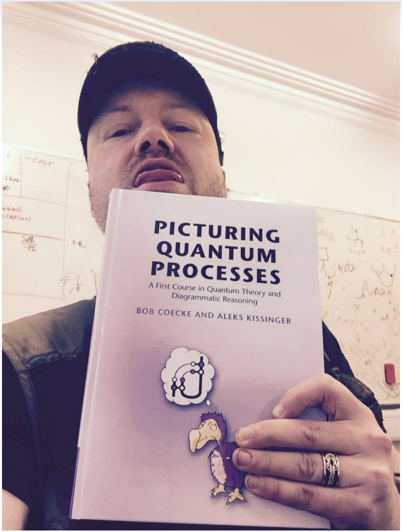
Picturing Quantum Processes

Aleks Kissinger and Bob Coecke

Radboud University and Oxford University

ESSLLI Toulouse 2017



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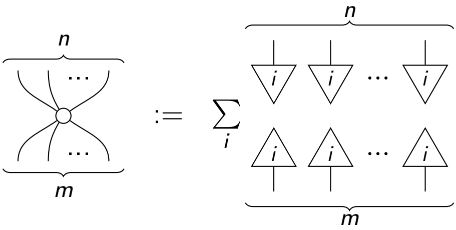
Chapters 11-13:

Quantum Foundations, Computation, and Resources

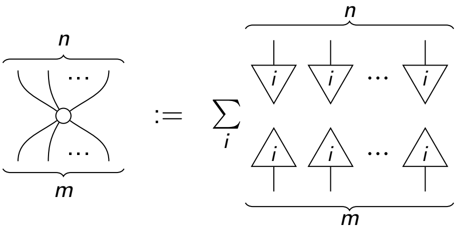
Mermin once summarized a popular attitude towards quantum theory as "Shut up and calculate". We suggest a different slogan: "Shut up and contemplate"!

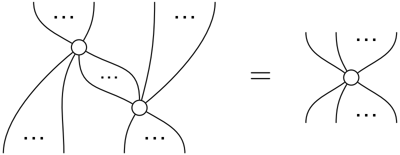
— Lucien Hardy and Rob Spekkens, 2010.

Spiders

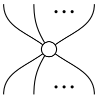


Spiders

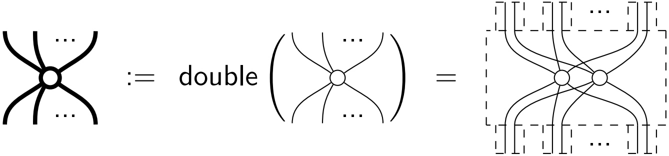




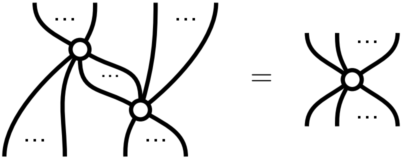
(Classical) spiders are linear maps:



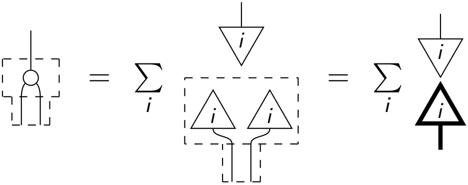
...so we can double them to get *quantum spiders*:



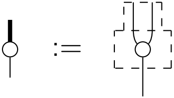
...which also fuse together:



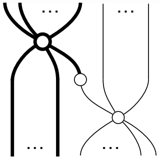
Measuring is a spider:



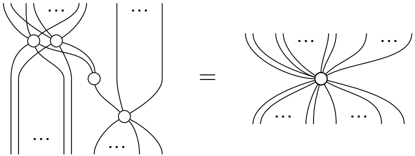
...and so is encoding:



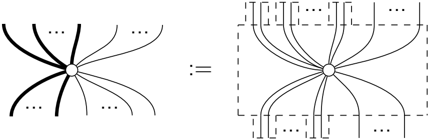
They connect classical to quantum:



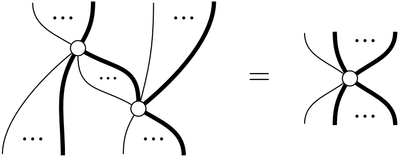
...giving something new:



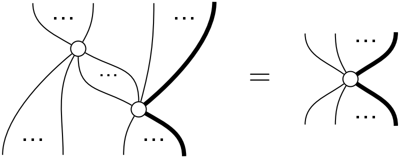
Bastard spiders!

