



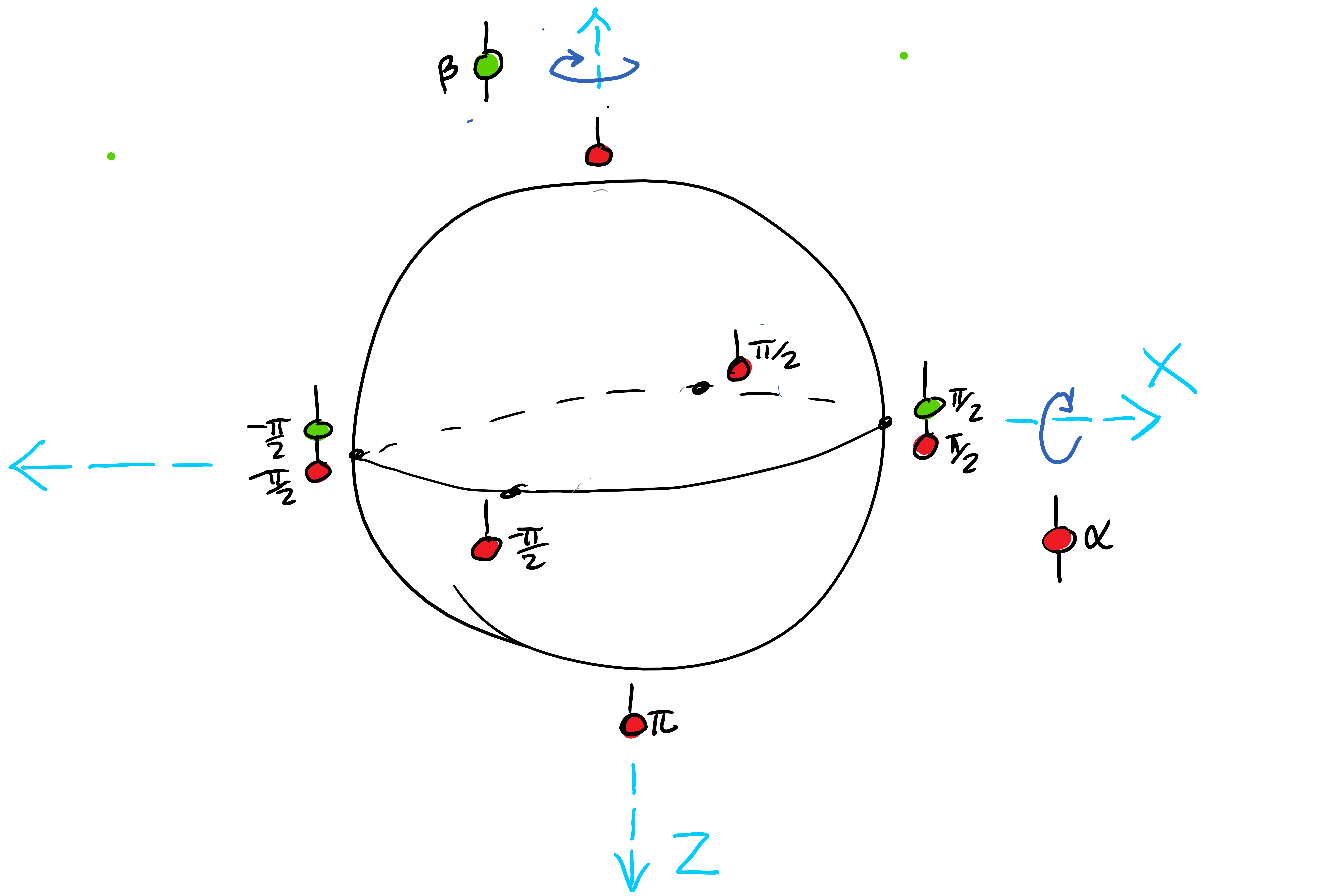
β
α
β
α
β
α
X
π
Z


U
γ
β
α

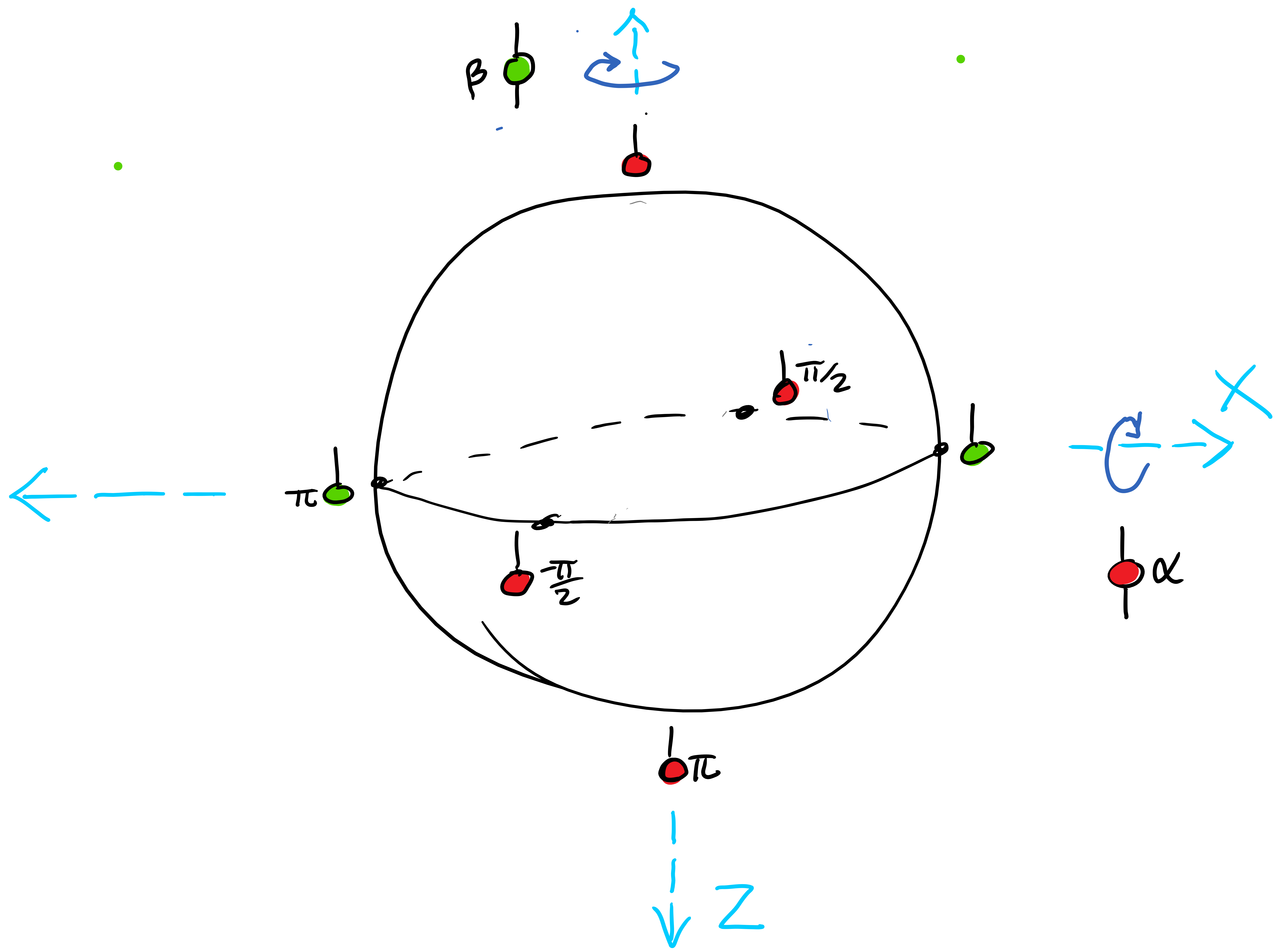

β
X
α
π
Z



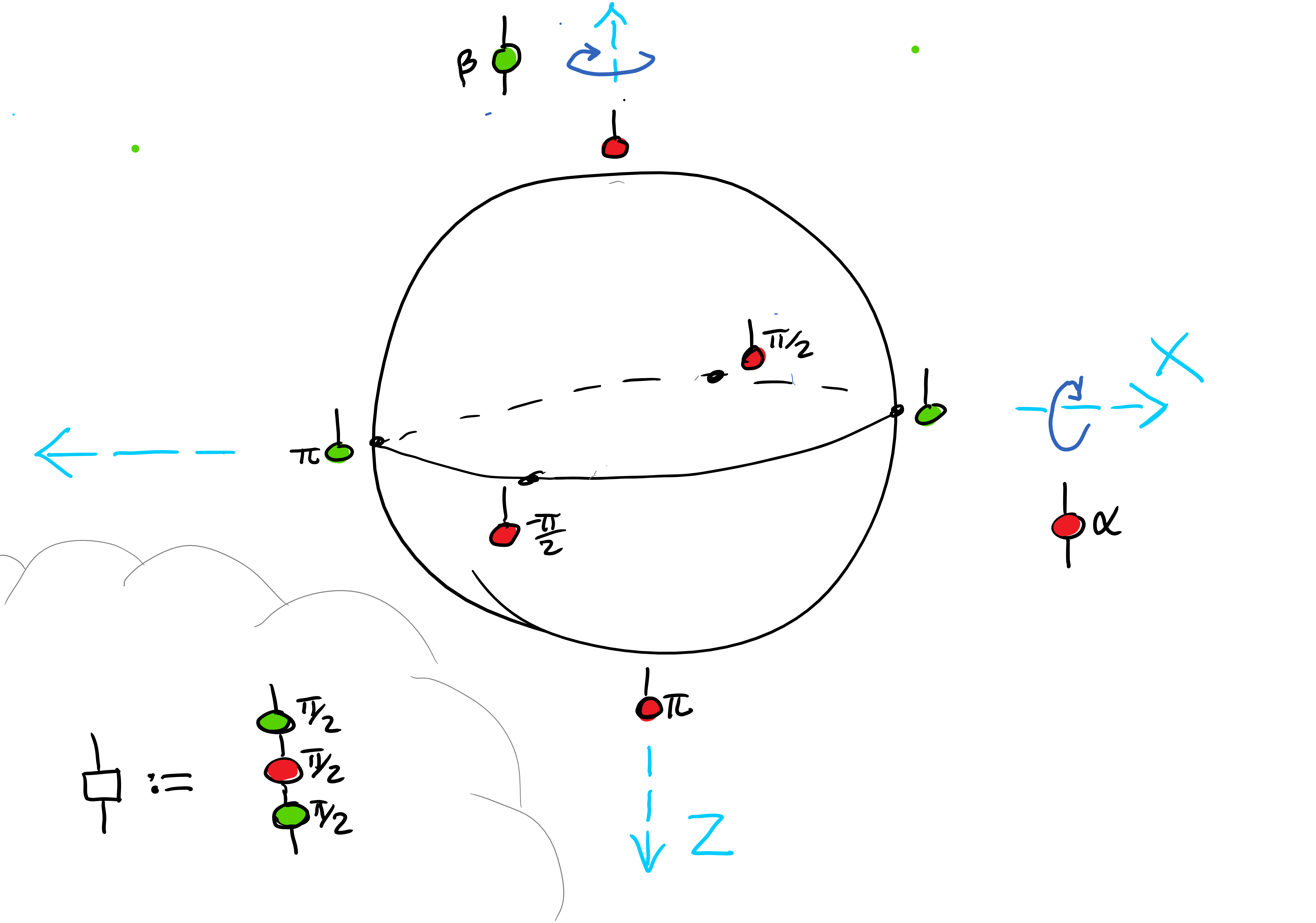
β
π/2
−π/2
π
X
Z
α



β
π/2
π/2
π/2
-π/2
-π/2
-π/2
α
π
X
Z



β
π/2
π
-π/2
α
X
π
Z

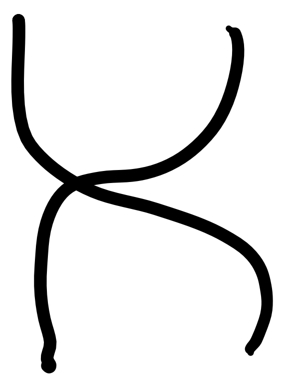


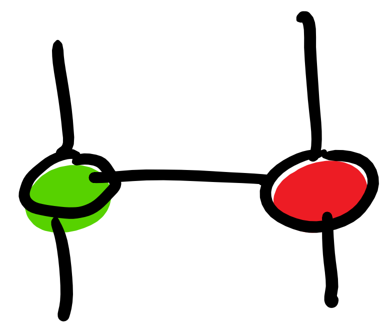
β
π/2
π
−π/2
α
X
Z
π
:=
π/2
π/2
π/2

MULTI-QUBIT LOGIC

BIGGER UNITARIES

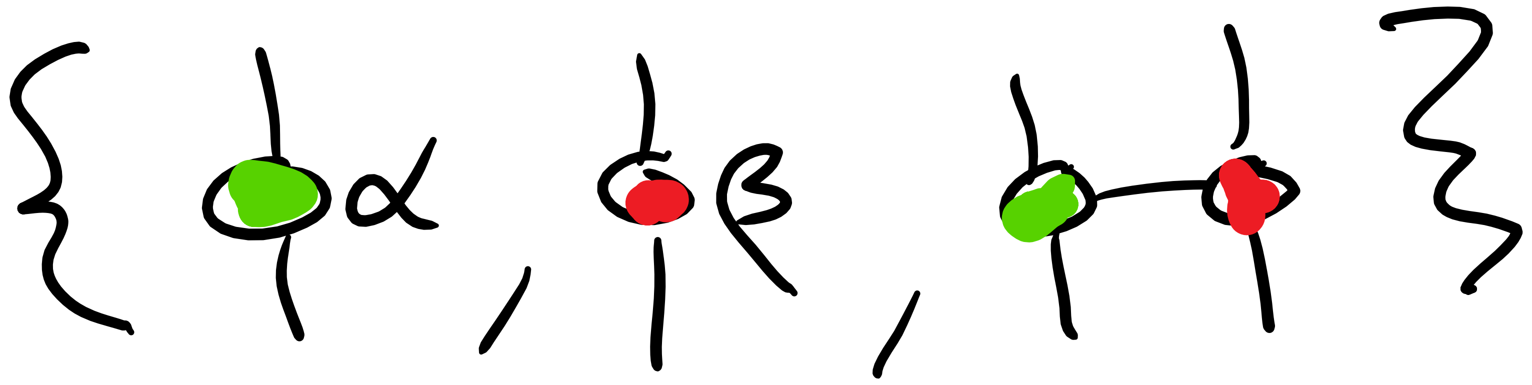
e.g.

SWAP → 

