Matrix Calculations

Assignment 6, Tuesday, March 14, 2017

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email
John van de Wetering	HG00.065	`wetering@cs.ru.nl`
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 6*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-6.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, March 20, 12:00 sharp!

Goals: After completing these exercises successfully you should be able to compute eigenvalues and eigenvectors of matrices, use this data to diagonalise matrices, and raise matrices to powers. The total number of points is 20.

1. **(3 points)**

   We consider the weather forecast predictions for the next day: a rainy day R, a cloudy day C or a sunny day S. Assume predictions follow this distribution rule:

Forecast R	70% stay at R	20% go to C	10% go to S
Forecast C	20% go to R	60% stay at C	20% go to S
Forecast S	20% go to R	40% go to C	40% stay at S

   (a) Provide the transition matrix $\boldsymbol{A}$.

   (b) If there is a 50% probability of rain today and 10% probability of sun, what is the probability that it will be cloudy the day after tomorrow?

2. **(10 points)** Consider the following "student transition matrix", denoting the fraction of RU students that will stay at / leave the RU and the fraction of non-RU students that will come to / not come to the RU:

$$\boldsymbol{S} = \begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix}$$

(a) Find eigenvalues and eigenvectors of $\boldsymbol{S}$.

(b) Let the eigenvectors form a basis $\mathcal{B}$. Write $\boldsymbol{S}$ as a diagonal matrix $\boldsymbol{D}$ with respect to basis $\mathcal{B}$.

(c) Compute the basis transformation matrices $\boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}}$ and $\boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$ and show that:

$$\boldsymbol{S} = \boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}} \cdot \boldsymbol{D} \cdot \boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$$

(d) What is the second iteration of the student transition matrix? (Use the diagonal matrix to compute this.)

(e) What will happen if the number of iterations goes to infinity (find $\lim_{n\to\infty} \boldsymbol{S}^n$)?

3. **(7 points)** Consider the following matrix:

$$\boldsymbol{F} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix}$$

(a) What is the characteristic polynomial of $\boldsymbol{F}$?

(b) Find the eigenvalues of the matrix.

(c) Compute the eigenvectors corresponding to each of these eigenvalues.