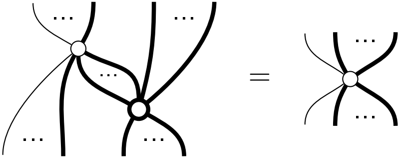


Bastard spiders fuse together:



They also absorb other kinds of spider:

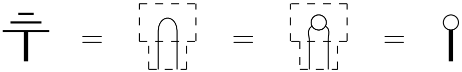




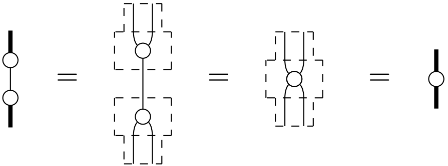
Useful bastards

Bastard spiders arise naturally in the interaction between classical and quantum data, e.g.

- Discarding:

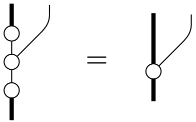


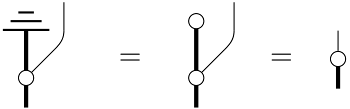
- Decoherence:

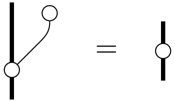


Useful bastards (cont'd)

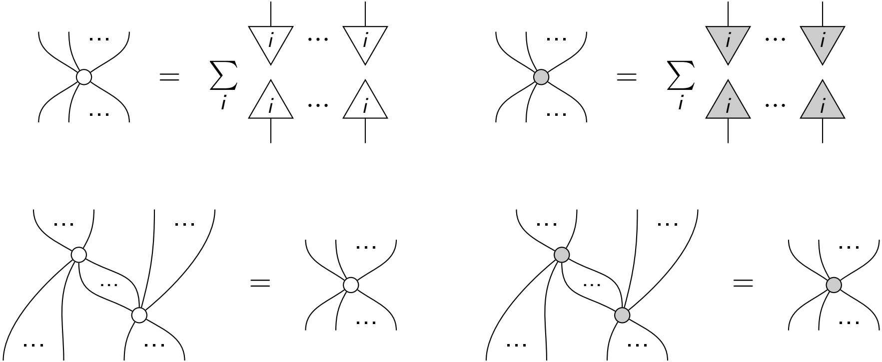
- Non-demolition measurement:



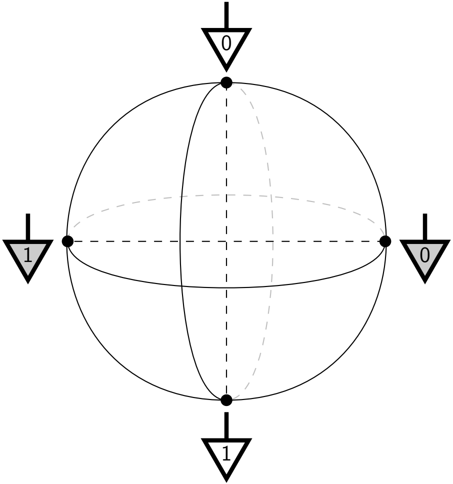




Multicoloured spiders



Complementary bases

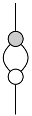


Complementarity

Theorem. Two bases are complementary (a.k.a. mutually unbiased):

$\forall i, j \quad :$  $= \frac{1}{D}$

if and only if:

 $= \frac{1}{D}$ 

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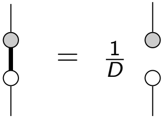
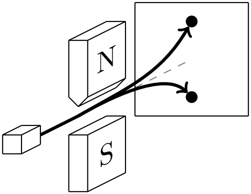
Complementarity