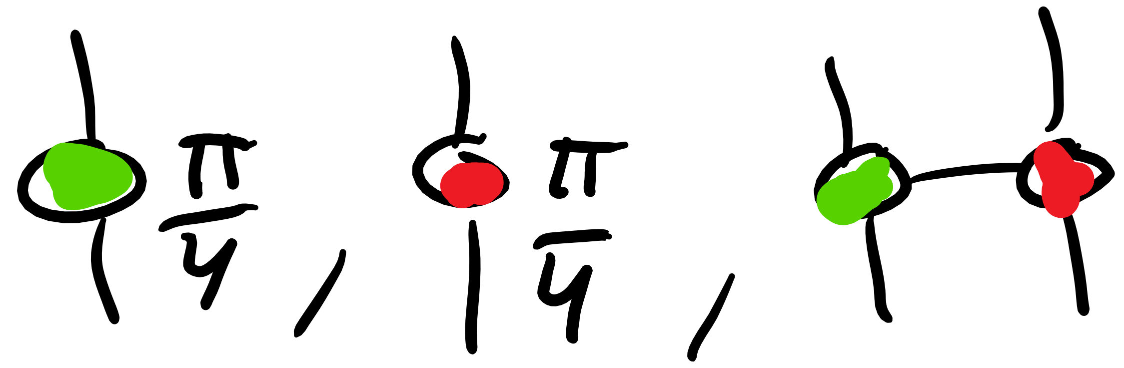
CNOT → 

DEF A *quantum circuit* is a unitary built from a fixed set of basic *gates*.

<u>Def</u> A <u>quantum circuit</u> is a unitary built from a fixed set of basic <u>gates</u>.

<u>Thm</u> The set  is universal for unitaries.

<u>DEF</u> A <u>quantum circuit</u> is a unitary built from a fixed set of basic <u>gates</u>.

<u>THM</u> The set $\{$  $\}$ is approx universal for unitaries.

Bits

2 States:

