Matrix Calculations

Assignment 4, Tuesday, February 21, 2017

Exercise teachers. Recall the following split-up of students:

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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 4*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-4.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, February 27, 12:00 sharp!

Goals: After completing this assignment, you should be able to determine if a set of vectors forms a basis, express a vector in terms of a given basis, translate linear maps to/from matrices, and do matrix multiplication.
The total number of points is 20.

1. **(6 points)** Determine if the following sets of vectors form a basis for the given vector space. If they do, prove that they are *linearly independent* and *spanning*. If they do not form a basis, explain why not.

   (i) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$ for $\mathbb{R}^3$

   (ii) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$ for $\mathbb{R}^3$

   (iii) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} \right\}$ for $V := \{(x, y, 2y, 0) \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^4\}$

2. **(4 points)** For the following matrices:

$$\boldsymbol{A} := \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{B} := \begin{pmatrix} 4 & -3 \\ 2 & 0 \end{pmatrix} \qquad \boldsymbol{C} := \begin{pmatrix} 1 & 3 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

   compute $\boldsymbol{A} \cdot \boldsymbol{B} \cdot \boldsymbol{C}$ and $\boldsymbol{C} \cdot \boldsymbol{A} \cdot \boldsymbol{B}$. Show intermediate calculations.

3. **(4 points)**

Consider the following 2 bases for $\mathbb{R}^3$:

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\} \qquad \mathcal{C} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

Write the following vectors with respect to $\mathcal{B}$ and $\mathcal{C}$:

$$\boldsymbol{v} := \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad \boldsymbol{w} := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

That is, write each vector in each of these two forms:

$$\begin{pmatrix} * \\ * \\ * \end{pmatrix}_{\mathcal{B}} \qquad \begin{pmatrix} * \\ * \\ * \end{pmatrix}_{\mathcal{C}}$$

4. **(6 points)**

This exercise is about transforming linear maps to/from matrices.

(a) Give the matrix corresponding the linear map:

$$f((x_1, x_2, x_3, x_4)) = (x_1 + x_2 + 2x_4, 2x_1 + 3x_2 + x_3 + 6x_4, x_1 + x_4).$$

in terms of the standard bases for $\mathbb{R}^4$ and $\mathbb{R}^3$.

(b) Consider the following matrix, written in terms of the standard bases:

$$\boldsymbol{A} = \begin{pmatrix} 4 & 1 & 3 & -5 \\ 1 & 3 & 3 & 7 \end{pmatrix}$$

give the linear map associated to $\boldsymbol{A}$.

(c) Consider the vector space

$$V = \{ \begin{pmatrix} x \\ -x \\ y \end{pmatrix} \mid x, y \in \mathbb{R} \} \subseteq \mathbb{R}^3$$

and the linear map $f : V \to \mathbb{R}^2$ given by:

$$f(\begin{pmatrix} x \\ -x \\ y \end{pmatrix}) = \begin{pmatrix} x + y \\ y \end{pmatrix}$$

Give the matrix associated to f, using the bases:

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \subset V \qquad \mathcal{S} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \subset \mathbb{R}^2$$