Diagrammatic Reasoning and Quantum Computation

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ACA, Kalamata

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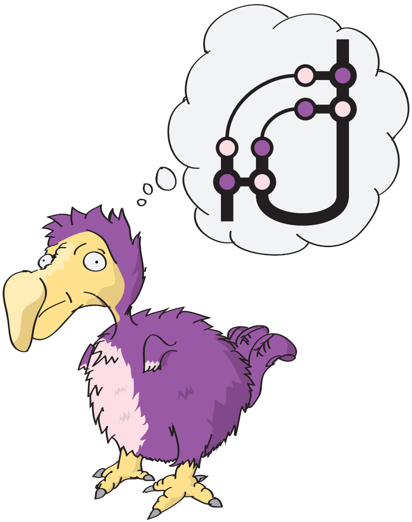
Quantum Group | Department of Computer Science, University of Oxford ⇒ Radboud University

Picturing Quantum Processes

A first course in quantum theory and diagrammatic reasoning

Bob Coecke & Aleks Kissinger

CUP 2015



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then replace a sub-term:

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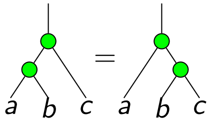

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becomes:

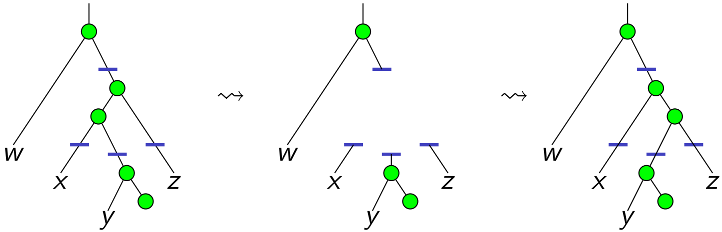

Diagram substitution

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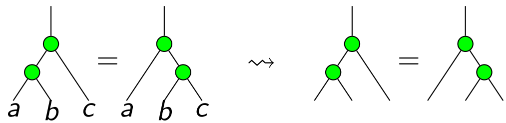

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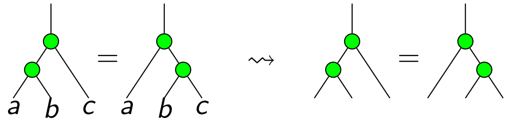
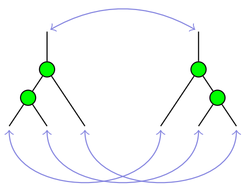


- The role of variables is replaced by the fact that the LHS and RHS have a *shared boundary*:



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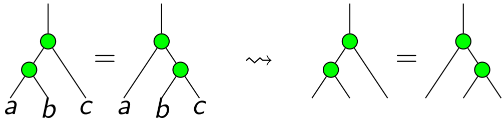

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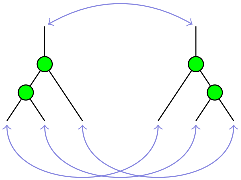

- This treats inputs and outputs symmetrically

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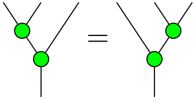
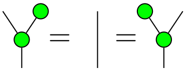
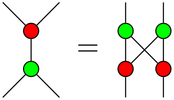
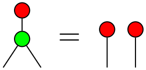
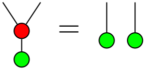




- The most interesting structures consist of algebras *interacting* with coalgebras:

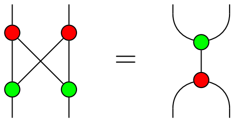






Equational reasoning with diagram substitution

- Again, we use equations to perform substitutions, but on graphs rather than just trees



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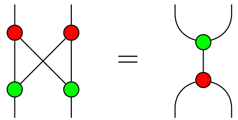
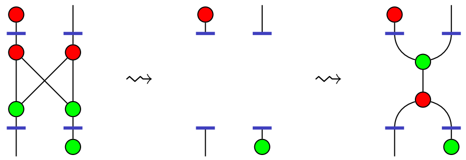
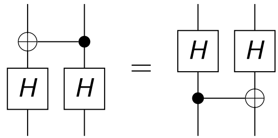


- For example:



Example: Quantum circuit rewriting



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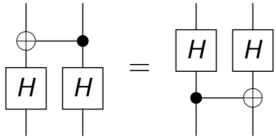