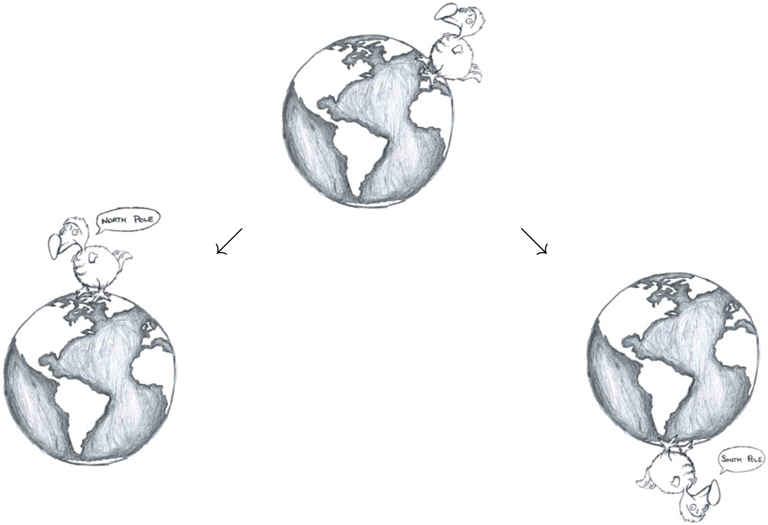


(encode in ○) THEN (measure in ●) = (no data flow)

Stern-Gerlach



Stern-Gerlach



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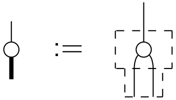

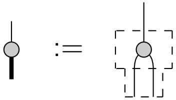

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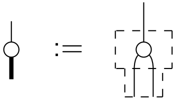
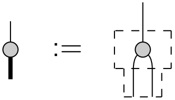
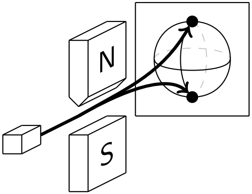
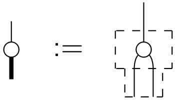