logic gates :=

Any function $\{0,1\} \to \{0,1\}$

CAN BE READ AT **ANY** TIME

Qubits

a whole sphere of states:

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unitary matrices $\Leftrightarrow$ rotations

CAN ONLY BE READ BY **QUANTUM MEASUREMENT**

QUANTUM MEASUREMENTS

* get classical data out of a system, probabilistically.
* destroy the system in the process.

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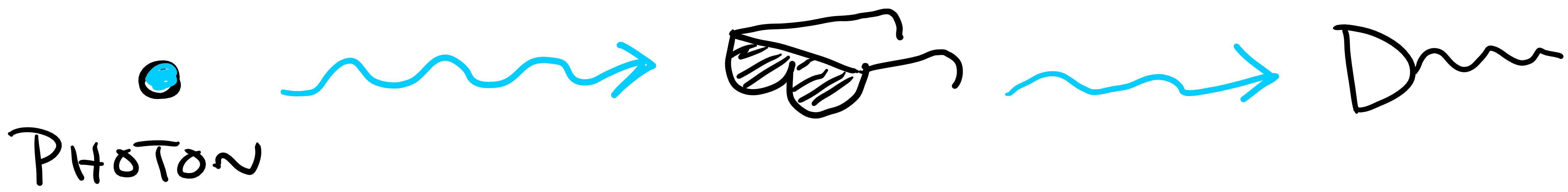