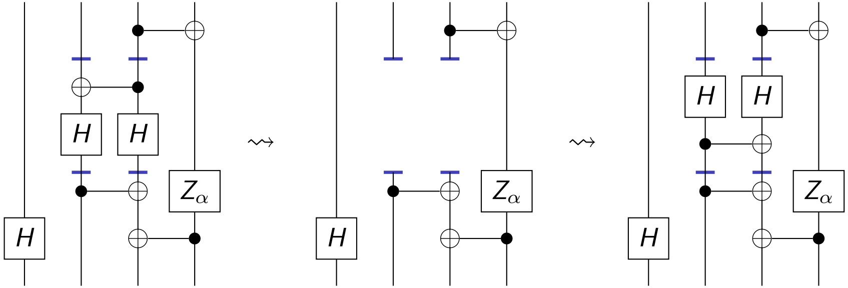

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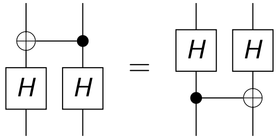

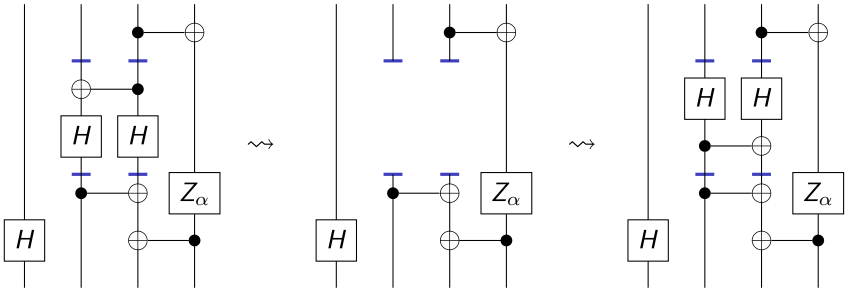

So, we can define an *equational theory* for quantum circuits, using rewriting.

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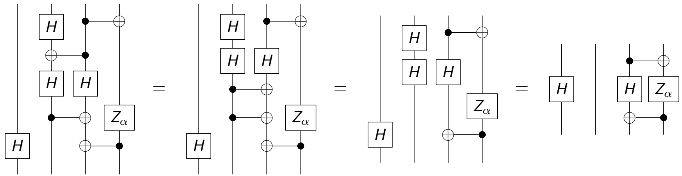
- circuit optimization:



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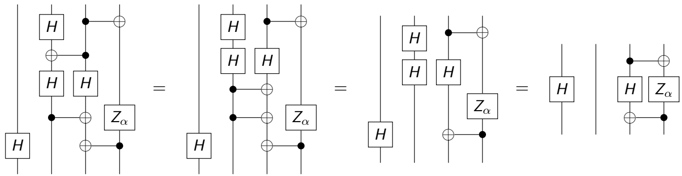
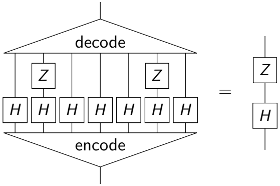


- verify equivalence (e.g. when adding error-correction)



- (automated) translation to other gate sets and paradigms
- exploit *algebraic invariants* to prove properties about *computations*

A complete set of gate identities

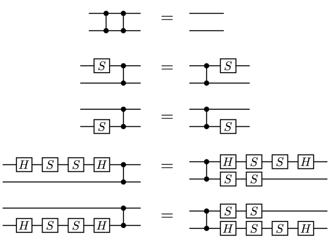
- These equations are complete for *Clifford circuits*:

$$\omega^8 = 1 \qquad \text{(C1)}$$

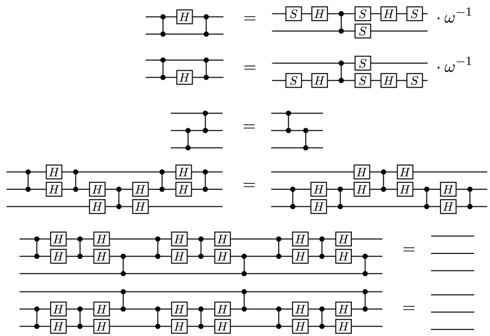
$$H^2 = 1 \qquad \text{(C2)}$$

$$S^4 = 1 \qquad \text{(C3)}$$

$$SHSHSH = \omega \qquad \text{(C4)}$$



(C5) (C6) (C7) (C8) (C9)



(C10) (C11) (C12) (C13) (C14) (C15)

(Selinger 2013)

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- The ugly:
    - proof of completeness is *extremely* complicated ($>$ 100 pages long! though mostly machine-generated)

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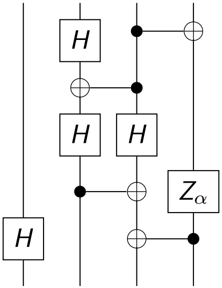