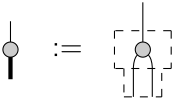


Z-measurement



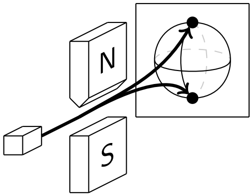
Stern-Gerlach

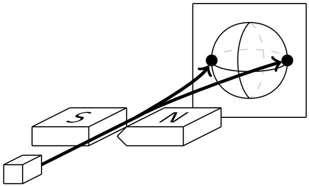




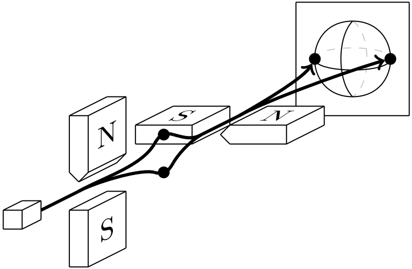
Z-measurement

X-measurement

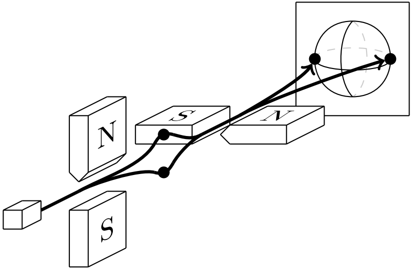


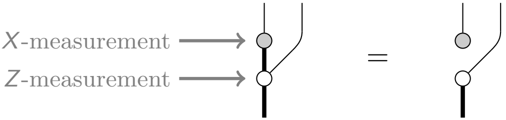


Stern-Gerlach

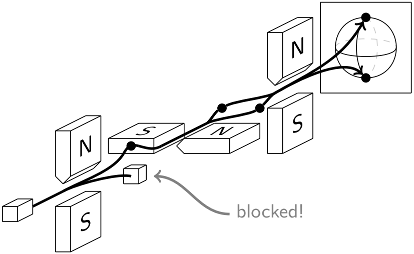


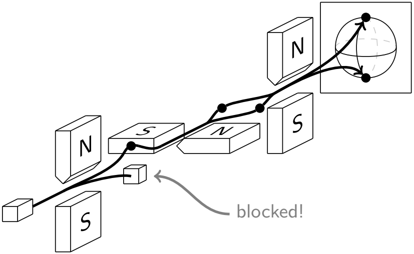
Stern-Gerlach

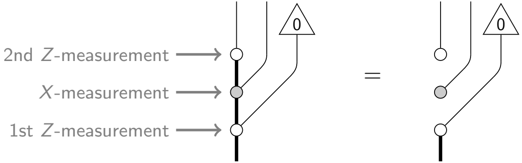




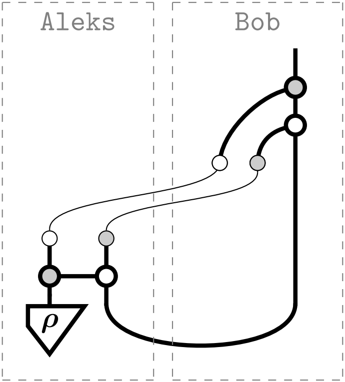
Stern-Gerlach



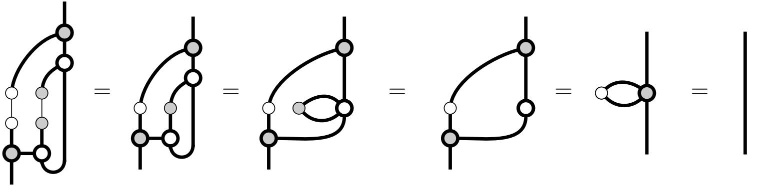

N
S
N
S
S
N
blocked!


2nd Z-measurement
X-measurement
1st Z-measurement
0
=
0

Teleportation with spiders



Teleportation with spiders



Power of the Graphical Language

We now have a fairly powerful language, it is natural to ask:

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Q: Is it *universal*?

Can we express any map in terms of spiders?

Power of the Graphical Language

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Q: Is it *universal*?

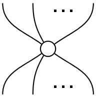
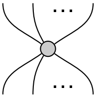
Can we express any map in terms of spiders?

Q: Is it *complete*?

Can we prove every equation between maps
using some set of spider-equations?

Universality

Q: Can we write any linear map as a diagram of:

 and  ?

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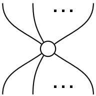
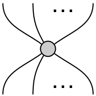
 and  ?

A: Clearly not! e.g. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

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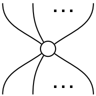
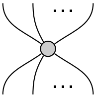
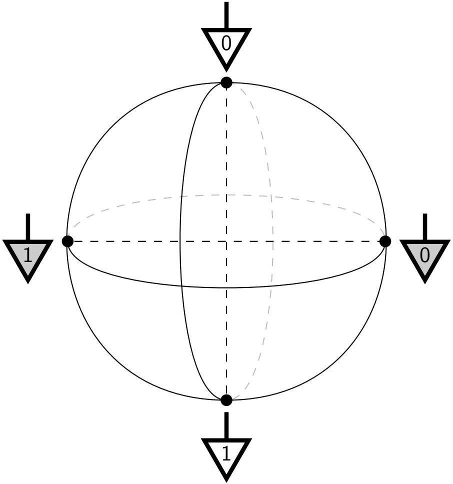
 and  ?