QUANTUM MEASUREMENTS

* get classical data out of a system, probabilistically.
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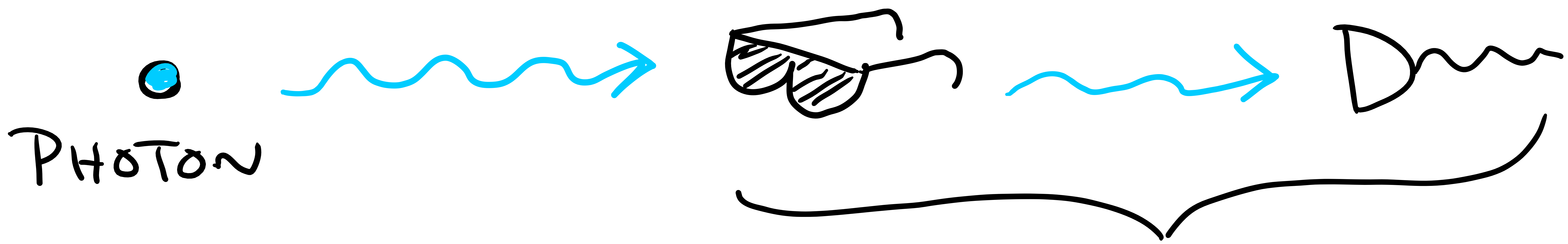
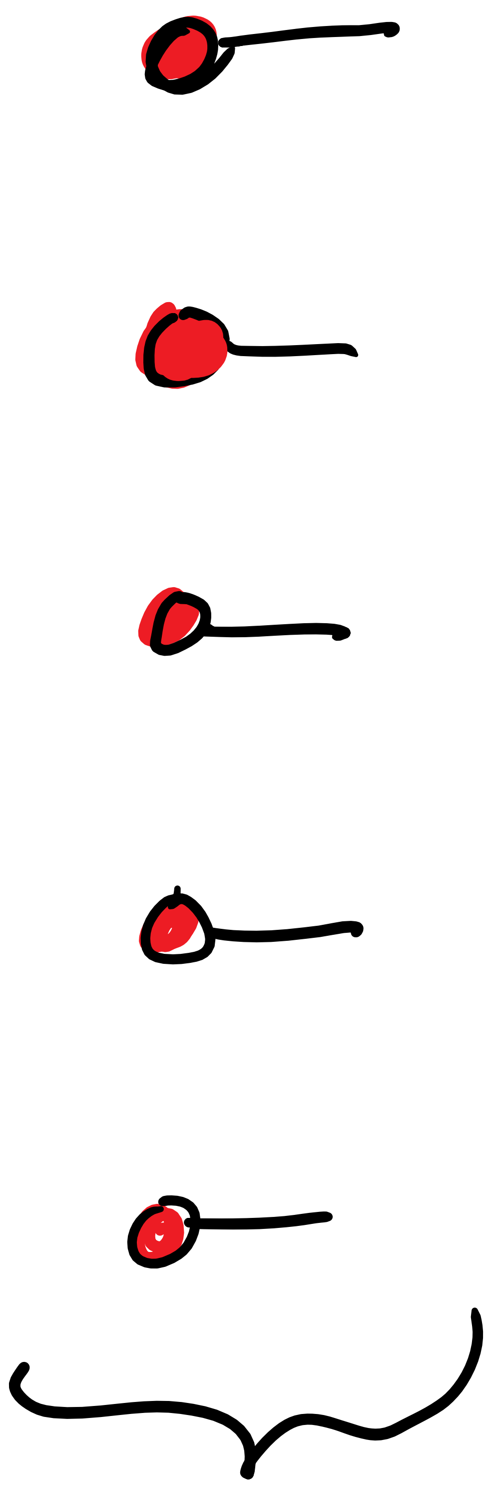