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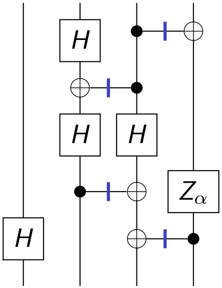
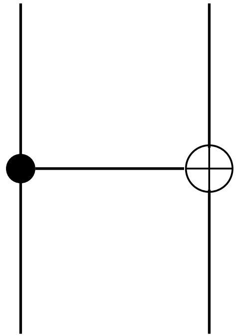


Decomposing CNOT



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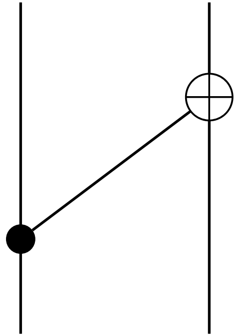

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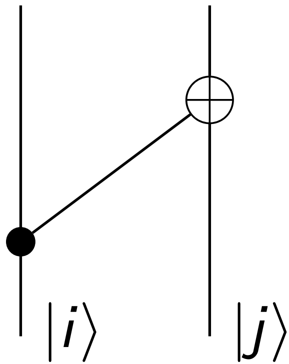

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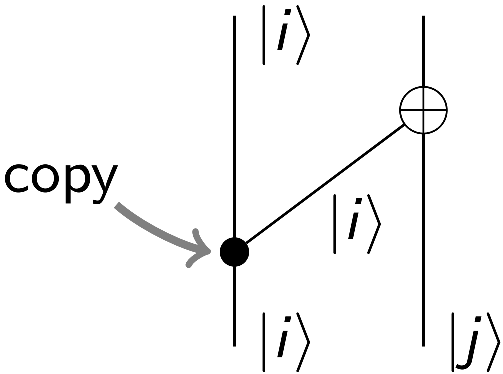

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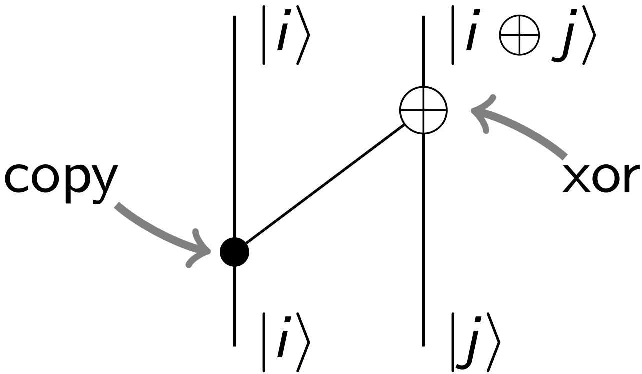
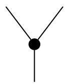
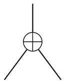


‘Copy’ maps



$$\begin{cases} |0\rangle \mapsto |00\rangle \\ |1\rangle \mapsto |11\rangle \end{cases}$$



$$\begin{cases} |00\rangle \mapsto |0\rangle \\ |01\rangle \mapsto |1\rangle \\ |10\rangle \mapsto |1\rangle \\ |11\rangle \mapsto |0\rangle \end{cases}$$

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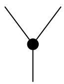

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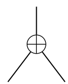

$$\begin{cases} |{+}{+}\rangle \mapsto |{+}\rangle \\ |{-}{-}\rangle \mapsto |{-}\rangle \end{cases}$$

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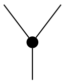

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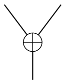

$$\begin{cases} |+\rangle \mapsto |++\rangle \\ |-\rangle \mapsto |--\rangle \end{cases}$$

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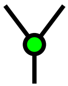

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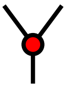

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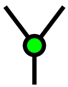

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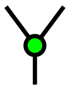

$$\begin{cases} |0\rangle \mapsto |00\rangle \\ |1\rangle \mapsto |11\rangle \end{cases}$$

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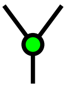

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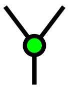

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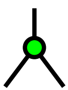

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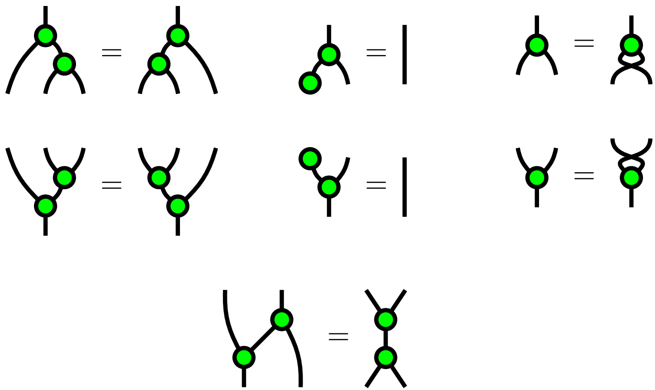
Algebraic identities...