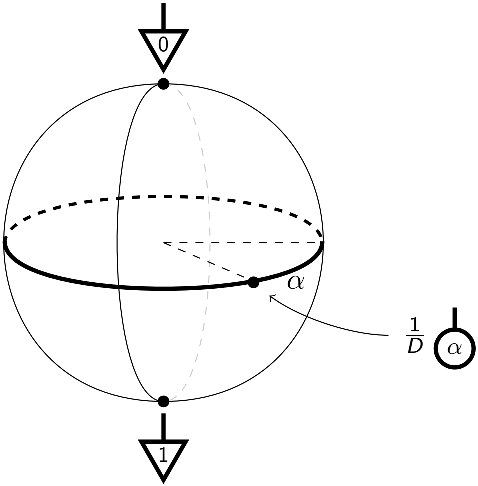
A: Clearly not! e.g. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

Q2: How much more do we need?

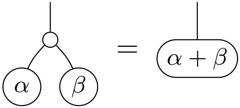
Phases



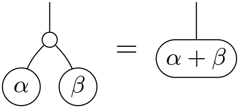
Phases

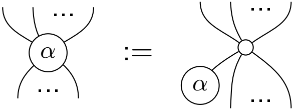


Phase spiders



Phase spiders





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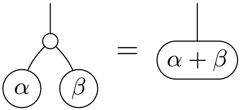
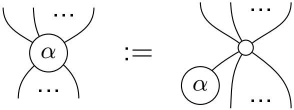
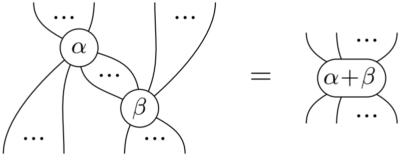


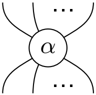




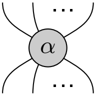
Universality

Universality

Theorem. Any linear map with 2D inputs and outputs can be expressed as a diagram of:



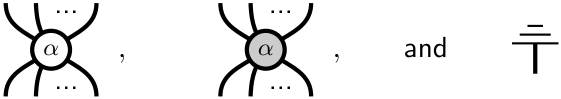
and



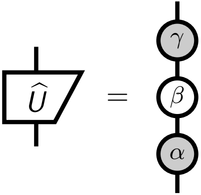
Universality

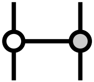
Theorem. Any linear map with 2D inputs and outputs can be expressed as a diagram of:

Corollary. Any **quantum map** from qubits to qubits can be expressed as a diagram of:

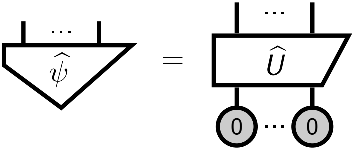


Proof. (idea)

Any single-qubit unitary: 

+ CNOT: 

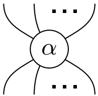
...gives any unitary, which gives any state:



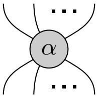
...which gives any map by process-state duality.

Completeness

Q. Can we find a *complete* set of equations to describe the behaviour of:



and



?

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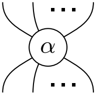
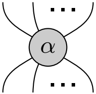


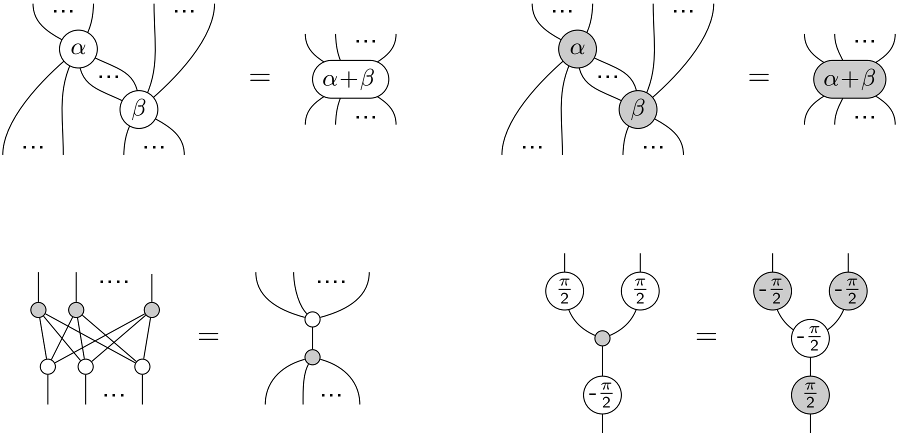
and



?