"NO CLICK" =: 0

"CLICK" =: 1

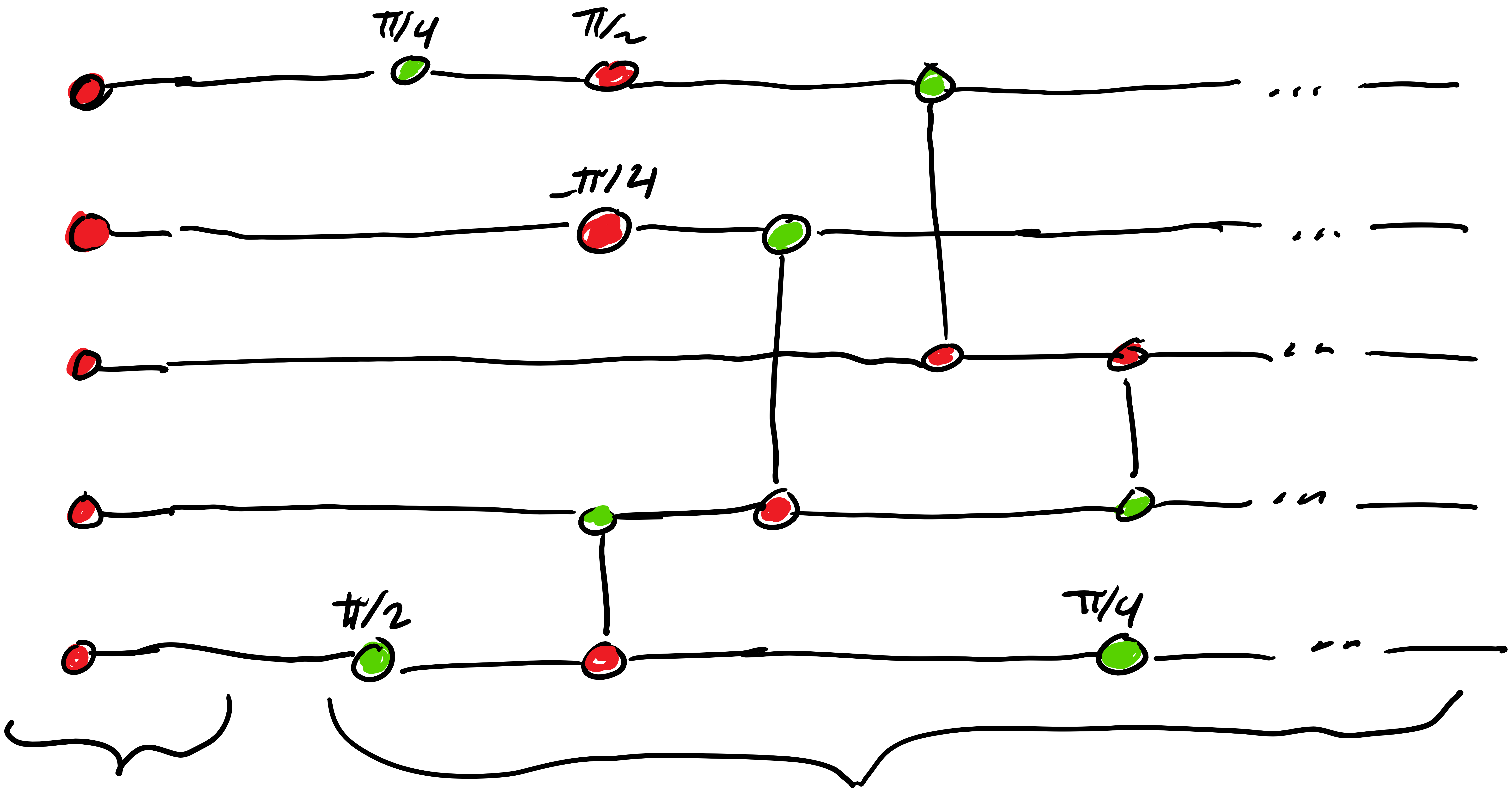
Quantum Circuit Model

Quantum Circuit Model



1. prepare a fixed state

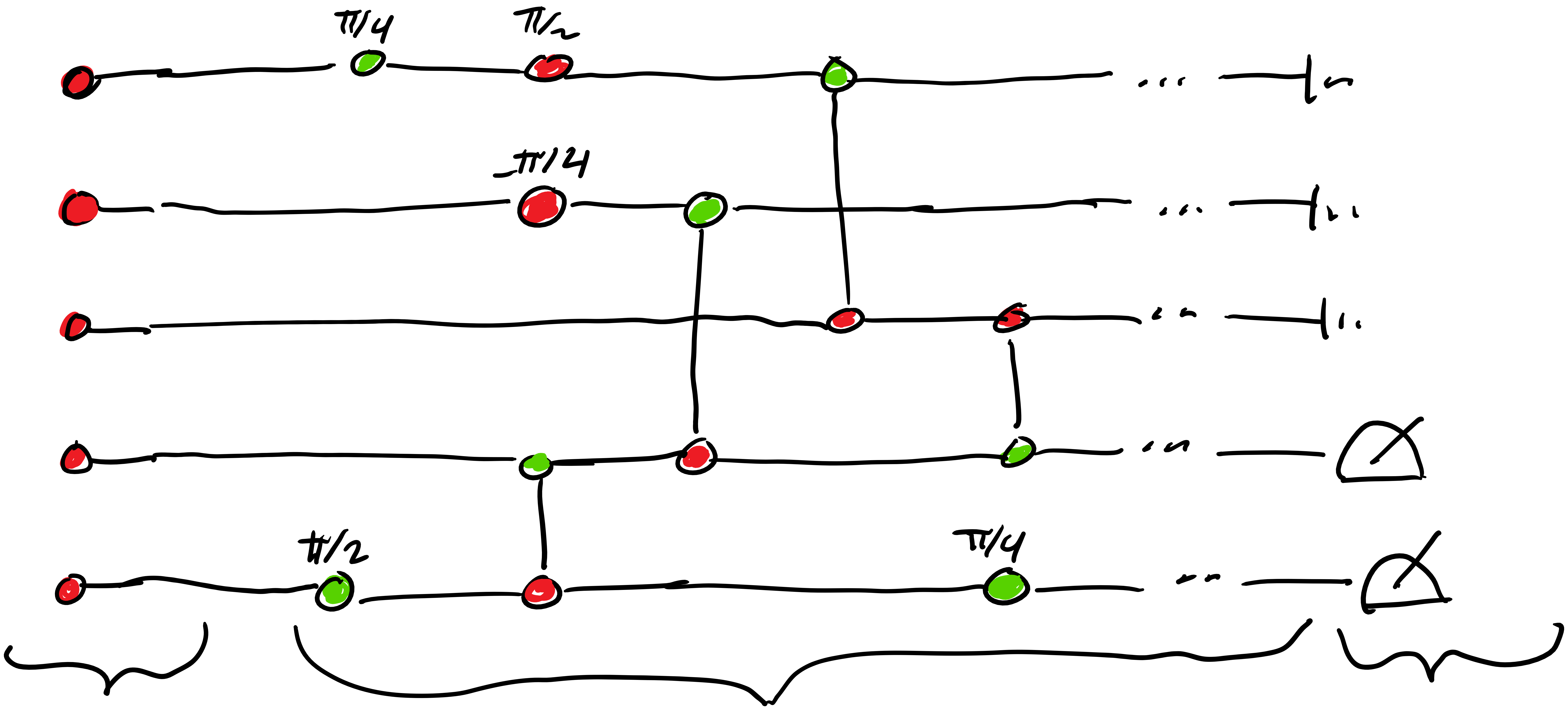
QUANTUM CIRCUIT MODEL



1. prepare a fixed state

2. apply quantum gates

Quantum Circuit Model

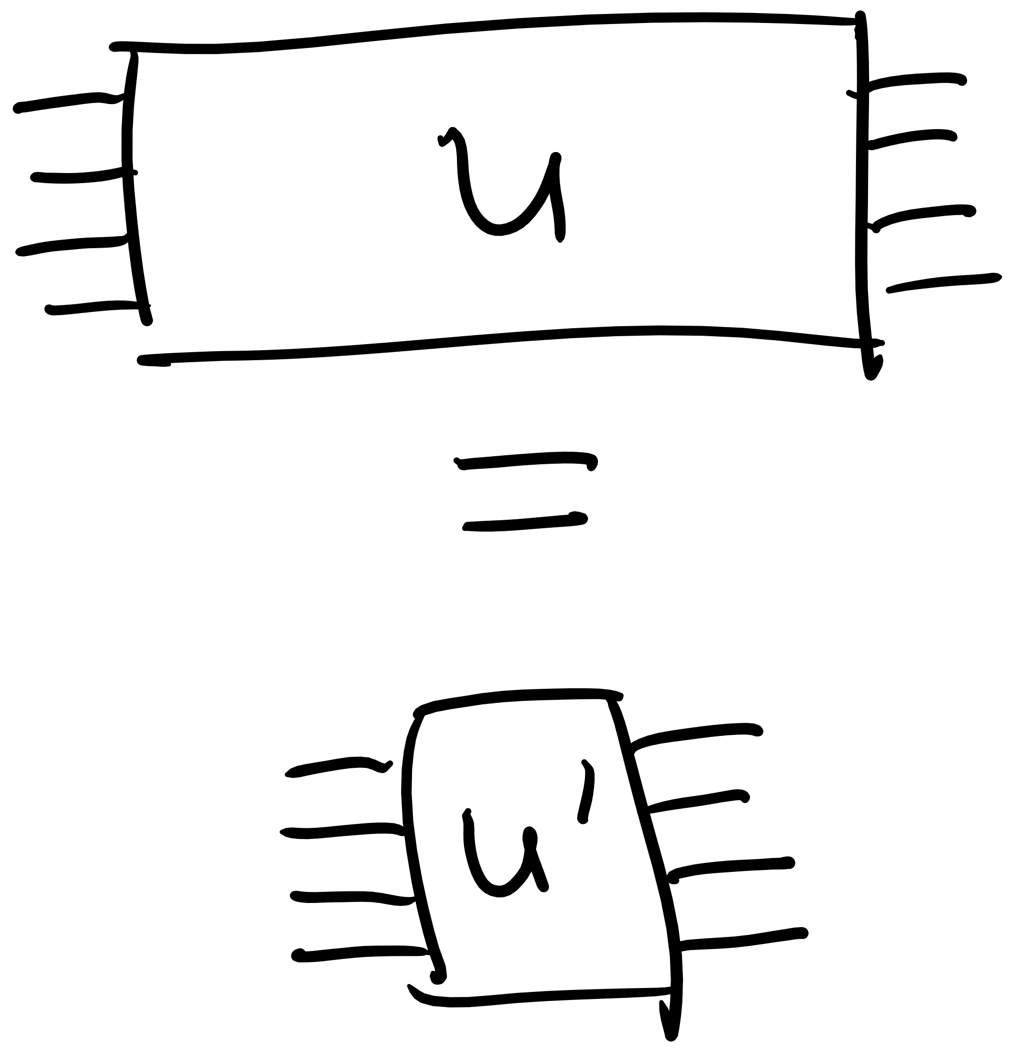


1. prepare a fixed state

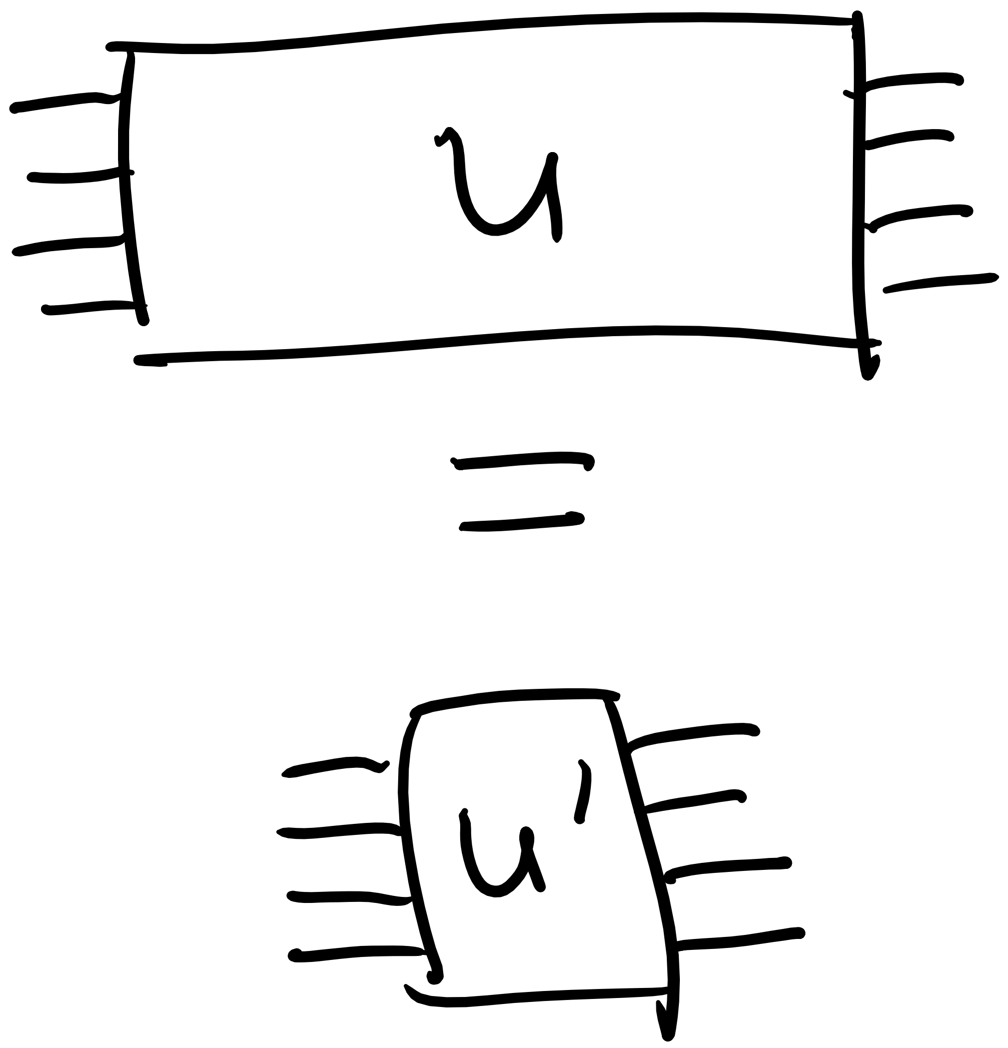
2. apply quantum gates

3. measure some outputs