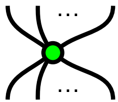
These satisfy 8 identities:



...making them a *commutative Frobenius algebra*.

But luckily...

...you don't need to remember all that! The only thing to remember is, for:



$$:= \begin{cases} |0..0\rangle \mapsto |0...0\rangle \\ |1..1\rangle \mapsto |1...1\rangle \end{cases}$$

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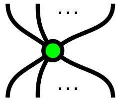

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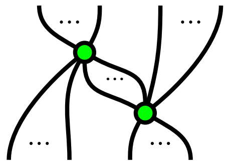

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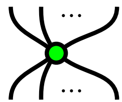

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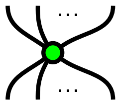

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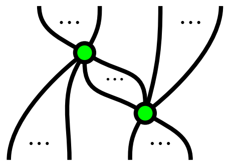

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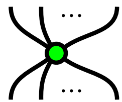

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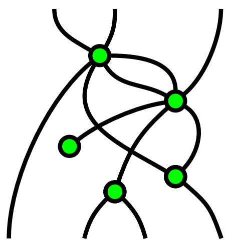

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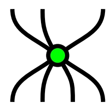

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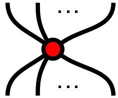

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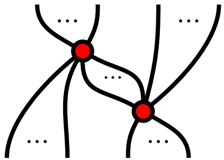

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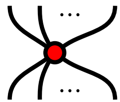

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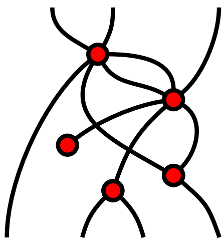

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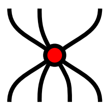

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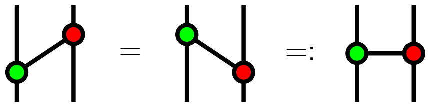

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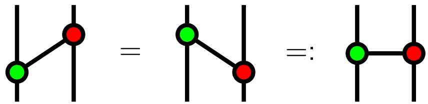
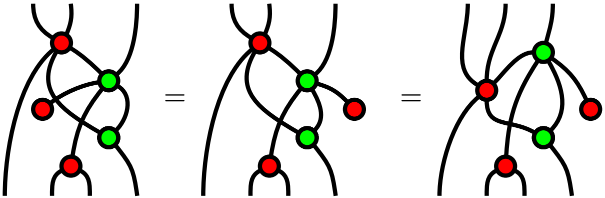


...in fact, **only topology matters**:



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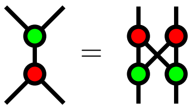
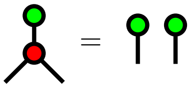
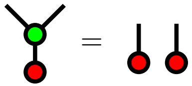
Red + green spiders also satisfy:







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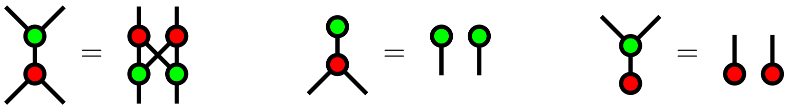
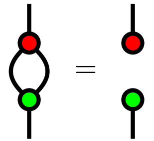
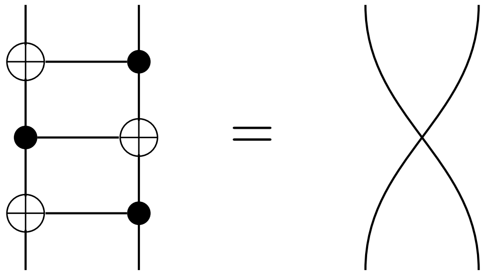


...from which we can derive:



make the overall structure into a *Hopf algebra*

Circuit calculation



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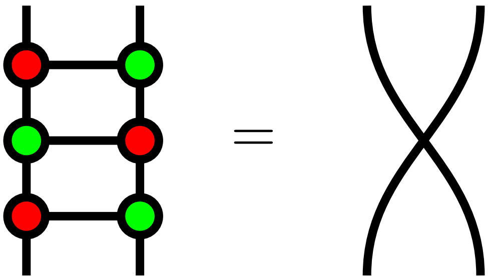

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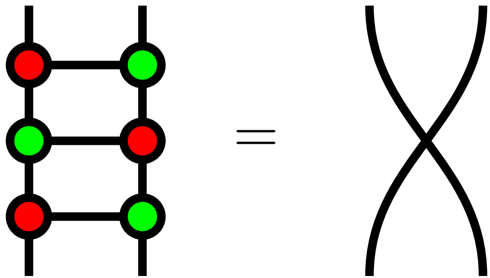
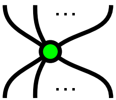