A. Yes! The *ZX-calculus*.

ZX-calculus



ZX-calculus

Theorem. The ZX-calculus is complete for *Clifford diagrams*, i.e. diagrams where $\alpha \in \{0, \pi, \pm\frac{\pi}{2}\}$.

ZX-calculus

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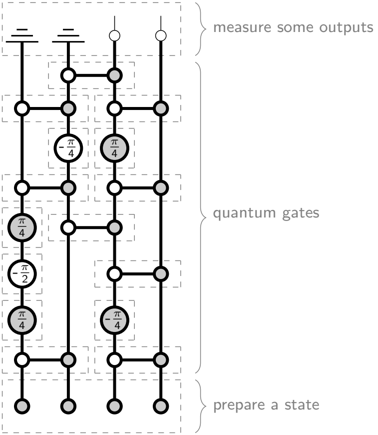
Theorem. (Hot off the press! [1,2]) A bigger version of the ZX-calculus is complete for *all diagrams*.

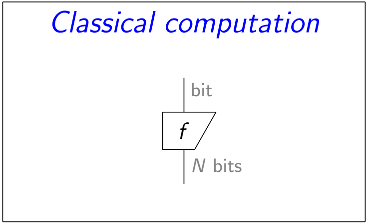
[1] Jeandel, Perdrix, Vilmart. 31 May, 2017. arXiv:1705.11151
[2] Wang & Ng. 29 June, 2017. arXiv:1706.09877

Application: Quantum Computing

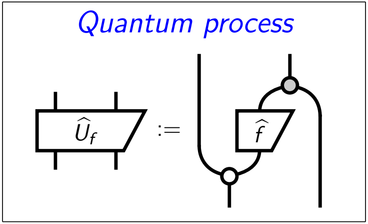
The quantum *circuit model*:



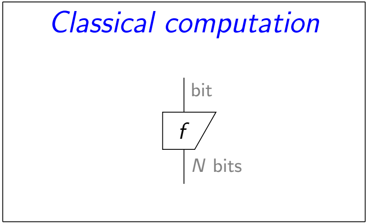
Quantum algorithms



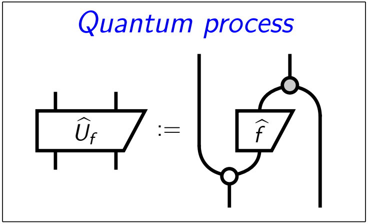
⇒



Quantum algorithms



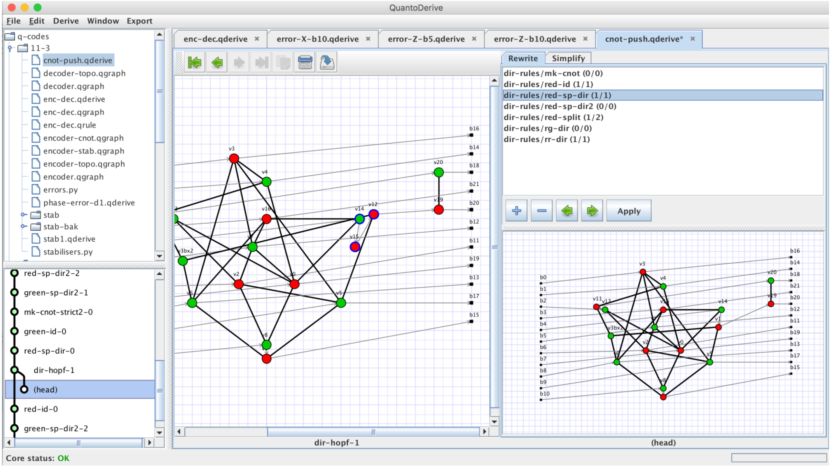
$\Rightarrow$



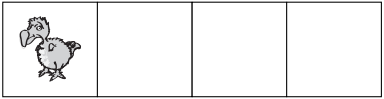
$\Rightarrow$ **Deutsch-Jozsa**, **Bernstein-Vazirani**, **quantum seach**, and **hidden subgroup** (e.g. factoring) algorithms.