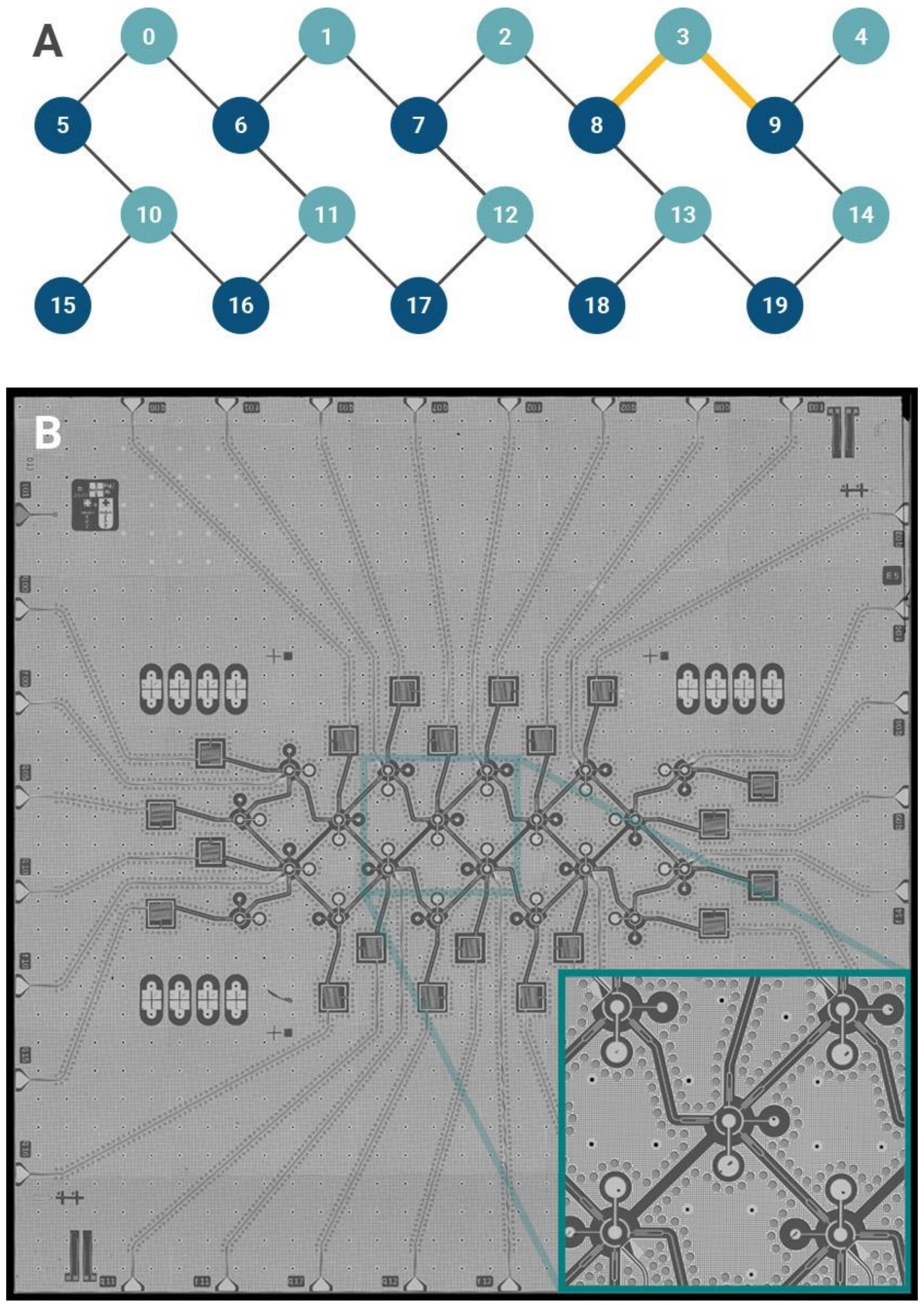
QUANTUM CIRCUIT OPTIMISATION



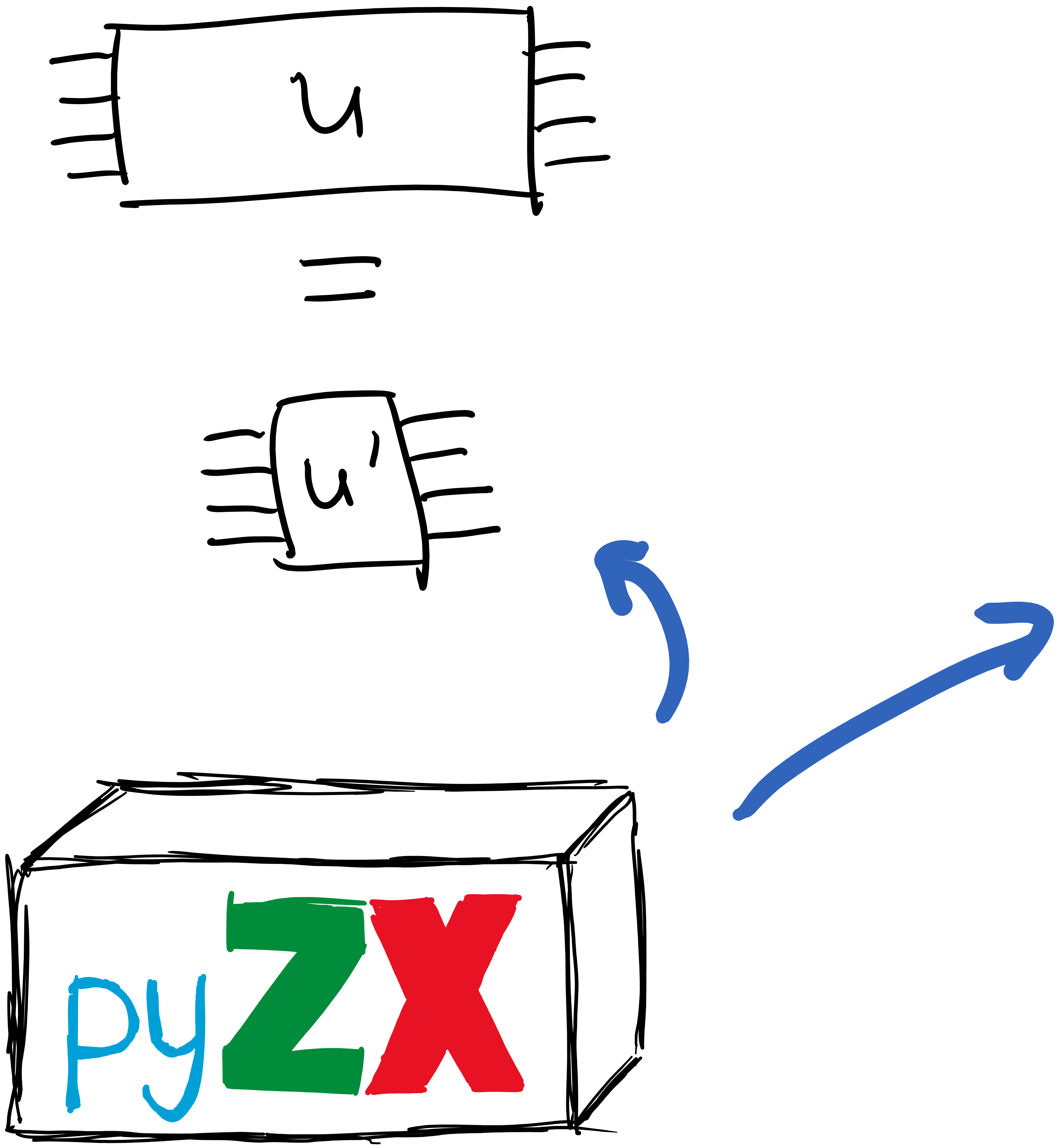
Quantum Circuit Optimisation



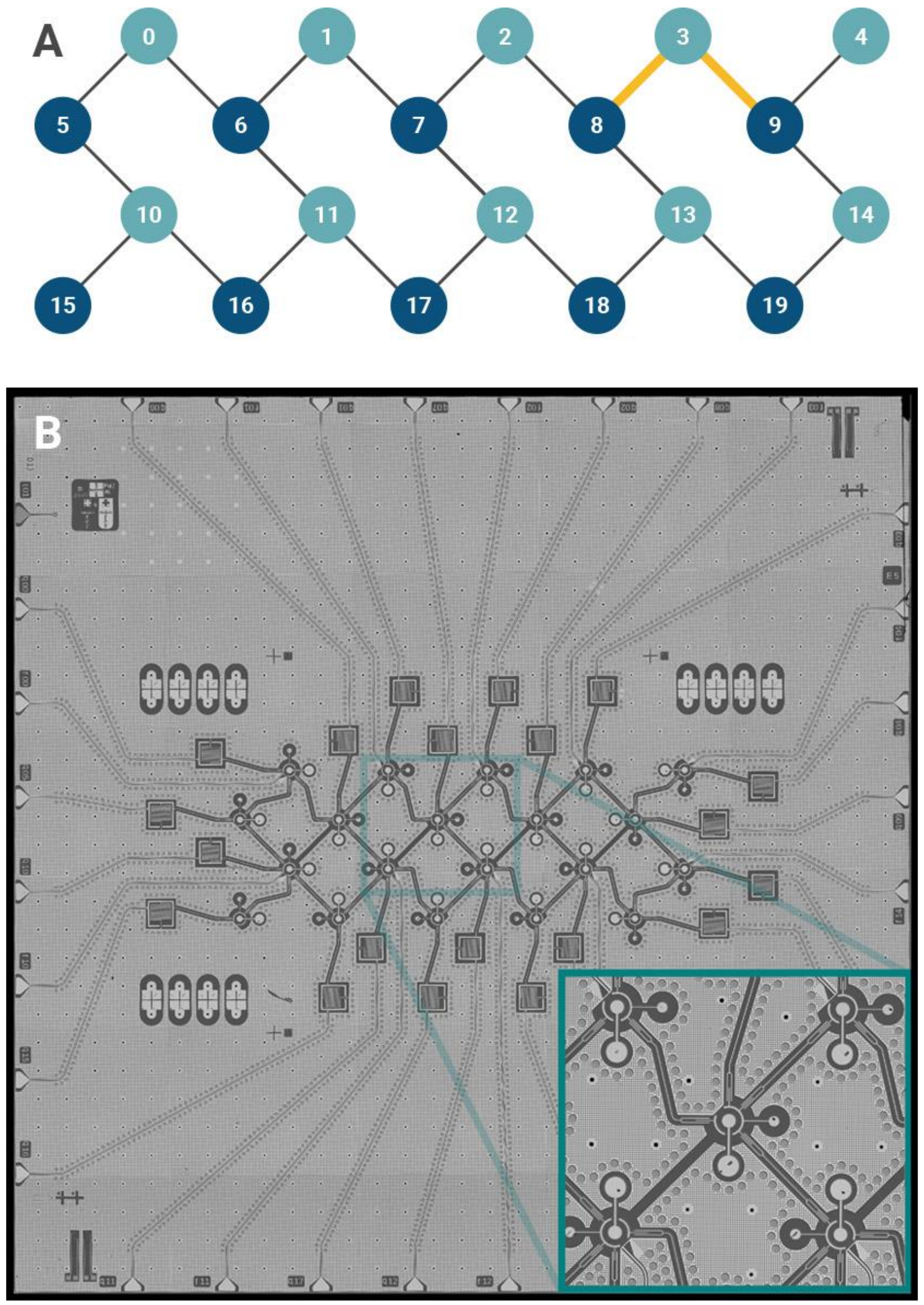
Routing



QUANTUM CIRCUIT OPTIMISATION



ROUTING



Thanks!

- *Picturing Quantum Processes: A First Course in Quantum Theory and Diagrammatic Reasoning.* Coecke & Kissinger. CUP (2017)
- ZX completeness:
  - *The ZX-calculus is complete for stabilizer quantum mechanics*, Backens, NJP (2014).
  - *A Complete Axiomatisation of the ZX-Calculus for Clifford+T Quantum Mechanics*. Jeandel, Perdrix, Vilmart. LICS (2018).
  - *A universal completion of the ZX-calculus*. Ng and Wang. arXiv:1706.09877 (2017).
  - *A Near-Optimal Axiomatisation of ZX-Calculus for Pure Qubit Quantum Mechanics*. Vilmart. arXiv:1812.09114 (2018).
- Simplification & circuit optimisation with ZX:
  - *Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus*, Duncan, Kissinger, Perdrix, van de Wetering, arXiv:1902.03178 (2019).
  - *Reducing T-count with the ZX-calculus*, Kissinger, van de Wetering, arXiv:1903.10477 (2019).
- Many more papers about ZX: https://zxcalculus.com/publications.html
- Formalisation of diagram rewriting as DPO:
  - *Open Graphs and Monoidal Theories*. Dixon, Kissinger, arXiv:1011.4114 (2010).
- http://quantomatic.github.io
- http://github.com/Quantomatic/pyzx