$(\Leftarrow \quad)$

Making spiders universal



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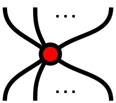

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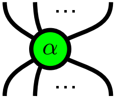

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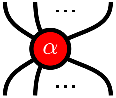

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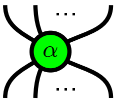

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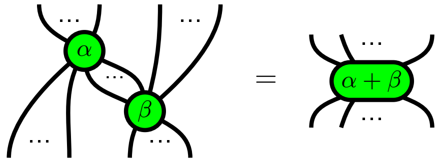

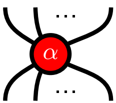

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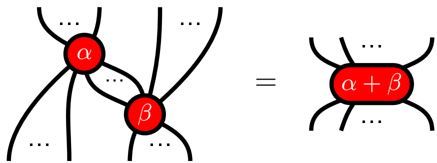


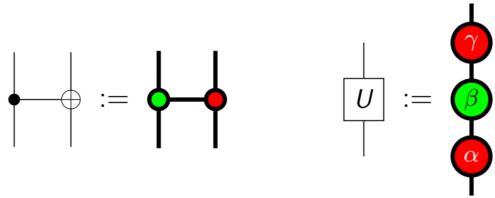
Making spiders universal

Theorem

Phased spiders are universal for qubit quantum computation.

Proof.

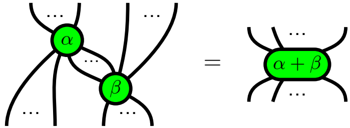
Let:

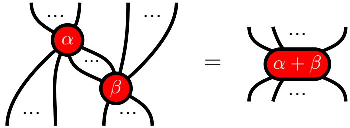


□

The ZX-calculus

The **ZX-calculus** consists of the two spider-fusion rules:





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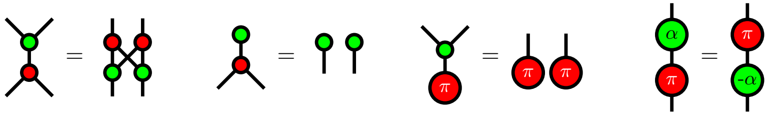

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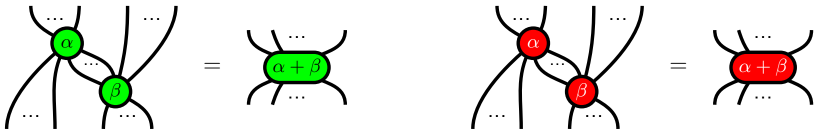

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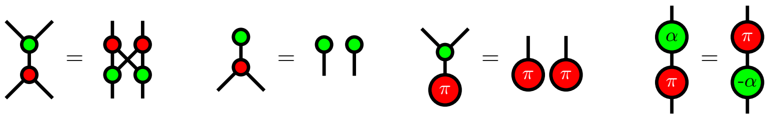

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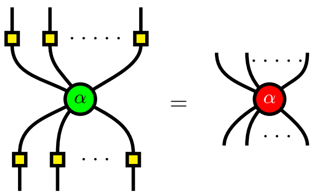

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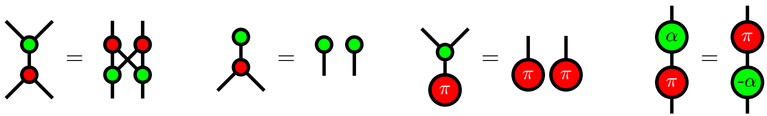

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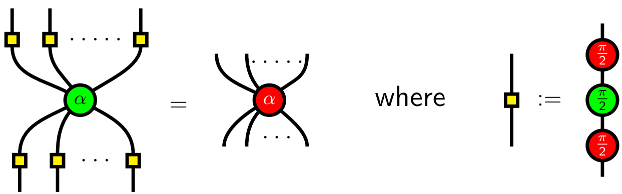
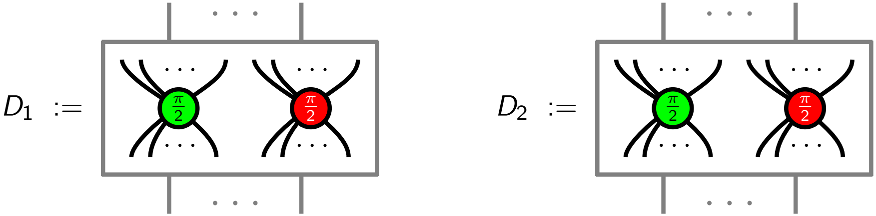