Automation

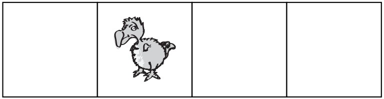
Quantomatic:



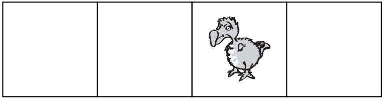
Spekkens' toy model



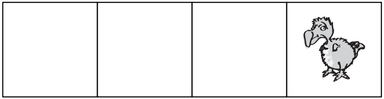
$:: \quad * \mapsto \{(0,0)\}$



$:: \quad * \mapsto \{(0,1)\}$



$:: \quad * \mapsto \{(1,0)\}$



$:: \quad * \mapsto \{(1,1)\}$

Spekkens' toy model

$$Z :: (a, b) \mapsto (a, b \oplus 1)$$

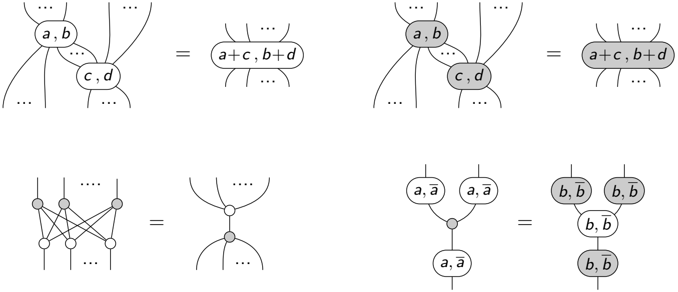
$$Z :: (a, b) \mapsto (a, a \oplus b)$$

$$H :: (a, b) \mapsto (b, a)$$

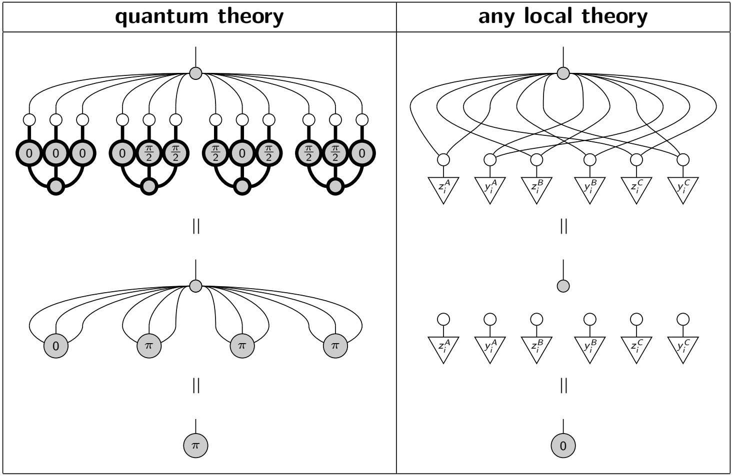
$$CZ :: ((a, b), (c, d)) \mapsto ((a, b \oplus c), (c, a \oplus d))$$

$$\text{MEASURE}(a, b) \;\; := \;\; \text{reveal } a, \text{ randomize } b$$

Spekkens' toy model

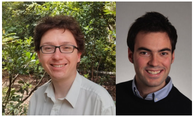


GHZ/Mermin non-locality



Thanks!

Coecke, Abramsky, Backens, Duncan, Edwards, Gogioso, Hadzihasanovic, Heunen, Lal, Dixon, Merry, Pavlovic, Perdrix, Quick, Selinger, Zamdzhiev, ...

Special thanks ⇒ 

Jamie Vicary and David Reutter - qubit.zone simulators

www.cambridge.org/pqp quantomatic.github.io