Completeness

Theorem (Backens 2013)

The ZX-calculus is complete for Clifford ZX-diagrams:

$$[\![D_1]\!] = [\![D_2]\!] \implies D_1 =_{ZX} D_2$$



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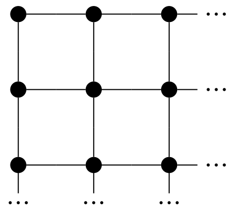

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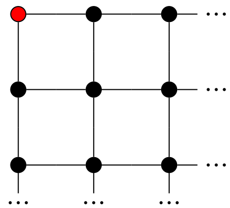

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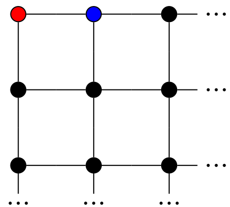

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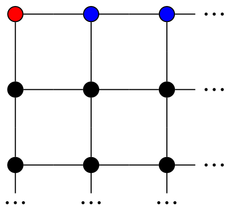

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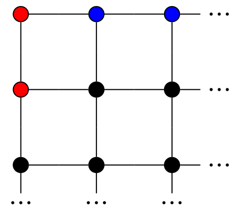

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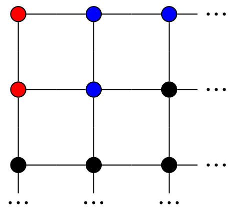

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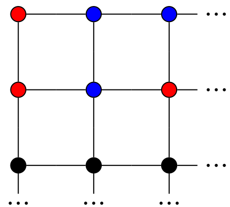

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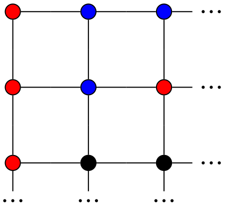

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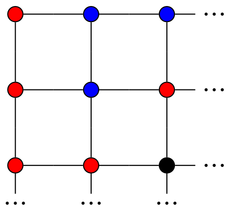

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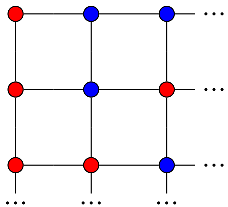

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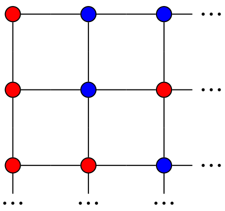
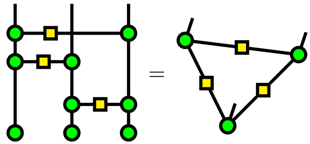


- But crucially, the **choices of measurements** can depend on **past measurement outcomes**. This is called **feed-forward**, and it's where all the magic happens.

Graph states and cluster states

- Graph states are prepared by starting with many qubits in the $|+\rangle$ state and creating entanglement with controlled-Z operations:



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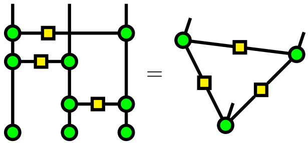
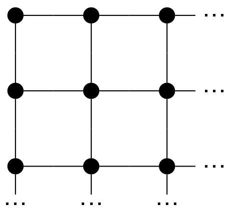
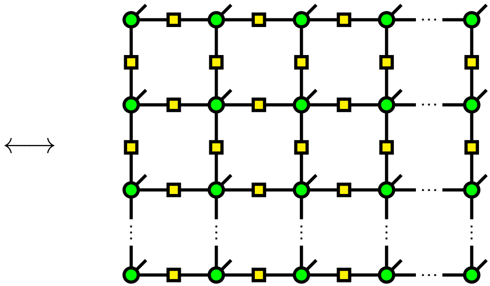


- Since controlled-Z's commute, the only relevant part is the graph:





Measurements and feed-forward

- Compute with single qubit ONB measurements of this form:

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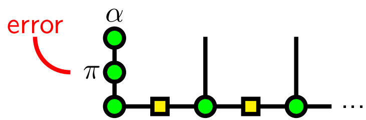
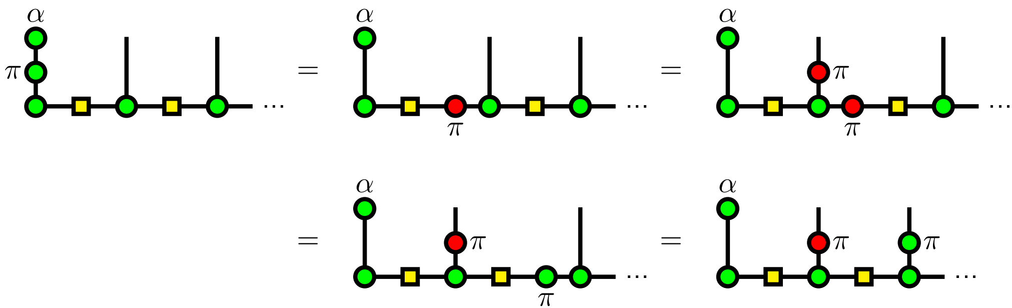
- We want to get the first outcome and treat the second outcome as an error:



Measurements and feed-forward

- We can propagate the error out using the ZX-rules:



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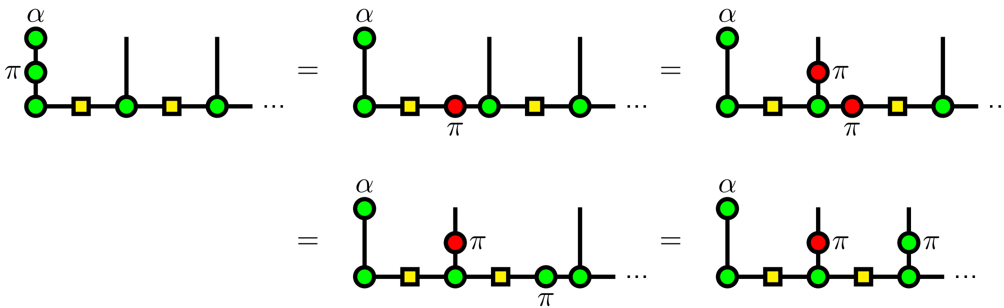
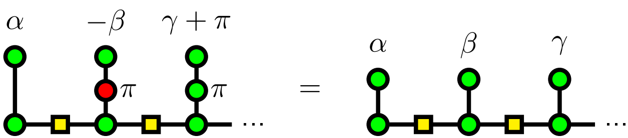


- If we know an error occurred, we can modify our later measurement choices to account for it:



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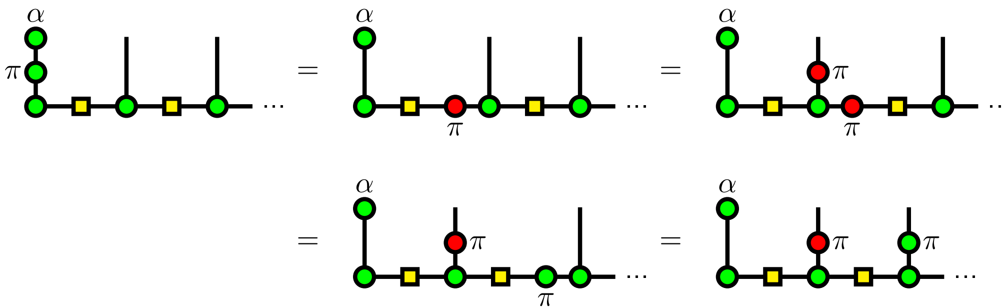

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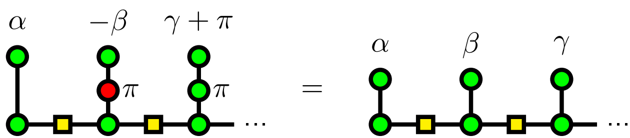
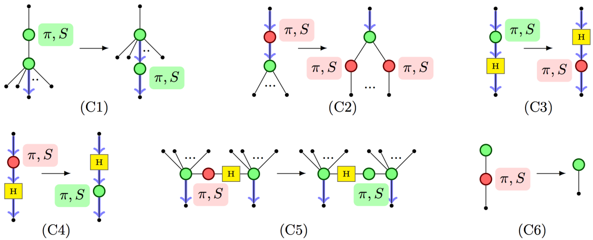


- $(\Leftarrow$ $)$

Notable results

Notable results: MBQC

- Duncan & Perdrix used the ZX-calculus to offer a new technique for transforming MBQC patterns to circuits, which has some advantages over other known methods, e.g. not requiring ancillas.[1]



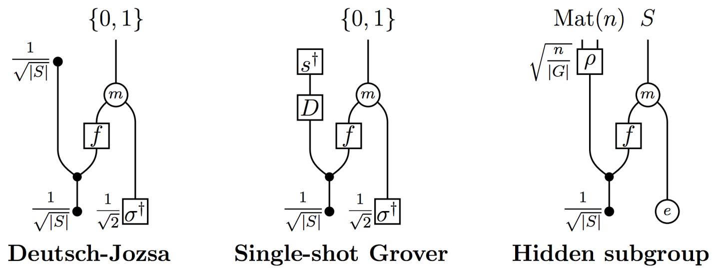
- For more details, Ducan has written a self-contained introduction to MBQC from the diagrammatic/ZX point of view, which is available on the arXiv.[2]

[1] Rewriting measurement-based quantum computations with generalised flow. R. Duncan, S. Perdrix, ICALP 2010.
personal.strath.ac.uk/ross.duncan/papers/gflow.pdf

[2] A graphical approach to measurement-based quantum computing. R. Duncan. arXiv:1203.6242

Notable results: quantum algorithms

- Vicary gave graphical characterisations of standard quantum algorithms[3]



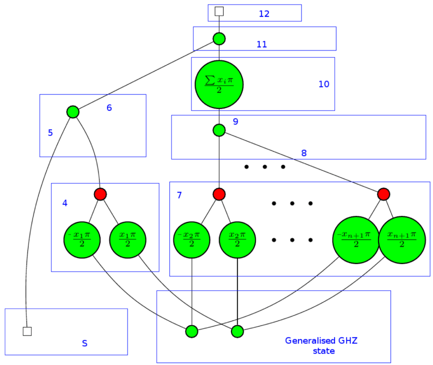
- ...a framework since used by Vicary & Zeng to develop *new* algorithms as generalisations[4]

[3] The Topology of Quantum Algorithms. LICS 2013, J. Vicary. arXiv:1209.3917

[4] Abstract structure of unitary oracles for quantum algorithms. J.Vicary, W. Zeng. arXiv:1406.1278

Notable results: quantum protocols

- Coecke, along with 3 Wangs and a Zhang give graphical proof of QKD[5]
- Hillebrand gave rewriting proofs of many ($\sim$ 25) quantum protocols.[6]
- Zamdzhiev used ZX-calculus to verify 3 kinds of quantum secret sharing.[7]



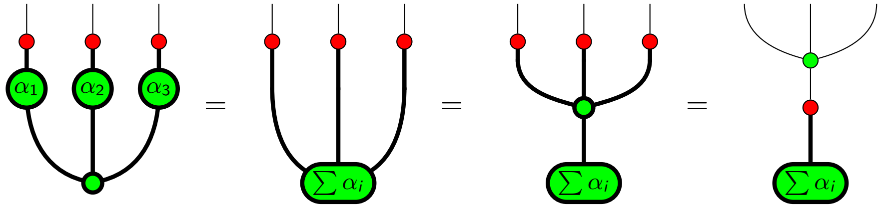
[5] Graphical Calculus for Quantum Key Distribution. B. Coecke, Q. Wang, B. Wang, Y. Wang, and Q. Zhang. QPL 2011.

[6] Quantum Protocols involving Multiparticle Entanglement and their Representations in the zx-calculus. A. Hillebrand. Masters thesis, Oxford 2011. `www.cs.ox.ac.uk/people/bob.coecke/Anne.pdf`

[7] An Abstract Approach towards Quantum Secret Sharing. Masters thesis, Oxford 2012. `www.cs.ox.ac.uk/people/bob.coecke/VladimirZamdzhievThesis.pdf`

Notable results: quantum non-locality

- AK, Coecke, Duncan, and Wang gave diagrammatic presentation of GHZ/Mermin non-locality argument[8]



- ...which has since been generalised to arbitrary dimensions and quantum-like theories[9]

[8] Strong Complementarity and Non-locality in Categorical Quantum Mechanics. B. Coecke, R. Duncan, A. Kissinger, Q. Wang. LICS 2012.

[9] Mermin Non-Locality in Abstract Process Theories. QPL 2015

Where do we go from here?

- Completeness (Clifford + T, full)

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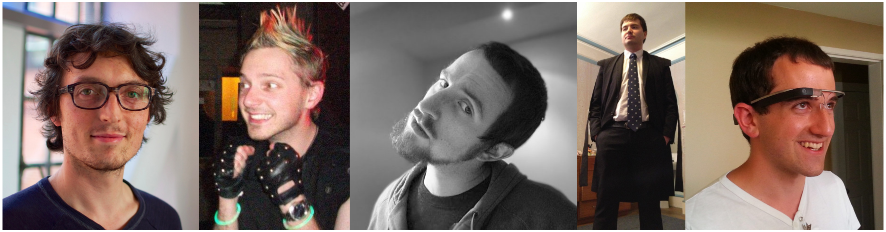
- Automation: implementation of Clifford decision procedure, theory synthesis
- **Bigger algorithms**, more **sophisticated protocols**, and generally more **expressiveness** of the diagrammatic language

Thanks!



- Quantomatic is joint work with Lucas Dixon, Alex Merry, Ross Duncan, Vladimir Zamdzhiev, and David Quick
- See: `quantomatic.github.io`